Mathematics curriculum material and teacher documentation: from textbooks to shared living resources

Editors: G. Gueudet, B. Pepin & L. Trouche

Foreword: R. Straesser

Introduction: the editors

Section 1: Teacher resources

The first section of the book focuses on the kinds and nature of resources for mathematics teachers from a practical, methodological and theoretical point of view. It examines what is, or is not, available for teachers' professional activity. It introduces the question of what kinds of changes are afforded by digital resources. (The conceptualisations proposed precise the view on teachers' professional development that results from a focus on thnir interactions with resources- not sure what you want to say here).

Chap. 1 (J. Adler): Knowledge resources in and for school mathematics teaching

Chap. 2 (G. Gueudet & L. Trouche): Teachers' work with resources: documentational geneses and professional geneses

Chap. 3 (M. A. Mariotti & M. Maracci): Resources for the teacher after a semiotic mediation perspective

Reaction to section 1: Bill Barton, Vice President of ICMI, University of Auckland, New Zealand.

Section2: Text and Curriculum resources

The second section of the book focuses on the characteristics of curriculum material, in particular how the cultural context shapes the materials. It raises questions about the design of curriculum materials, and about their integration, appropriation, and transformation by teachers in and for their everyday teaching. It examines the various factors shaping the nature of the resources, their design and their use, with a specific focus on cultural factors, illuminated by international comparative studies.

Chap. 4 (K. Ruthven): Constituting Digital Tools and Materials as Classroom Resources

Chap. 5 (J. Remillard): Modes of Engagement: Understanding Teachers' Transactions with Mathematics Curriculum Resources

Chap. 6 (B. Pepin): Working with teachers on (educative) curriculum materials to develop mathematical knowledge in/for teaching

Chap. 7 (W. Schmidt): The Cumulative Effects of Middle School Tracking: How Content Coverage Varies

Chap. 8 (C. Proust): Teachers' writings and students' writings': school material in Mesopotamia

Reaction to section 2: ?

Section3: Use of resources

The third section of the book focuses on the use of resources by teachers and students, in-class and out-of-class. It studies the influence of the resources' characteristics on teacher and student activity. Furthermore, it considers the interactions between the various educational agents, and the effects of these interactions on the development and design of resources.

Chap. 9 (C. Kieran): Researcher-designed resources and their adaptation within classroom teaching practice

Chap. 10 (D. Forest & A Mercier): Classroom's video data and teaching resources

Chap. 11 (S. Rezat): The role of mathematics textbooks within the documentational genesis

Chap. 12 (M. Trigueros & D. Lozano): Teachers teaching mathematics with Enciclomedia

Chap. 13 (P. Drijvers): Teachers transforming resources into orchestrations

Reaction to section 3: Luis Radford, Professor of Mathematics Education, Laurentian University in Ontario, Canada

Section4: Collaborative use

The fourth section of the book focuses on the collaborative aspects of teacher documentation. Teachers are in contact with various groups and communities, and this dimension is particularly considered here. Concepts are introduced, which illuminate the influence of the nature of the groups and communities in this respect, the particularities of the processes of documentation within collectives, and individual-collective relationships.

Conclusion: the editors

Section 1: Teacher resources

Author: Jill Adler

Title: Knowledge resources in and for school mathematics teaching

This chapter builds on a previous study of teachers' 'take-up' from an in-service teacher education programme in South Africa (Adler, 2000) where resources in use in school mathematics practice were problematised. With a theoretical orientation drawn from social practice theory, we developed a conception of re(sourcing) practice that extended beyond material resources to include human and cultural resources. In focus in that study were material (e.g. chalkboards) and cultural resources (language, time). Out of focus were human resources; teachers themselves and their knowledges in use. The question of mathematical and other professional knowledge for/in the practice of mathematics teaching is a key focus of the QUANTUM research project (see Adler & Davis, 2006; Adler & Huillet, 2008). Five case studies of secondary school mathematics teaching in South Africa have explored the question: what mathematical and other knowledges do teachers draw on to ground or legitimate school mathematical knowledge in their classroom practice (Bernstein, 1996)? This chapter draws on two of these cases, and through these elaborates knowledges in use as (re)sources for school mathematics practice. The sections in the paper will include:

- A discussion of the conceptualisation of re(source), the research from which it emerged, how it relates to documentational genesis (Gueudet & Trouche), and how the initial conceptualisation and its illumination through the concept of 'transparency' requires elaboration when attention shifts to knowledge as resource. This is achieved through the concept of 'ground', and explained in terms of the operation of pedagogic judgement – argued as central to the work of teaching;

- A brief discussion of mathematics teacher education in South Africa as the context in which a conceptualisation of teachers acting with resources – of teachers re-sourcing their practice – shifts away from deficit to enabling discourses of teaching and teachers; and a discussion of the school mathematics curriculum, its hybrid form and multiple orientations to mathematics; thus demanding that teachers draw (flexibly) on mathematical principles, procedures and conventions, everyday metaphors and everyday experiences, in order to ground whatever they intend to be learned;

- A discussion of mathematical and other professional knowledge for teaching in the field, particularly the significant work of Deborah Ball and colleagues (Hill et al, 2008) and their extensive study of relationships between teachers' knowledge, quality instruction and student learning. In contrast, the QUANTUM cases study the way ground comes to function in professional practice, so levering up insights into knowledge-in-use;

- Extracts from the two contrasting case studies follow, together with the methodology used. These illuminate objects of attention in focus, the domains of knowledge and practice that the teachers drew on to ground these, and the teaching tasks that supported their practice. The contrast lies in how ground came to function in each case, revealing different knowledges in use that enable elaboration of the conceptualisation of re(sources).

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Authors: Ghislaine Gueudet and Luc Trouche

Title: Teachers' work with resources: documentational geneses and professional geneses

The teachers draw on *resources* in many ways, in diverse places and moments. These resources (books, software, Internet, students reactions…) are crafted (adapted, revised, reorganised…) within interwoven design and implementation processes. This *documentation work* is central in teachers' professional activity.

We present in this chapter a theoretical approach dedicated to the study of this documentation work and of its consequences on the development of teachers' professional knowledge (Cooney 1999). It is inspired by the instrumental approach (Rabardel 1995, Guin *et al.* 2005) developed within mathematics didactics for the study of students' learning in technology-rich environments. We devote a specific interest to digital resources, and to the evolutions of teachers' activity yielded by such resources (chap. Lozano and Trigueros). We naturally also consider more traditional resources like textbooks (section 2); in our perspective, even a student's production, a discussion with a colleague, are resources for teachers, according to the general meaning attributed by (chap. Adler) to this concept.

We consider that teachers develop *documents*, associating recombined resources and professional knowledge. Teachers' craft knowledge (in particular their curriculum script, chap. Ruthven) shapes the documents produced; conversely, the features of the resources shape the knowledge developed (Cooney 1999). Teachers' documents are organized in structured *systems*, according to the structure of their professional activity. *Documentational geneses* are continuous processes; they are key components of the teachers' *professional geneses*.

We also expose a specific methodology for the study of teachers' documentation work, in class and out of class. We present case studies, drawing on data collected with this methodology.

This chapter extends the results stated in (Gueudet & Trouche 2009), precising and complementing the concepts, deepening the methodology and exploiting new data.

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Authors: Maria-Alessandra Mariotti and Mirko Marracci

Title: Resources for the teacher after a semiotic mediation perspective

Artefacts, and in particular ICT tools, are commonly considered as resources susceptible of enhancing the teaching-learning activity. But while the potentialities of artefacts for learning have been extensively studied with a main focus on their possible use by the students, there has been the tendency to underestimate the complexity of the teacher's role in exploiting these potentialities.

The Theory of Semiotic Mediation (TSM) (Bartolini Bussi and Mariotti, 2008) addresses this issue.

Within the frame offered by the TSM the use of an artefact is considered both from the perspective of the students who use the artefact for accomplishing given tasks, and from the perspective of the teacher who uses it as a means to achieve specific educational goals. Drawing on a Vygotskian perspective (Vygotskij, 1998) the TSM put into evidence that the use of an artefact for accomplishing a mathematical task in a social context may lead to the emergence of meanings, which, on the one hand, are related to the actual use of the artefact, and, on the other one, may be related to the mathematical meanings relevant to the use of the artefact and to the task. The complex of relationships among use of the artefact, accomplishment of the task, personal meanings and mathematical signs is called the *semiotic potential of the artefact* with respect to the given task. In a mathematics class context, when using an artefact for accomplishing a task, students can generate meanings which can be put in relationship with mathematical meanings. But the construction of such relationship is not a spontaneous process for students. On the contrary it should be assumed as an explicit educational aim by the teacher.

In the chapter "Artefact as tool of semiotic mediation: a resource for the teacher", assuming a TSM perspective we developed an analysis of how the teacher can exploit the *semiotic potential of the artefact* and trigger, through its use, the semiotic mediation process at the core of the teaching-learning process. Taking an instrumental approach (Rabardel, 1995) and exploiting the notion of utilization schemes we described how an artefact may function as *an instrument of semiotic mediation* for the teacher. More specifically, the focus was on the teacher's actions mediated by the artefact and concerning the *orchestration of the classroom discussion*.

The discussion developed in this chapter is meant to have a wider scope. In fact we aim at studying not only how the teacher uses a particular ICT tool, but also how he uses other specific artefacts which can be put in relationship with the use of the ICT tool considered. We are referring in particular to written texts: e.g. texts written by students in relation to their experience with the ICT tool, or texts drawn from historical sources.

In synthesis a system of artefacts will be considered in this study: a specific ICT tool, written productions which can be put in relationship with the actual use of the artefact, in particular historical sources. With this respect, Wartofsky's distinction between primary, secondary and tertiary artefacts is expected to contribute to clarify the relationship among the mentioned artefacts, and how the teacher can productively use them.

Taking into consideration a system of artefacts approaches our study to the study of the process of the documentational genesis (Gueudet and Trouche, chapter 3). Comparing the two studies could shed light on the differences and similarities between the two perspectives.

Finally, beyond illustrating the functioning of the semiotic mediation process, the elaboration provided in the chapter intends to contribute to a key issue concerning the notion of mediation, and specifically of semiotic mediation: the importance of identifying different forms of mediation to be related to teaching-learning process, and of expressing them in an explicit way so that they can be communicated and shared in the community of the teachers.

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Section 2: Text and Curriculum resources

Author: Kenneth Ruthven

Title: Constituting Digital Tools and Materials as Classroom Resources

This chapter develops a conceptual framework that identifies key structuring features of classroom practice, showing how they shape the integration by teachers of digital tools and materials as classroom resources within everyday practice. An important *interpretative flexibility* surrounds any technology in the course of its appropriation as a practical tool. In particular, teachers interpret educational resources and mediate their students' use of them. Equally, however, teachers necessarily incorporate such resources into wider systems of classroom practice which depend on *craft knowledge*, the largely reflex system of powerful schemes, routines and heuristics which teachers bring to their classroom work, often tailored to their particular circumstances.

Structuring features of classroom practice

Five key structuring features of classroom practice are examined, showing how they relate to the incorporation of digital tools and materials as classroom resources:

- *Working environment* focuses on the physical surroundings in which lessons are conducted, and the associated infrastructure and organisation;

- *Resource system* focuses on the combined operation of the mathematical tools and curriculum materials in classroom use, particularly on their compatibility and coherence of use (Amarel, 1983);

- *Activity format* focuses on the templates for action and interaction which frame the contributions of teacher and students to a particular type of lesson segment. (Burns & Anderson, 1987);

- *Curriculum script* focuses on the loosely ordered model of relevant goals and actions which serves to guide a teachers' teaching of a particular topic, interweaving ideas to be developed, tasks to be undertaken, representations to be employed, and difficulties to be anticipated (Leinhardt *et al.,* 1991);

- *Time economy* focuses on how teachers seek to manage the 'rate' at which the physical time available for classroom activity is converted into a 'didactic time' measured in terms of the advance of knowledge (Assude, 2005).

An investigative lesson with dynamic geometry

The conceptual framework sketched in the last section is used to analyse the practitioner thinking and professional learning surrounding a lesson incorporating the use of dynamic geometry (Ruthven, Hennessy & Deaney, 2008). This highlights the adaptation of teaching practices and development of craft knowledge associated with the teacher's appropriation of digital tools and materials as resources for teaching and learning mathematics.

In particular, this case study shows how the teacher had adapted his working environment to exploit the proximity of ordinary classroom and computer laboratory, shifting between rooms according their suitability for particular types of activity format, so developing a new type of activity structure, justifiable within the time economy, providing what he regarded as a more desirable structure for an investigative lesson. Likewise, he had formulated a relationship between manual construction tools and dynamic geometry software in which they form, if not a wholly coherent resource system, one in which they serve complementary functions, sequenced within his curriculum script to advantage mathematical learning, and so justifiable in terms of time economy.

This conceptual framework prioritises and organises constructs developed in earlier research, and has proved a useful tool for analysis of already available case-records. It has potential to be employed not just in relation to secondary mathematics teaching, but to other school phases and curricular areas; indeed, much of the earlier research from which it draws its different central concepts has such a range. Equally, it has potential to to be employed in relation to integration of a wide range of technologies in education beyond the particular digital ones on which it has focused. Further studies are now required in which data collection (as well as analysis) is guided by the conceptual framework, so that it can be subjected to fuller testing and corresponding elaboration and refinement.

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Author: Janine Remillard

Title: Modes of Engagement: Understanding Teachers' Transactions with Mathematics Curriculum Resources

In the last decade a number of studies have documented a variety of personal, professional, and classroom-based results from teachers using curriculum resources as tools (e.g., Remillard et al. 2009). Some scholars have focused on theory building related to teachers' use of mathematics curriculum resources (Adler, 2000; Brown, 2009; Gueudet & Trouche, 2009; Remillard, 2005). There is general agreement that the process of using a curriculum resource is not a straightforward process of implementation and that curriculum use involves an interaction between the teacher and the resource. In this chapter, I draw on Ellsworth's (1997) conception of modes of address and Rosenplatt's (1982) theory of transaction with texts to examine the relationships that teachers develop with curriculum resources and how they come to be.

Modes and forms of address

In film studies, a mode of address captures who the film's designers think the audience is, what they want, and how they read. All films (or texts) have an intended audience and are written to capture and appeal to, to speak to that audience. Similarly, curriculum materials are written with particular teachers (readers) in mind. They are written to both appeal to those readers' needs and desires and to affirm them in order to keep the text-teacher relationship intact.

The *form* of address of a curriculum resource refers to the physical and visual form it takes up, the nature and presentation of its contents, the means through which it, addresses teachers. The form of address is what teachers actually see, examine, and work with when using a curriculum resource. I argue that when it comes to curriculum materials, form takes on its own significance and meaning. And while it is a critical component of the mode of address, form deserves a particular analytic focus. In my examinations of mathematics curricula and my studies of teachers using these resources, I have found that the large number of characteristics that make up the form of address can be loosely classified into four categories: *look, structure, voice,* and *genre*.

Modes and forms of engagement

Modes of engagement refer to how teachers engage with or meet the forms of address of the text. My thinking about this idea has been influenced by Rosenblatt's (1982) theory of transacting with text— that readers not only interpret visual signs, but they infuse meaning into them. This stance focuses the reader's attention. The mode of engagement refers to what teachers do as they transact with a text. It is supported by and reflects the stance. In my research, I have found that a teacher's mode of engaging a curriculum resource includes four primary forms or kinds of reading: *what she reads for; which parts she reads; when she reads; and who she is as a reader.*

Shifting Modes of Engagement

I see the domain of modes of engagement, how teachers engage and read curriculum materials, to be a fruitful place for learning to take place. At this point, we understand little about the processes through which teachers might learn to engage with curriculum resources in substantially new and unfamiliar ways.

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Author: Birgit Pepin

Title: Working with teachers on (educative) curriculum materials to develop mathematical knowledge in/for teaching

Working with mathematics curriculum materials to create learning opportunities for pupils, and at the same time to develop mathematical knowledge for teaching is an important part of the work of teaching. This chapter explores the 'journey' of two secondary school teachers in England, John and Paul, as they work together in a group of teachers and with a university academic, on the analysis, development and teaching of mathematics curriculum materials. The main aim of this project was to work with these teachers to develop their mathematical knowledge in/for teaching, that is (1) to assist teachers to analyse, select, and amend mathematically rich problems; and (2) to assist teachers to 'implement' these into their pedagogical practice (and maintain the richness) as they guide students through these problems.

Underpinning this study is the assumption that teacher learning involves teacher autonomy and agency when analysing, choosing, changing and transforming materials, devising alternatives, and 'enacting' the materials (Ben-Peretz, 1984). Paris (1993) emphasizes that teacher agency in curriculum matters involves "the creation or critique of curriculum, an awareness of alternatives to establish curriculum practices, the autonomy to make informed curriculum choices, an investment in self, and ongoing interaction with others." (p.16) Moreover, in the UK the recent Williams Review (Williams, 2008) drew attention to the need, and challenge, of strengthening mathematical knowledge for teaching system wide, and they recognise the scale of professional development initiatives that is required to secure that knowledge. As new mathematics curriculum materials are being and have been developed in many countries (e.g. US: NCTM, 1989; UK: DfES, Standard Units, 2005), teacher learning is considered an important aspect and part of the 'enactment' of these materials (e.g. Remillard & Bryans, 2004).

The study built on previous work with teachers, textbooks and other curriculum materials (e.g. Pepin, 2009; Haggarty & Pepin, 2002; Pepin & Haggarty, 2009). Over a period of nine months the work with teachers consisted of two phases:

- Curriculum exploration and transformation (deepening teacher knowledge of mathematics concepts and teaching mathematics 'with understanding'; development of analysis schedule for curriculum materials; analysing mathematical tasks; selection of tasks and activities for lessons);

- Link to practice and lesson analysis ('enacting' the curriculum materials; reflection on enactment- videoed lesson; sharing of work and discussions).

There is evidence that the project has had positive benefits to:

- Teacher knowledge with respect to 'pupil learning mathematics with understanding';
- Teacher selection and analysis of mathematics curriculum material;
- Teacher confidence of amending and enriching material;
- Teacher reflectivity with respect to the enactment of curriculum material.

Results show that this project has helped teachers to spend time on developing their knowledge for/in teaching, by thinking about and analyzing curriculum material (some of it educative), developing the material further, and by 'enacting' the material and reflecting on the processes. The project has succeeded in raising teachers' awareness, and knowledge, of the educative nature of curriculum material, and what that may mean for their pedagogic practice. It is argued that we need to help teachers learn from and work with all types of curricular materials –whether they are educative and well designed, or otherwise- as they prepare for their teaching. This goes beyond 'curriculum delivery', and involves developing strategies to use the support offered by the school environment and uncovering ways to support their learning with the help of available curriculum materials. Moreover, the study has added further evidence to the debate regarding the benefits of providing teachers with opportunities, and time, to use curriculum materials to assist teachers' learning and practice- well-designed materials must be a resource for teacher learning (and not only pupil learning).

It is also evident that the process of interacting with new material is complex, and it is often neither explicit nor public. Teacher benefit from opportunities to analyse, examine, enrich or amend new curriculum materials with their colleagues. This involves a process of 'mutual transformation'- transforming the curriculum material, as well as potentially transforming the teachers' notions of what can be done in the classroom, their pedagogic thinking. Adding to this new resources, such as digital resources, the web of interaction becomes even more complex. Further research is needed that takes us away from the dualistic thinking of 'teachers and texts', to more sophisticated processes and forms of analysis that include the working environment, the resource system, the activity format, and the curriculum script (Ruthven, 2008), and consider and focus more the actual processes that link the different 'ingredients', rather than the identification of factors.

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Author: William H. Schmidt

Title: The Cumulative Effects of Middle School Tracking: How Content Coverage Varies

This chapter reports the use of textbooks to capture the diversity of mathematics coverage resulting from tracking in US middle schools. It is broadly accepted that textbooks are a good reflection of the implemented curriculum in most countries and a particularly accurate reflection of the implemented curriculum in the US (e.g. Valverde *et al.* 2002). Using textbooks to estimate the amount of demanding

mathematics that students were supposed to learn during their careers in middle and high schools provides a window into the nature and the effects of tracking in middle school mathematics in the US.

Textbook data were collected from mathematics teachers of a national sample of 3,116 US public school students from grades 7 through 12 as part of the Longitudinal Study of American Youth. Mathematics textbooks used by these teachers were collected and coded using a modified document analysis procedure developed by the 1995 TIMSS International Curriculum Analysis study for cross-national curriculum comparisons (Schmidt *et al.* 1997).

The document analysis procedure captures both content and the performance expectation intended by the textbooks. Not surprisingly, this provided a cumulative summary of mathematics content that typical students received in secondary school. But in addition to qualitative descriptions of the topics, the procedure allows us to also estimate the amount of coverage intended or potentially implementable. We examined different approaches to quantitatively summarize the data in order to characterize students' exposure to mathematics content. The results were tested against our expectation of the long term effects of tracking in middle schools.

Contrary to tracking theory's assertion that matching student ability with course type would improve students' achievement growth, many studies and results from LSAY have shown that tracking actually increases the achievement inequalities. Studies have shown that students' eighth grade math course positioned them to take higher-level mathematics courses. By grouping students by the type courses they took in 7th and 8th grades, we compared different approaches to defining the "amount" of mathematics they were exposed to in the subsequent years. Each of these approaches captured different characteristics of the mathematics contents represented in the textbooks. For example, the extent of coverage was measured by the number of pages (or proportion of pages), the level of performance expectation (ranging from recall to reasoning) and the level of content complexity as measured by the grade level when the topic was introduced and generally taught in the school curriculum internationally (International Grade Placement Index).

The results showed that students who enrolled in Algebra prior to 9th grade were exposed to almost three times as much of mathematics contents as students when did not take algebra in secondary school. The gap in exposure persisted when contrasting these students with other students who had algebra in high school though to a smaller extent. We also examined the diversity of exposure within each group.

In conclusion, the chapter demonstrates a method to characterize and summarize instructional materials that can be used to distinguish different types of curriculum. It produced indices that could be used in quantitative analyses to account for curricular differences in student achievement.

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Author: Christine Proust

Title: Teachers' writings and students' writings': school material in Mesopotamia

By nature, school drafts of Mesopotamia were meant to destruction. But, thanks to clay support of writing and ancient recycling practices, they reached us in vast amount. These school tablets were mainly produced along a quite short period, between 18th and 17th century B. C. They contain principally exercises for learning writing, Sumerian language and mathematics. Although they are very distant from us, these sources bear witness of phenomena linked with those which are examined in this book: change of knowledge medium, from memorisation to writing; standardisation processes, notably in the field of writing and metrology, in which scribe schools played a key part; emergence of a set of ideological references specifically linked to a scholar milieu. This contribution will rely on school sources, particularly mathematical ones, in order to bring to light some aspects of these phenomena. In particular, it will endeavour to identify authors, users, function and status of different types of mathematical writings produced in the frame of scribes' schools. In a methodological point of view, approaches will be diverse. School texts, which content is very repetitive, give away information to historians only if they are tackled in various dimensions: textual one, of course, but also material, archivistic, contextual.

This contribution will develop the following parts.

The first part will present the sources and historical context. School tablets were unearthed in many places of a very large geographical area including all or part of the present Iraq, Iran, Syria and Lebanon. But the great majority of them come from Nippur, religious and cultural capital of Mesopotamia. The context in which the young scribes were taught is not always clear. It was probably not the same anywhere and anytime. According to the sources used, context seems institutional, familial, religious, or is like a kind of apprenticeship (Charpin 2008, Veldhuis 1997, Michel 2008). At Nippur, school context is clearly institutional and secular, and stakes of education are very important. Sources allow us to reconstruct a genuine curriculum, strongly structured, and which was exported in whole cuneiform Ancient Near East (Veldhuis 1997 ; Proust 2007). The second part will be devoted to analyse the nature of transmitted knowledge in the level of elementary education, mainly at Nippur: learning by heart of a set of stereotyped lists which follow on from each other in a fixed and unchanging order; interactions between mathematics and writing technology; structure of lists, linked both to art of memorisation and to art of writing; typology of tablets, which bears evidences of pedagogical methods; possible existence of specialisation in the curriculum. The third part will focus on teachers' writings. Who wrote mathematical texts? Who they wrote for? Answers are various according to place, and sources are incomplete. Students' writings are stereotyped, and they are attested in a lot of duplicates; school tablets were found in a well known archaeological context. It is not the case for teachers' writings, which are more difficult to identify because transition between elementary education and erudition is gradual; moreover, erudite tablets come from illegal dugs and their provenience is unknown. We shall stress the ambivalent nature of mathematical texts: they lie both within in a teacher-pupil relation and within communication between peers. (Bernard, et al. forthcoming). The last part will discuss documentation in scribes schools. For historians, « documentation » means « sources ». Here, the meaning is not the same. What is documentation used by teachers in scribes' school? We find: reference documentation (norms, tools), mainly memorised; written texts, which are problems repertoires (paradigms) both devoted to education and to elaboration of new knowledge.

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Section 3: Use of resources

Author: Carolyn Kieran

Title: Researcher-designed resources and their adaptation within classroom teaching practice

As byproducts of the research enterprise, mathematics education research has, over the years, yielded numerous resources, many of which have been made accessible to practitioners. However, little is known about the ways in which teachers take on such research-based resources and adapt them to their own needs. In 2000, Adler proposed that, "mathematics teacher education needs to focus more attention on resources, on what they are and how they work as an extension of the teacher in school mathematics practice" (Adler, 2000, p. 205). However, some of the resource-related research in the ensuing decade has focused more on the resources alone (Ainley & Pratt, 2005), rather than more broadly on how the resources are used and under what circumstances. More recently, Gueudet and Trouche (2009) have enlarged the theoretical frame for researching such issues by placing them "within the study of teachers' professional evolution, where the researchers' attention is focused on the resources, [and] their appropriation by the teacher or by a group of teachers working together" (p. 200). This broadening of research perspective on resource use draws upon the earlier work of, for example, Cohen, Raudenbush, and Ball (2003), who proposed that *resources* include not just "conventional resources (teachers' formal qualifications, books, facilities, class size, and time)," but also "personal resources (practitioners' will, skill, and knowledge) and environmental and social resources (state guidance for instruction, academic norms, professional leadership, and family support)" (p. 127). Gueudet and Trouche (2009), whose research has included a focus on teachers' use of digital resources, have also emphasized that the most elementary components to be considered for the analysis of a resource include: the material component, the mathematical component, and the didactical component.

Building on the developments of the resource-oriented research of the last decade or so, this chapter will sketch a resource-based frame for analyzing the features of a set of resources as designed by a team of researchers and the ways in which these resources are adapted by teachers in their classrooms (bearing on Gueudet & Trouche's resource components, and the 'activity format' and 'curriculum script' of Ruthven), as well as the nature of the factors shaping these adaptations (drawing from, but extending, the Cohen et al., 2003, description of personal resources). A set of CAS-supported resources was designed by the researchers according to the task-technique-theory framework, inspired by Chevallard's (1999) anthropological theory of didactics. Teachers were invited to use these resources over a five-month period in their grade 10 algebra classrooms. The analysis to be presented in this chapter focuses on researcher observations of the salient ways in which three of the teachers adapted, either intentionally or not, pieces of these resources – adaptations that were related to factors such as: teacher's sense of self in the instructional setting; teacher's mathematical sensitivity; teacher's mathematical expectations for the pupils; teacher's view of the role of, and related use of, various other semiotic resources in the classroom (see also the chapter by Mariotti and Marracci); and teacher's use of collective student reflection and student communication of ideas as an additional resource.

An underlying assumption of mathematics education research has been that of a direct beneficial effect of well-designed, researcher-crafted, curriculum resources and semiotic tools on student learning. The research presented in this chapter suggests that the line between the two is not as straight as has been assumed. The ways in which teachers adapt available resources are complex and varied, and serve to both enrich, as well as weaken, the intended usage of the resources as originally designed by the researchers.

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Authors: Dominique Forest and Alain Mercier

Title: Classroom's video data and teaching resources

We draw on a case study to examine the way in which the professor can use material and symbolic elements from the milieu, especially the contributions of the pupils memorized on the blackboard, like resources for his own action. We consider in this example the importance of language, body and space fittings as mean of supporting student's joint attention. We connect non-verbal aspects to linguistic interactions, by specifying them to didactic content.

We refer to the framework of Guy Brousseau, and the theory of didactic situations in mathematics (Brousseau 1998). A central support for the development of this theory was the COREM[1], an « observation school» in Bordeaux, France. Lessons have been video recorded at the COREM during more than twenty years, and some of them are accessible to education researchers. This provision is supported by the INRP[2] in the VISA[3] European project[a].

We analyze a lesson corresponding to the beginning of a teaching of rational numbers, and more precisely to the invention, by the students, of a means of indicating sizes much lower than the unit. Our objective is to understand what the teacher has to do, during the lesson.

The analysis of the teacher's practice is principally based on the Joint action theory in didactics (Sensevy & Mercier 2007). For studying non-verbal behaviors, we use proxemic analysis (Hall 1963, Forest 2009). In this lesson indeed, the teacher, after fitting students' propositions as resources on the blackboard, draws on these propositions to present the notations created as new mathematical objects. We will see that this work about language is not only a communication work, in the usual and telegraphic meaning of transmitting signal from transmitter to receiver (Shannon 1948), but requires to design a fitting [agencement] of elements.

Enlightened by these analysis, video data as resource appears as carrying promises for researchers as well as for teachers. For the research, semiotic and dynamic constructions like the ones observed here are only visible by using video data. The same remark can be made about using video data for professional development, by giving to see didactic techniques in their corporal inscription, and not only linguistic ones.

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Author: Sebastian Rezat

Title: The role of mathematics textbooks within the documentational genesis

In the chapter we will elaborate on the role of mathematics textbooks within the documentational genesis (Gueudet & Trouche 2009).

Mathematics textbooks are considered to play a major role in the teaching and learning of mathematics (Valverde et al. 2002; Remillard 2005). Since mathematics textbooks are regarded as potentially implemented curriculum (Valverde *et al.* 2002) the scientific engagement focused almost exclusively on the analysis of the contents of textbooks in the past. But, from the perspective of documentational genesis the ways teachers use mathematics textbooks to create educational opportunities become a central issue. The few studies that examine the use of mathematics textbooks by teachers point to the fact that mathematics textbooks are one of the most important resources that teachers use for designing their lessons (c.f. i.e.: Bromme & Hömberg 1981; Johansson 2006). In the

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[1] Center for Observation and Research on Mathematics Teaching, associated with observation school "Jules Michelet", Talence, France

[2] National Intitute for Pedagogical Research, France

[3] VISA, Videos of teaching and learning situations, http://visa.inrp.fr/visa

chapter an overview of studies investigating teacher's use of mathematics textbooks for designing their lessons will be given. Furthermore, the results of these studies will be reinterpreted in terms of documentational genesis.

In many countries, the mathematics textbook is a resource that is shared by teachers and students. Thus, it is likely that there is an interaction of teacher's and student's use of mathematics texbooks. In the chapter we will summarize findings from a study of student's utilization of mathematics textbooks as instruments for learning mathematics (Rezat 2008) with a focus on examples of the interaction of teacher's and student's use of mathematics textbooks. Based on the findings of this study it is argued that the student's use of resources must be considered an important aspect within teachers' documentational genesis. The teacher has to take into account the student's use of the resources in designing educational opportunities. It will be argued that considering students' use of resources in the documentational genesis is an important aspect of teachers' professional development. The use of resources by students in terms of instrumental genesis will be integrated into the model suggested by Gueudet and Trouche (2009). This leads to a more complex model of the documentational genesis which incorporates the interactive aspect of teachers' and students' use of resources in shaping the enacted curriculum.

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Authors: Maria Trigueros and Dolores Lozano

Title: Teachers teaching mathematics with Enciclomedia

In this chapter we intend to describe teachers' documentation work and documentation systems when they use interactive resources provided to them by the national program Enciclomedia in Mexico.

Encilomedia intends to complement already existing materials in primary school classrooms –such as the mandatory textbooks - with computer programs and teaching resources that are to be used with an interactive whiteboard. It is a large-scale project, involving more than 7000 schools.

Enciclomedia's programs have been designed with the intention of motivating students to engage in mathematical activities by inviting them to take part in games and by providing them with interesting contexts. Interactive whiteboards also motivate students to take part in the activities. Teachers themselves often resort to the programs in Enciclomedia to experiment with different solutions, and some of them now begin their lessons by working with an activity within Enciclomedia, without giving a formal explanation to the students beforehand. The use of Enciclomedia has, therefore, changed teaching practices to some extent.

In this chapter, we will examine teachers' documentation systems as they interact with Enciclomedia. In order to do this, we will analyse information about teachers' interactions with Enciclomedia obtained from different sources. We will investigate the way in which teachers work with Enciclomedia in the classroom through lesson observations. We describe how different teachers use the digital resources and the interactive whiteboard in different ways articulating them with other resources, such as teaching guides also provided by the project. We will also report on the way teachers describe their work with Enciclomedia during interviews. Additionally, we will analyse data obtained during teachers' workshops, where teachers not only learn more on the way to use the resources provided by Enciclomedia, but they also share with each other their own experience and their practices in the

classrooms. Finally, we will also examine teachers' responses to questionnaires and interviews where they have reflected on the way they put together different resources when they plan their lessons.

In previous research (Tigueros and Lozano, 2007; Trigueros, et. Al., 2007; Lozano et. Al., 2006), we have used an enactivist framework (Maturana and Varela, 1992) and the instrumentation approach (Rabardel, 1999). In this work we will try to review our experiences in terms of the documentation approach. We will highlight the uses teachers make of different resources and the evolution of articulation between the conception of Enciclomedia resources and their use by teachers and by groups of teachers.

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Author: Paul Drijvers

Title: Teachers transforming resources into orchestrations

Nowadays, teachers are confronted with a myriad of resources that are available through technological means, and through internet in particular. However, resources do not transform into teaching in a straightforward way. Teachers may perceive difficulties in orchestrating education which makes use of technological tools and resources. As Robert and Rogalski (2005) point out, teachers' practices are both complex and stable. The availability of technology amplifies the complexity and, as a consequence, challenges the stability of teaching practices (Lagrange and Monaghan, 2009). Teaching techniques that are used in 'traditional' settings can no longer be applied in a routine-like manner when technology is available. The question addressed in this chapter, therefore, is how teachers orchestrate the use of digital resources in teaching practice and how the transformation of resources into can be related to their views on mathematics education and their teaching practices in 'traditional' lessons.

In previous research, we identified different types of orchestrations that teachers develop while engaged in teaching with technological resources (Drijvers et al., submitted). In the present chapter, we consider a case study of a mathematics teacher who is not very experienced in using technology in his teaching, but who volunteered in a pilot project set up by the publisher of his text book series. The orchestrations this teacher puts into action are related to his views on mathematics education, are compared to the previously identified types of orchestrations and are backed up with findings of the whole sample of 60 participating teachers. The results suggest that teachers take their regular practices as points of departure for their ways of transforming resources into orchestrations and that their choices are related to their views on mathematics education and their views on technological resources and tools therein.

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Section 4: Collaborative use

Author: Carl Winsløw

Title: A comparative perspective on teacher collaboration: the cases of lesson study in Japan and of multidisciplinary teaching in Denmark

We depart from two situations which a priori are not related at all: first, the rather well known pratice of lesson study in Japan, particularly well described in the case of primary school mathematics teachers(see e.g. Miyakawa and Winsløw, in press); and the first experiences with a recent reform of Danish upper secondary school, which includes a number of new teaching formats in which students should combine and contrast a (small) number of disciplines, formats which cause particular difficulties for mathematics teachers, according to a recent official evaluation of the reform (EVA, 2009). The common denominator, which gives rise to the questions raised in this chapter, is that of *teacher collaboration on designing new, concrete instances of teaching.* In the Danish case, the limits of the educational background of the teachers *impose* collaboration, at least at a global level; in the former case, collaboration at very local levels can be said to be integrated in the educational and cultural background of the teachers. Comparing the two contexts is not a question of evaluating them against a (probably non-existing) common scale, but rather to identify the differences and the effects of institutional constraints which seem to cause them, and discuss the solidity of these constraints. To do this, we apply the model of *didactical determination* proposed by Y. Chevallard (2002) and further developed in the context of international comparison (Artigue & Winsløw, submitted) as well as a more recent model of *epistemic systems* (Winsløw, in press) on which the various levels of determination act.

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Authors: Ghislaine Gueudet and Luc Trouche

Title: Communities, documents and professional geneses: interrelated stories

We extend in this chapter the presentation of a *documentational approach of didactics* whose main concepts have been already exposed (first chapter Gueudet & Trouche), in considering teacher's activity as essentially social.

Teachers' collective work is always an outcome of a cultural and social history, as it appears clearly for the Japanese Lesson Studies (chap. Winslow), it can be fostered by associations or institutions, and can also be spontaneous. We particularly examine how these social aspects are illuminated and spurred by the development of digital medium.

We use the framework of *communities of practice* (Wenger 1998) to grasp the constitution processes of professional collectives. The *participation/reification* dialectic allows to encompass the complexity of joint documentation processes in emerging teachers' groups. We illustrate these processes through some particular devices of mathematics teachers' collective documentation work (Gueudet and Trouche 2008). As (Sensevy chapter) does, we evidence that participation requires to recognize a common project, and that reification cannot be understood as a crystallisation of congealed objects, but as a breeding of living entities. We propose the concept of *community documentational genesis* to embrace the mirror development of a community and of its documention.

We more precisely study what is produced by a collective documentation work within a community. In traditional environments, the textbooks constitute a document template (chap. Pepin), guiding teachers' documentation. In digital environments, such templates do not exist, but *models* appear in the course of the collective documentation work, both as its produce and as a condition for its

development. We analyze the genesis of such models, related to the history of organizations and resources and we question the notion of *meta-design* (Ficher & Oswald 2003).

We analyze the relationships between individual and community documentation, particularly from the point of view of documentation systems. We study the way in which these systems are influenced by collective work and question the relationships between documentation and professional geneses. We formulate consequences of our analyzes in terms of *methodological assistants* to support teachers' documentational geneses, particularly in the case of teachers' training devices.

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Author: Gérard Sensevy

Title: Patterns of didactic intentions: community of practice, institution, collective thought?

This contribution comes within the framework of the Joint Action Theory in didactics (Sensevy & Mercier, 2007). In this theory, the teacher's and students' work may be viewed under the description of two articulated moments: the *in situ* didactic activity, when the teacher have the students playing the didactic game; the preparation of this activity, when the teacher is building the game to be played. With this respect, this paper is devoted to understand the system of resources which shapes the didactic intentions. This system could be viewed as an interface between the building of the game and the effective game that the teacher institutes. We study particular teachers'practices while they are implementing in their classes specific courses.

In the first part of this chapter, elaborating on Baxandall's work (1991), we first describe a general frame aimed at understanding intentions and the way they are founded in the resources milieu. We specify this frame to didactic transactions by determining how one can understand the didactic intentions by using a strategic rule/strategy doublet aimed at taking up and reorganizing the brief/charge doublet proposed by Baxandall.

In the second part of the chapter we use the theoretical structure we worked out previously.

We attempt to show how teacher's intentional structures are shaped in the assemblage of certain factors : the epistemic prefiguration by the text of Knowledge; the way this text is reelaborated in the collective work of studying and comprehending; the didactic memory of the teacher in its different components; the monitoring she produces on the spot according to the respect of fundamental didactic equilibria; the way the collective discussed the teacher's didactic implementation. Thus, by following the intentional structure as we found it in our study, we try to acknowledge the system of resources which enable her to get her didactic agency.

In the third part of this chapter, we highlight the following fact: one cannot understand the teacher's action we studied without referring it to the collective whose she is a member. We thus characterize some ways of doing of this collective, by describing some of its peculiarities. In doing so, instead of employing the rather interesting notion of community of practice, we prefer using the notions of institution (Douglas, 1986), institutional milieu, and though style (Fleck, 1934/1986; Sensevy et al., 2008)) to give an accurate account of the production of resources and of their enactment in the intentions. We try to draw some conclusions from this epistemologic and theoretical choice.

In the last part of the chapter, which try to delineate a kind of normativity, we envision how such a collective, as we describe it through its impact on specific individuals, could inspire new ways of working, which gather teachers and researchers. We show didactic and anthropologic implications of such an endeavor.

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Author: Jana Visnovska and Paul Cobb

Title: Mathematics teachers as instructional designers: what does it take?

In this chapter, we draw on five-year interventionist professional development study that we conducted with a group of middle-school mathematics teachers in the USA (Cobb, McClain, Lamberg, & Dean, 2003; Dean, 2005; Visnovska & Cobb, 2009). The final fifth year of the study involved a performance assessment in which the teachers collectively designed an instructional unit on statistics in which they sought to both capture what they learned in the professional development group and to address the objectives for middle-school statistics prescribed by the standards and objectives for mathematics in their State. This performance assessment provided a window into the teachers' documentation work (Gueudet & Trouche, 2009), especially to the resources the teachers needed to engage in this type of design work effectively.

Although some changes in teachers' documentation work necessarily occur whenever teachers use new instructional materials for the first time, designing resources to proactively support the emergence of *specific* shifts (e.g., towards practices that the research on learning indicates are effective) remains a non-trivial task. This is in part due to the complexity of resources involved in a productive instructional design. To illustrate such complexity, we outline the set of resources that the teacher group used routinely in the last year of our study, and draw contrasts with the resources used in the group's documentation work two years earlier. We draw on the process of documentation genesis as described by Gueudet and Trouche (2009) to explain how these seemingly same resources (e.g., instructional sequence in statistics, co-participating teachers, the research team members, classroom video) were used to generate significantly different meanings in the group activities over time. We thus illustrate and substantiate the argument that teachers themselves, in working with and re-working different resources, play the key role in developing the sophisticated documents that are needed to facilitate their instructional improvement efforts (cf., Rabardel & Bourmaud, 2003).

We then focus on the process by which "working with and re-working different resources" contributed to the increasing sophistication of the documents developed by the teachers in our study. Specifically, we highlight that the group constituted a professional teaching community (Dean, 2005; Wenger, 1998) characterized by a shared purpose, repertoire of ways of reasoning with tools, and productive norms of mutual engagement. We illustrate how the specific practices that became normative in the group made it possible for the teachers to problematize their underlying assumptions about teaching and learning mathematics and thus gradually re-work the very basis for their instructional decisions. Importantly, in this particular group, the development of productive norms of mutual engagement alone took 19 months (Dean, 2005).

To conclude, we seek to problematize the rhetoric of *teachers as instructional designers*, cautioning that it is often used to suggest that teachers should be competent to design coherent instructional sequences immediately and without ongoing support. As the case that we present illustrates, the complexity involved in effective instructional design indicates that such rhetoric is both romantic and unviable. Teachers as effective instructional designers and the associated professionalization of teaching is an accomplishment rather than a starting point.

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