The International Commission on Mathematical Instruction (ICMI)

ICMI Study 19: Proof and Proving in Mathematics Education

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The ICMI 19 Study on proof and proving in mathematics education will discuss the different meanings of the term proof and bring together a variety of viewpoints. In professional mathematics today, proof admits a range of subtly different meanings: for example, giving an axiomatic formal presentation; using physical conceptions, as in a proof that there are only five Platonic solids; deducing conclusions from a model by using symbolic calculations; or using computers in experimental mathematics. For mathematicians, proof varies according to the discipline involved, although one essential principle underlies all its varieties: To specify clearly the assumptions made and to provide an appropriate argument supported by valid reasoning so as to draw necessary conclusions. Its simplicity, however, is disguised in the subtlety of the deep and complex phrases "to specify the assumptions clearly", "an appropriate argument" and "valid reasoning".

The study will consider the role of proof and proving in mathematics education. It will discuss several issues, among them: the use of proof and proving in the classroom to promote understanding, which in no way contradicts their role in mathematics; the complexity of teaching proof as consisting of explicit chains of inference following agreed rules of deduction, and the use of formal notation, syntax and rules of manipulation; the use of proof in terms of developmental proof, which grows in sophistication as the learner matures towards coherent conceptions; teachers' views about the essence and forms of proofs and how they interpret and implement curricular tasks that have the potential to offer students opportunities to engage in proving, how they diagnose students' difficulties in proving and design instructional interventions to help overcome these difficulties.

The study will have three major features: 1) Proof and proving in school curricula have the potential to provide a long-term link with the discipline of proof shared by mathematicians. 2) Proof and proving can provide a way of thinking that deepens mathematical understanding and the broader nature of human reasoning. 3) Proof and proving are at once foundational and complex, and should be gradually developed starting in the early grades.

The study consists of three components: 1) an invited *Study Conference*, 2) a *Study Volume* and 3) a *Study Website.*

1) The *Study Conference* will be held in Taipei, Taiwan, from May 10 to May 15, 2009. As is the normal practice for ICMI studies, participation in the study conference is by invitation to the authors of accepted contributions. The *Study Conference* is a working one; every participant is expected to be active. The printed proceedings, available at the conference, will contain the refereed submissions of all participants and will form the basis of the study's scientific work.

2) The *Study volume*, a post-conference publication, will appear in the New ICMI Study Series (NISS). Participation in the conference does not automatically ensure inclusion in the book. The

Study volume will be based on selected contributions as well as on the outcomes of the Conference. We expect it to be an edited book which can serve as a standard reference in the field.

3) The *Study website*, http://www.icmi19.com accessible before, during and after the conference, contains information on the conference and is updated periodically. The Study themes are set out in the Discussion Document.

International Program Committee

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Plenary activities:

Jonathan Borwein: Digitally assisted discovery and proof in mathematics

Giuseppe Longo: The reasonable effectiveness of mathematics and its cognitive roots

Judith V. Grabiner: Why proof? Some lessons from the history of mathematics

Arthur Jaffe: On the Unreasonable Effectiveness of Physics in Mathematics

Panel:

Proof within the western and the eastern cultural traditions, starting from a discussion of the Chinese book "The Nine Chapters": Implications for mathematics education.

Chair: Niels Jahnke (Germany)

Discussants: Karine Chemla (France); Wann-Sheng Horng (Taiwan); Ko-Wei Lih (Taiwan); Man-Keung Siu (Hong Kong)

Working groups:

1. Cognitive development of proof (Co-Chairs: David Tall and Oleksiy Yevdokimov)

2. Argumentation (Chair: Viviane Durand-Guerrier)

3. DGS/Experimentation (Chair: Ferdinando Arzarello)

4. Proof in the school curriculum, knowledge for teaching proof (Chair: Fou-Lai Lin)

5. The Nature of proof for the classroom (Co-Chairs: Tommy Dreyfus, Niels Jahnke and Wann-Sheng Horng)

6. Proof at the tertiary level (Chair: Annie Selden)

In addition, the expected 150 participants will present their research papers.

A report on ICMI Study19 and its outcomes will be presented at the 12th International Congress on Mathematical Education in 2012.

For additional information on proof and proving, visit the following sites:

http://www.icmi19.com

http://fcis.oise.utoronto.ca/~ghanna/philosophyabstracts/index.htm

http://www.lettredelapreuve.it/

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