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Abstracts

Teaching with Primary Historical Sources: Should it Go Mainstream? Can it?

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Many are now teaching mathematics directly with primary historical sources, in a variety of courses and levels. How far should this be taken? Should we adapt or redesign standard courses to a completely historical approach, chiefly from primary sources? If so, what are the obstacles to achieving this? Materials? Instructor training and attitudes? Class time? Textbooks? Classroom pedagogy? What should and can we do about such things? We attempt to provide answers to these questions, and illustrate with a sample student project based on Pascal's *Treatise on the Arithmetical Triangle* how numerous core course topics can be learned via a primary historical source.

Dialogism in Mathematical Writing: Historical, Philosophical and Pedagogical Issues

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The notion of dialogism was introduced by the Russian semiotician Mikhail Bakhtin. For him, every sentence or every discourse must be understood as a rejoinder in a dialogue: it is an answer to other sentences, or discourses and it is intended to be received by somebody. My purpose in this paper is to explore the meaning and the implications of this notion for mathematical texts. The consequences for historical works are clear, in the sense that one should pay attention to the nature of texts (letters, papers, books), to the texts known to the authors, and so on. In a philosophical perspective, the notion of dialogism leads to a reflection on mathematical proof. At the classroom level, dialogism is an interesting notion for the reading of ancient texts by pupils and thinking about the persons for whom the pupils write.

The Process of Mathematical Agreement: Examples from Mathematics History and an Experimental Sequence of Activities

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This article presents the basic evidence that allow us to give account of a process of production of knowledge, that we have called *process of mathematical agreement* (Martìnez-Sierra, 2005, 2008), that allows to establish a statement at the same time as its truth. The truth of the statement can be interpreted as "agreed truth;" in the sense that it is set from the necessity to make a theoretical corpus. We will present three examples from mathematics history, that show that it is possible the production of the meaning of: 1) the fractional exponents, 2) the square root of negative numbers as precursor on the meaning of complex numbers and 3) the radian and the trigonometric functions. In order to experiment the process of mathematical agreement finally we present the results of an experimental sequence that has the objective to favor the acceptance and the operativeness of the square root of negative numbers.

Researching the History of Algebraic Ideas from an Educational Point of View

Luis Puig, Universidad de Valencia

In our research on the teaching and learning of algebra, we have observed that, when dealing with arithmetic-algebraic word problems or with the solving of equations, pupils use a stratified sign system, with strata that come from their previously acquired vernacular and arithmetic language, and also from the concrete models used in the teaching sequences. Those observations have led us to study the sign systems prior to symbolic algebra, and texts in history in which arithmetic-algebraic problems were dealt with through the use of geometrical representations and cut and paste methods. We present here some results of our examination of cut and paste methods in texts from three moments in history: Old Babylonian algebra, al-Khwārizmi's algebra, and Jordanus de Nemore's *De Numeris Datis*. In Old Babylonian algebra, cut and paste methods constitute an important part of the language in which arithmetic-algebraic problems were solved. In al-Khwārizmi's algebra, cut and paste methods are used to justify the correctness of the algorithmic rules that solve the canonical forms. In Nemore's *De Numeris Datis*, cut and paste methods do not appear explicitly. However, we develop a reading of some of Nemore's proofs, which uses a didactical artifact built on cut and paste methods. This reading enables us to understand Nemore's proofs by translating them into a more concrete representation (the didactical artifact), inspiring then a teaching model for algebraic identities and second degree equations based on this capability of the didactical artifact.

Equations and Imaginary Numbers: A Contribution from Renaissance Algebra

Giorgio T. Bagni, University of Udine

In this article I investigate Bombelli's introduction of imaginary numbers in his Algebra (1572). My interpretation draws on the ontosemiotic approach to representations, Radford's socio-cultural approach, and Peirce's semiotic perspective. I argue that Renaissance mathematics and the emergence of new methods to deal with algebraic problems and the introduction of abbreviations and symbolism were influenced by the general sixteenth century cultural movement. I claim that the analysis of the mathematical practices in their own cultural context is relevant in order to understand the conceptualization of important mathematical objects such as imaginary numbers. In particular, historical and epistemological reflections and the historical understanding of the signifying relationships between signs (i.e., their semiosic chain) are important in order to understand the origins of mathematical contents and their contemporary meaning , and to address relevant teaching and learning questions.

The Multiplicity of Viewpoints in Elementary Function Theory: Historical and Didactical Perspectives

Renaud Chorlay, IREM de l'Université Paris

The so-called rigorization of Analysis in the 19th century is a standard topic in the history of mathematics, and has indeed provided material for didactical research works centered either on notions (e.g., limit, continuity) or on shifts in levels of abstraction (Advanced Mathematical Thinking). For four years, members of the "history of mathematics" group of the French IREM network endeavored to establish new connections between historical and didactical questions. For the Paris group, the starting point was the identification of four viewpoints on functions: point-wise, infinitesimal, local and global. We gathered historical material—sometimes standard, sometimes less well-known—showing typical interactions between these viewpoints at different stages of the rigorization process. We also tried to identify the contexts in which these viewpoints first emerged, then were explicitly differentiated one from the other. We eventually devised two epistemological models—the "world of quantity" and the "world of sets"—in order to describe two distinct forms of "functional thinking." These high-level descriptive tools helped us gain new insights into didactical questions relevant for the teaching of Analysis at elementary or advanced level. After a short case-study, we will present the main features of the epistemological models. We shall eventually consider more general teaching perspectives.

From History to Research in Mathematics Education: Socio-Epistemological Elements for Trigonometric Functions

Gabriela Buendia Abalos and Gisela Montiel Espinosa, Centro de Investigación en Ciencia Avanzada y Tecnología Aplicada del IPN

In this research paper, the history of mathematics goes beyond its motivational contribution, and far beyond being a source of problems or a necessary element of the culture that the school imparts as part of a citizen's education. From a socio-epistemological standpoint, the mathematical knowledge related to the trigonometric function, what is taught and learned, is all questioned, prompting a historical search through the social practices that condition humans' construction of mathematical knowledge. The analysis of the *uses* and *significations* of mathematical knowledge in specific socio-cultural situations will allow us to show that this knowledge is not made up of isolated concepts and conceptual structuring but instead presents a juncture born out of the development of certain practices. The proposal is to formulate epistemologies of practices, called *socio-epistemologies*, in which historic aspects enable the formation of an initial base of significations for the trigonometric function and its meaningful and articulate introduction into mathematics education. In this way, they contribute elements for the re-design of the school mathematics discourse.

Harmonies in Nature: A Dialogue Between Mathematics and Physics

Man-Keung Siu, University of Hong Kong

The customary practice in school to teach mathematics and physics as two separate subjects has its grounds. However, such a practice deprives students of the opportunity to see how the two subjects are intimately interwoven. This paper discusses the design and implementation of an enrichment course for school pupils in senior secondary school who are about to embark on their undergraduate study. The course tries to integrate the two subjects with a historical perspective.

Exposure to Mathematics in the Making: Interweaving Math News Snapshots in the Teaching of High-School Mathematics

Batya Amit, Nitsa Movshovitz-Hadar, and Avi Berman, Technion—Israel Institute of Technology

This paper elaborates on introducing math news in the classroom. The goal is to overcome the gap between the ever growing nature of mathematics resulted by mathematicians' on-going struggle for establishing new results and the stagnated nature of school curriculum, that yields a wrong image of mathematics as a discipline in which all answers are known, leaving little room for further exploration. Interweaving snapshots of mathematical news is proposed as an appropriate pedagogy. The challenge and dilemma involved in such an intervention are considered. Preliminary results of an on-going experimental study are also reported briefly. The significance of this study resembles that of studies concerned with integrating the history of mathematics in the curriculum, as today's news is tomorrow's history.

History, Figures and Narratives in Mathematics Teaching

Adriano Demattè and Fulvia Furinghetti, University of Genova

This article relates to a project aimed at acting on students' image of mathematics through the use of the history of mathematics. We report on an experiment in which students of secondary school (aged 14–18) were presented with pictures taken from ancient documents. In these pictures there were (direct or indirect) references to mathematics. We asked students to interpret what was happening in the pictures and, in particular, to focus on the mathematical aspects. The results of this activity were narratives in which students produced conjectures suitable to construct a story based on the pictures. The main educational implications of the work here presented is the possibility of fostering students' perception of mathematics as a science operating inside the socio-cultural context, to wide their image of mathematics, and to stress the links with other school disciplines.

Pedagogy, History, and Mathematics: Measure as a Theme

Luis Casas and Ricardo Luengo, Universidad de Extremadura

We present an educational innovation project that was developed in the Region of Extremadura in Spain, but which can serve for teachers in any part of the world to implement similar experiences tailored to their geographic and cultural contexts. The objective of the activity was to illustrate and appreciate the usefulness of mathematics in one of the most mundane of activities: using weights and measures. We investigated the historical use our ancestors made of the units and instruments of measurement, from ancient systems to the metric system. With this focus of interest as a pedagogical resource, we designed activities for pupils and teachers that were ethnographic (collecting information from the elderly, recreating tools that have fallen into disuse), historical (review of ancient texts), and more mathematical (calculation of measures, construction of devices, and the solution of related problems). The work is complemented with an interactive CD which includes all the information and activities of the project. These activities made it possible to integrate various areas of the curriculum, to foster collaborative work among teachers, and to engage in the school tasks people from the social context close to the pupils.

Students' Beliefs About the Evolution and Development of Mathematics

Uffe Thomas Jankvist, Southern University of Denmark

The paper is an empirical study of students' beliefs about the history of mathematics. Twenty-six students in an upper secondary mathematics class were exposed to a line of questions concerning the evolution and development of mathematics in the form of a questionnaire and follow-up interviews. In the paper it is argued that the beliefs literature, in general, lacks a discussion of goals dealing with, for instance, desirable beliefs among students in order to provide them with a more coherent image of mathematics as a discipline. A couple of descriptions from the Danish literature and upper secondary regulations are provided as an example of such a dimension. The concrete student beliefs from the research study are evaluated against these descriptions.

Changes in Student Understanding of Function Resulting from Studying its History

Beverly M. Reed, Kent State University

This study examines the mathematical learning that occurred when students studied the history of the concept of function. Students experienced an in-depth study of the history of functions during a 5-week unit in the junior-senior level History of Mathematics course. They completed a series of worksheets, readings, and problems. The research methodology was a teaching experiment and the framework for analysis of data was *APOS* (Action, Process, Object, Schema) Theory. All 17 students enrolled in the course completed an extensive initial questionnaire and 6 were selected to participate in an in-depth interview to reveal their understanding of the function concept. During the unit, each student wrote a series of reflections about his or her understanding. After the unit, students completed a second questionnaire and participated in another in-depth interview to discern the changes in their thinking about the concept. The findings support the notion that studying the history of a mathematical concept enables a deep reflection of ideas. Four of the six participants notably strengthened their function conceptions. Two moved an entire APOS level. Five of the six exhibited an increased ability to recognize a function in a given scenario. Growth was most profound in the area of graphical representations.

Integrating the History of Mathematics into Activities Introducing Undergraduates to Concepts of Calculus

Theodorus Paschos and Vassiliki Farmaki, University of Athens

The close relationship between Mathematics and Physics during their historical development is generally considered to offer motivational power to the educational praxis. In this paper, we discuss a *genetic didactic approach* to the teaching and learning of mathematics. It is an approach inspired by history in which the integration of genetic moments' in the history of Mathematics and Physics can lead to the development of activities for learning *mathematical topics*. In our case, we focus on designing activities which use historical elements from the mathematical study of motions in the later Middle Ages (14^{th} century: Merton College, N. Oresme) to introduce first-year undergraduates in the Department

of Mathematics at Athens University to the definite integral concept and the fundamental theorem of Calculus. The activities were based on motion problems (most of which concern the representation of velocity/time on Cartesian axes) in which velocity, time and distance covered are represented simultaneously: velocity and time as line segments, and distance as the area between the curve and the time axis. By relating the distance covered to the area of the corresponding figures, the problems help students grasp the connection between velocity and distance covered on the same graph, and to gain an essential understanding of the fundamental theorem of Calculus. The educational intervention formed part of a wider action research which sought to study the difficulties students face in bridging the gap between intuitive-informal and formal mathematical knowledge. The instructive approach was applied in an interactive milieu. In this paper, we present: (1) elements of the History of Mathematics and Physics which we used in designing the activities, (2) the didactic aims of the activities, (3) worksheets and excerpts from student interviews, and (4) observations on theoretical issues and results arising from analysis of the data collected.

A Multiple Perspective Approach to the History of the Practice of Mathematics in a Competence Based Mathematics Education: History as a Means for the Learning of Differential Equations

Tinne Hoff Kjeldsen, Roskilde University

The purpose of the present paper is to discuss how and in what sense history of mathematics and the use of original sources in mathematics education can be used as a means for the learning of mathematics without distorting or trivializing history. It will be argued that this can be pursued by adopting a multiple-perspective approach to the history of the practice of mathematics within a competence based mathematics education. Empirical evidence is provided by means of a student directed project work on the influence of physics on the development of differential equations. The project work is analyzed for its potential learning outcomes with respect to the development of students' insights into history of mathematics and the development of their mathematical competence.

History of Statistics and Students' Difficulties in Comprehending Variance

Michael Kourkoulos and Constantinos Tzanakis, University of Crete

This paper examines students' difficulties in understanding variance and standard deviation in introductory statistics, taking into consideration relevant elements of their historical evolution. The comparative consideration of students' behavior and relevant historical elements: (i) permitted us to improve the understanding of students' difficulties with these concepts, (ii) underlined some important deficiencies of the usual introductory teaching, (iii) pointed out significant elements that could enrich the introductory teaching of these concepts, in order to facilitate and enhance their understanding by the students.

Designing Student Projects for Teaching and Learning Discrete Mathematics and Computer Science via Primary Historical Sources

Janet Heine Barnett, Jerry Lodder, David Pengelley, Inna Pivkina and Desh Ranjan, Colorado State University and New Mexico State University

We discuss and present excerpts from student projects being developed and tested by an interdisciplinary faculty team for courses in discrete mathematics, graph theory, combinatorics, logic, and computer science. The goal of our work is to provide motivation, direction, and context for these subjects through student projects based directly on the writings of the pioneers who first developed crucial ideas and worked on seminal problems. Each project is built around primary source material close to or representing the discovery of a key concept. Through guided reading and activities, students explore the mathematics of the original discovery and develop their own understanding of the subject. We illustrate how project design elements support our pedagogical goals and discuss classroom implementation. Further evaluation and project development is underway, and two web sites provide expanded materials and information. Ongoing support is provided by the US National Science Foundation.

History of Mathematics for Primary School Teacher Education Or: Can You Do *Something* Even if You Can't Do Much?

Bjørn Smestad, Oslo University College

In my pre-service courses for primary and lower secondary school teachers, I include history of mathematics in several ways. In this article, I give examples of some of these, and discuss the choices I have made. In particular, I discuss to what extent it is possible to include bits of history of mathematics even to students with no prior knowledge of history of mathematics.

Reflections and Revision: Evolving Conceptions of a *Using History* Course

Kathleen Clark, Florida State University

A course in the history of mathematics is often required of students completing secondary mathematics teacher preparation programs. The course, "Using History in the Teaching of Mathematics," is required of all undergraduate secondary mathematics education majors at Florida State University[1] (FSU) and focuses on examining various middle school and high school topics from an historical perspective, while emphasizing essential mathematics and pedagogy related to such a perspective. Since 2006, I have investigated how pre-service mathematics teachers (PSMTs) draw upon their experiences with course activities to consider a topic historically and subsequently develop a teaching unit or model lesson (the capstone project in the course) for use in future secondary mathematics teaching. In the capstone project, students are required to examine their topic along several dimensions. For example, the teaching unit might ideally include cultural and humanistic influences and historical texts and problems. The study's data sources included the students' completed teaching unit or model lesson assignment and accompanying documentation required for the assignment, as well as student reflection journals documenting their historical, mathematical, and pedagogical progress during the course. I used my own weekly reflections on course activities, as well as the evaluation of the content of the capstone projects produced to consider further revisions for subsequent course offerings.

Mapping Our Heritage to the Curriculum: Historical and Pedagogical Strategies for the Professional Development of Teachers

Leo Rogers, Oxford University

The sudden changes in the English Mathematics curriculum in 2007/8 now require pupils to recognize "the rich historical and cultural roots of mathematics." Since for the past ten years neither the teacher training system nor government policy has paid any serious attention to mathematical culture or history, there is a serious lack of such knowledge on the part of the majority of teachers. This paper proposes a new approach to introducing relevant material from the history of mathematics into the classroom by introducing the concept of Heritage' through the use of a Concept Map together with a pedagogy that capitalizes on recent research.

Teachers' Conceptions of History of Mathematics

Bjørn Smestad, Oslo University College

In 1997, the history of mathematics was included in the curriculum goals for primary and lower secondary schools (ages 6–16) in Norway. However, studies suggest that the history of mathematics did not get the attention the curriculum mandated. To learn more about how this 1997 change in the curriculum may have been approached by mathematics teachers, I did an interview study of four Norwegian secondary and high school teachers. This is a phenomenological study aimed at gaining more knowledge about teachers' conceptions of history of mathematics, thereby supplementing the findings of earlier studies. The study shows that the teachers differ in what they consider the history of mathematics to be and in what the goal of incorporating it in teaching may be. They include the history of mathematics in very different ways and to different degrees. They have various opinions on how it works. The knowledge gained in this study may contribute to the discussion on how to successfully integrate the history of mathematics in the average mathematics classroom that is: how to engage "ordinary" teachers in this endeavor.

[1] The mathematics education program referred to here has since been suspended by Florida State University.

The Evolution of a Community of Malthematical Researchers in North America: 1636–1950

Karen Hunger Parshall, University of Virginia

This chapter explores various factors–both regional and transnational–which effected the emergence of communities of mathematical researchers in North America in the last quarter of the nineteenth century and their development in the twentieth. The analysis for the United States will hinge on a periodization defined largely by broader political and social influences; contemporaneous developments in Canada will be highlighted.

The Transmission and Acquisition of Mathematics in Latin America, from Independence to the First Half of the Twentieth Century

Ubiratan D'Ambrosio

This paper is part of a large research program on the transmission, acquisition and diffusion of mathematical knowledge as a result of the dynamics of cultural encounters. The basic issues are the relations between the producers and consumers of cultural goods and the factors involved in the transmission and acquisition of these goods. In this paper I focus on Latin America.

In Search of Vanishing Subjects: The Astronomical Origins of Trigonometry

Glen Van Brummelen, Quest University

When and how did trigonometry begin? These simple questions, asked by an innocent student, lead to complex answers—or even no answer at all. Shadowy false starts eventually culminated in Hipparchus's theory of chords, a mathematics born for only one purpose: the study of the heavens. This exploration of the origins of trigonometry reveals some of the difficulties behind the writing of the history of mathematics, and the need to consider carefully the cultural context of the emergence of mathematical ideas.