The experimental dimension in mathematical research problems

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November 12, 2007

EXPRIME

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- Claire TARDY, formatrice d'enseignants (IREM,IUFM)

Theoretical references :

- theory of situations of Brousseau
- mathematics
- the place of the mathematical objects (Quine, Frege, Gardies)

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Research problem

"In a mountain hike one needs a goal, but the main interest of hiking is not the goal but the walk"
Jean-Yves Girard (2007)

Using "research problems" in the classroom in order to develop

- scientific methods;
- heuristics;
- reasoning

They can, also give sense to some concepts
and make the learners construct mathematical notions by themselves

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Research questions

1. starting with "classical" research problems;
2. studying the mathematical concepts possibly taught through the resolution of the problem;
3. going from a mathematical situation to a situation for the students;
4. focusing on the experimental dimensions in problem solving

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Methodology

Elaborating problems or fields of problems

- linked with mathematical concepts
- allowing the students to shuttle between the experimental part of the research and the structured construction of mathematical knowledge

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Methodology

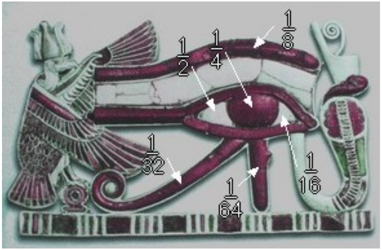
develop specific tools allowing us to understand and to analyse how the mathematical web is spun all around the objects which are mobilised in the problem solving

The school context

The results are based on observations experienced in classrooms with pupils and students between secondary school and university

Three examples

To break down 1 into the sum of fractions of numerator 1.



Can you find two distinct integers a **et** b **such as:** $1 = \frac{1}{a} + \frac{1}{b}$**?**

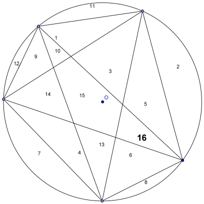
Can you find three distinct integers a, b **and** c **such as:** $1 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$**?**

Can you find four distinct integers a, b, c **and** d **such as:** $1 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$**?**

Continue...

Three examples

Regions inside the circle



To find the maximum number of regions inside a circle by joining n points of a circle;

Three examples

Polya urns.

An urn contains two balls: a white one and a black one. At each time a ball is drawn from the urn. The contents of the urn are then altered, puting one more ball in the urn of the same color of the drawn ball. Study the dynamic of the urn.

Experiment in mathematics

Solving problems: linking a mathematical situation with our own knowledge (Dias, 2005)

Experiment in mathematics

Our experiment field is constructed on our personal knowledge, on familiar objects and uses, more or less, familiar tools (particularly true for technological tools)

Experiment in mathematics

- $\frac{1}{a} + \frac{1}{b} = 1$ hence $a - b = ab$
- no $a + b = ab$
- then $-a - b = -ab$
- we have made a great headway! (*laugh*)
- I hope we are not going to have something like that in our next assesment!

V : The smallest fraction with integers is one half; you can't have a smaller one.

V : 1 isn't possible, the smallest fraction is $\frac{1}{2}$.

T : Can you explain?

V : For the second question: $a = 2, b = 3, c = 6$ for the third question: 3 $a = 2, b = 3, c = 9, d = 18$

R : Why is the first question impossible?

V : Write the first *(fractions with numerator 1)*

R : And why is it impossible?

V : In any case, a must be 2

R : Perhaps, is there an other fraction with a numerator 1 which gives 0.5...

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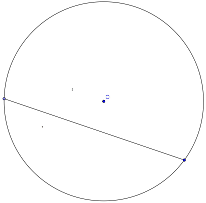
Experiment in mathematics

Experiments allow us to conjecture but also to **refute** conjectures;

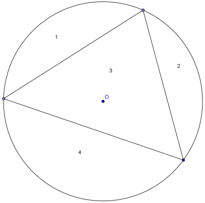
The number of regions for 2 points is :

n	2
nb	2



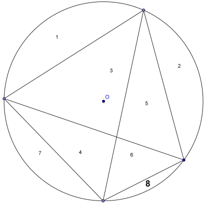
The number of regions for 3 points is :

n	2	3
nb	2	4



The number of regions for 4 points is :

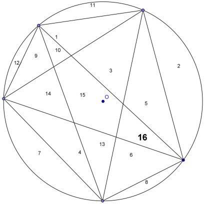
n	2	3	4
nb	2	4	8



Experiment in mathematics

The number of regions for 5 points is :

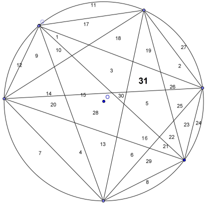
n	2	3	4	5
nb	2	4	8	16



Experiment in mathematics

The number of regions for 6 points is :

n	2	3	4	5	6
nb	2	4	8	16	31



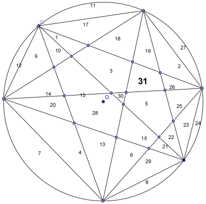
Experiment in mathematics

$$nb_n = 1 + \binom{n}{2} + \binom{n}{4}$$

Experiment in mathematics

It's also possible to do an experiment in the field of the graph theory

Completing the graph in order to make it planar



Hence we have to count:

- the vertices :

$$n + \binom{n}{4}$$

- the edges :

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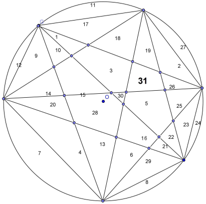

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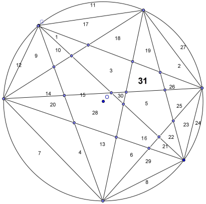

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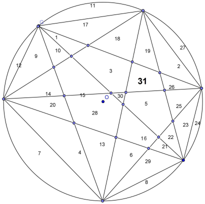

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Using the Euler's formula :

$$F = E - V + 2$$

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Using the Euler's formula :

$$F = n + \binom{n}{2} + 2\binom{n}{4} - n - \binom{n}{4} + 2$$

Using the Euler's formula :

$$F = \binom{n}{4} + \binom{n}{2} + 2$$

Using the Euler's formula :

Hence, the number of inside faces is:

$$F = \binom{n}{4} + \binom{n}{2} + 1$$

A difficulty is to transform a productive mathematical situation into a productive didactical situation

"Setting up a real experimental approach, in which the different steps are not bypassed needs appropriate situations and a demanding didactical management"
Michèle Artigue, 2007

Egyptian fractions

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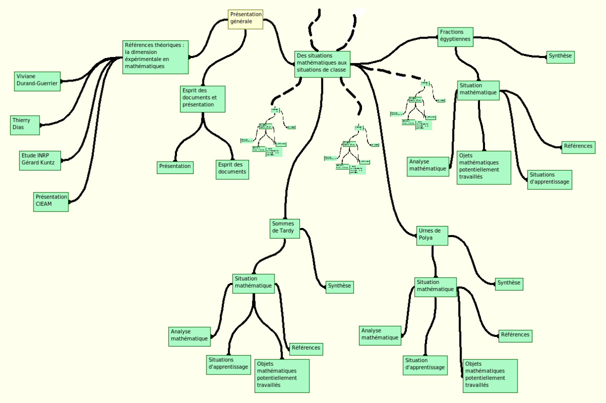
The aim of our work consists in suggesting class organisations linked with didactical variables of the mathematical situation



Go

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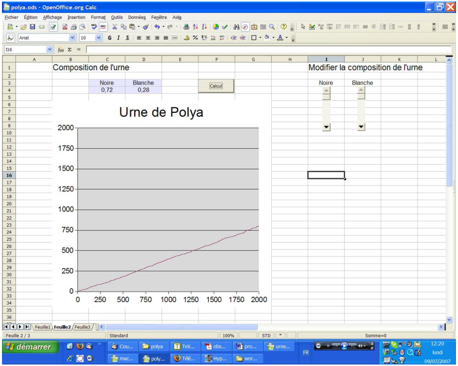
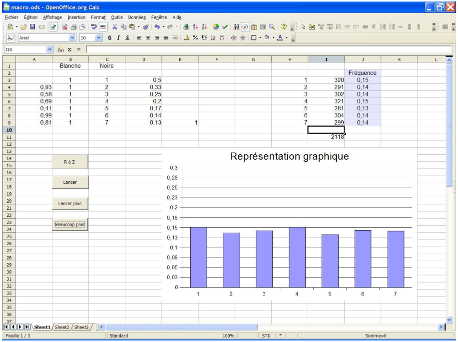


Polya's urn


macro.ods - OpenOffice.org Calc
Blanche
Noire
Fréquence
R à Z
Lancer
Lancer plus
Beaucoup plus
Représentation graphique
Sheet1
Sheet2
Sheet3
Feuille 1 / 3
Standard
100%
STD
Somme=0

Link between experiment and knowledge's institutionalization

- The experiments are local;
- Institutionalization has to take into account the different approaches;
- The mathematical analysis helps to understand these different approaches.

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* Distance between experiments and institutionalized objects;
  "*Are you sure there is a real law*" Student bac+1
* Distance between simulations and experiments

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