A RESOURCE TO SPREAD MATH RESEARCH PROBLEMS IN THE CLASSROOM

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Abstract : in this communication we intend to present a digital resource the aim of which is to give aid to teachers to use research problems in their classes; in a first part we are going to present the theoretical framework which was used by the team in the conception of the resource and the consequences on its model; we will present the results of a study dealing with the role and the impact of the resource used by teachers preparing lessons.

INTRODUCTION

Different works have shown the benefits of the use of research's problems (Polya, 1945), (Schoenfeld, 1999), (Brown and Walter, 2005), (Harskamp and Suhre, 2007), (Arsac and al., 1991), (Arsac and Mante, 2007), in the construction of knowledge and both the interest of teachers and the difficulties to deal with in the classrooms; moreover, the institutional injunctions of using research problem are important in France and are going to take part in the final evaluation of the secondary school (Fort, 2007).

As far as we are concerned, and in the framework of the Piagetian psychological theory, we assume that the construction of knowledge has to go through an adjustment to the milieu as we will define it in the next section, and in this context, research problems are elements of the "material milieu" that teachers offer to learners.

We also assume that amongst all hindrances of generalization of research problems in the classroom, the following points are decisive:

1. the important part of the experimental dimension in problem solving clashes with the main representation of mathematics amongst maths teachers but also in the society;
2. the focus on heuristics and reasoning skills in maths research problem is in contradiction with the institutional constraints of teaching maths notions, particularly regarding French maths curricula;
3. difficulties for teachers to pick out in the students' activity the mathematics part of their work, and, as a result the notions which can be institutionalized;
4. the difficulties teachers have to assessed such a work, the usual assessment modalities being not appropriate.

In this context, a team of researchers from different institutions (IREM de Lyon, IUFM de Lyon, INRP and LEPS[1]), has worked on the construction of a numerical resource the aim of which is to give aid to maths teachers in order to use research problems in their teaching. In this paper, we will present the main theoretical frameworks used in the construction of this resource and will show, through the results of a particular study, the role this resource can play in the activity of teachers from the preparation of a lesson to the implementation in the classroom.

THE THEORETICAL CHOICES

This resource was written to be a part of the milieu of the teachers in the meaning Brousseau (Brousseau, 1986a, Brousseau, 1997, Brousseau, 2004) and after him (Margolinas, 1995, Bloch, 1999, Bloch, 2005, Houdement, 2004) give to this concept. More precisely, learners learn through regulations of their links with their milieu. Going a bit deeper in this concept, Margolinas (Margolinas, 1995) described the structure of the milieu as a set of interlocked levels which can be described as follow:

Level	**Teacher**	**Pupil**	**Situations**	**Milieux**
3	Noosphere-T		Noospherian situation	Construction milieu
2	Builder-T		Construction situation	Milieu of project
1	Project-T	Reflexive pupil	Project situation	Didactical milieu
0	Teacher	Pupil	Didactical situation	Learning milieu
-1	Teacher in action	Learning pupil	Learning situation	Reference milieu
-2	Teacher observing	Pupil in action	Reference situation	Objective milieu
-3	Teacher organising	Objective pupil	Objective situation	Material milieu

Table 1 Structuring of the milieu

[1] IREM : Institut de Recherche sur l'Enseignement des mathématiques ; IUFM : Institut Universitaire de Formation des Maîtres ; INRP : Institut National de Recherche Pédagogique ; LEPS : Laboratoire d'Etude du Phénomène Scientifique, Université de Lyon.

In this table, the milieu of level *n* is the situation of level *n-1* and is made up of the existing relationships between *M*, *P* and *T*. Using the symmetry of the table and, in our case, proposing to the teachers a situation, (in the acceptation of the didactical theory of situations) in which the a-didactical situations of action had as aim to allow teachers to construct, by themselves, the knowledge necessary to conduct a situation of problem research in the classroom (Peix, 1998), we speak of the material and objective milieu of the teachers. In this study, the resource appears to be a part of the material milieu of the teacher and the question is: is it possible, for a teacher, to use the resource to facilitate his tasks:

- Organizing the material milieu of the pupils
- Understanding the objective milieu of the pupils and the links between their knowledge and conceptions
- Choosing the pertinent notions to be institutionalized in the reference milieu of the pupils, and anticipating the conflicts between misconceptions and tools to solve…

Moreover, the theoretical framework of cognitive ergonomics through its concepts and methods allows us to study the competencies of the teacher in his interaction with the worksystem, and more particularly in the relationship between the prescribed tasks and his activity. Lastly, and in the field of using a numerical resource in professional tasks, the concepts of utility, usability and acceptability have been sounded out in two different ways:

- by an evaluation by inspection in order to construct and organize the resource,
- by an empirical evaluation in a professional situation.

Utility is “the question of whether the functionality of the system in principle can do what is needed” (Nielsen, 1993)

Usability can be defined as: “the capability to be used by humans easily and effectively” (Shackel, 1991), but also “the question of how well the users can use that functionality” (Nielsen, 1993)

Acceptability refers to the decision to use the artefact, and answers the questions: is this artefact compatible with the culture, the social values, the global organisation in which the artefact has to be included.

PRESENTATION OF THE RESOURCE

Structure

It is possible to use this resource in different ways; theoretical texts about the experimental dimension in mathematics (Dias, 2005, Kuntz, 2007) can be read as well as different presentations made in conferences. It is also possible to understand the sense of the resource by reading a curriculum-vitae of the different steps and

reflections of its building. The different situations are outlined using a common structure:

- Maths situation out of the classical literature on open problems developed in particular in IREM de Lyon (nowadays, there are seven maths situation):
    - Egyptian fractions: break down 1 into the sum of fractions of numerator 1.
    - Trapezoidal numbers: study of the sum of consecutive whole numbers.
    - The river: study of the shortest distance between two points with constraints.
    - The number of zeros of n!: study of the digits of n! in different numeration systems.
    - The greatest product: study of the product of integers of fixed sums.
    - Polya's urns: study of the dynamic of the composition of an urn in a repeated experience.
    - Inaccessible intersection: find a line going through an inaccessible point.
- Maths objects that may be used to solve the given problem: for each of the situations, the a-priori analysis allows to extract the mathematics objects that are part of the mathematics situation and can be used in the process of resolution.
- Learning situation: how the maths situation has been transformed into a didactical situation? In this part of the resource, reports of real experiments can be read.
- References
- Synthesis: a ten pages synthesis of the situation allows teacher to familiarize themselves with the content of the section.
- Connected situations: how is it possible to protract the situation and what are the extensions in the maths researches nowadays?

Introduction of the resource

In order to confirm the hypothesis and to evaluate the utility, usability and acceptability of the resource, we built an experimentation with teachers from the first handover of the resource to the real experiment of a research problem in the classroom. In this section we are going to focus on the first handover in order to evaluate the usability of the resource.

The methodology of this part of the experimentation was built using an observation of teachers faced to a professional problem (preparing a lesson using research

problem) and discovering the resource as an artefact in the sense that the functioning of the resource has not been explained; the observer (the first author of this paper) recorded dialogues and in the same time recorded the computer screen.

There is a confrontation, for the same person, between the position of expert (a teacher preparing a lesson, hence choosing objectives, a problem linked to these objectives, organising time of the lesson …) and the position of beginner in two different ways: using research problem in his (her) preparation and using a new tool. The theoretical framework of the didactic situation theory gave us the possibility to observe the position of the resource in the milieu of the teacher and to observe why this resource gives a possibility to the teacher to have a look into the pupils' objective milieu as described above. The cognitive ergonomics framework gives us keys to analyse the activity of teachers in this professional situation. Moreover, the concept which is tested was the usability of the resource, using the following criterions:

- Possibility of learning the system
- Control of the errors
- Memorization of the functioning
- Efficiency
- Satisfaction

But also, and we will see why later, its acceptability.

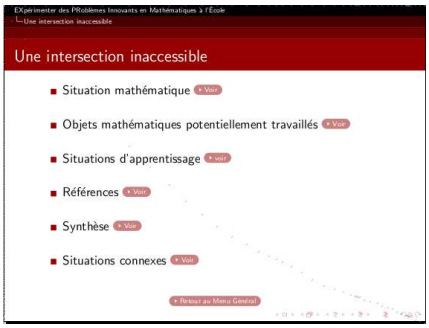
The first result that we can highlight is the very quick adaptability of the observed teachers in front of the resource. After à three minutes wandering, the teachers used the different path in the resource to find exactly what they want as it can be possible to see when teachers changed from one situation to an other. In the first time, the mouse hesitated on the screen, going from one button to the others before the click, and progressively, the structure became clearer and the adequacy between the given objective and the browsing into the resource became safer:

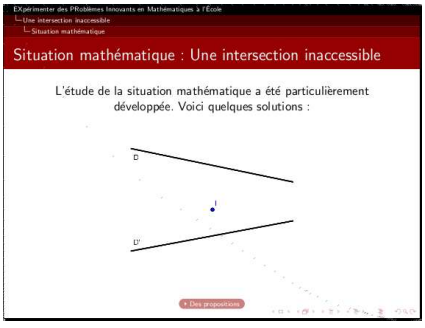
After nine minutes:

Are you interested?

Yes

(*click on "situation mathématique*[2]*"*





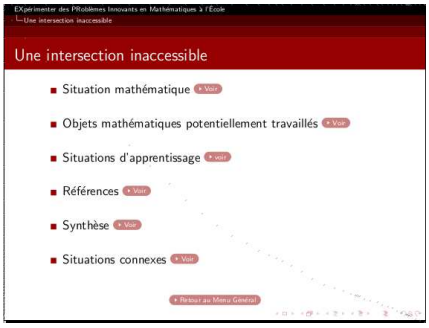
[2] Mathematical situation

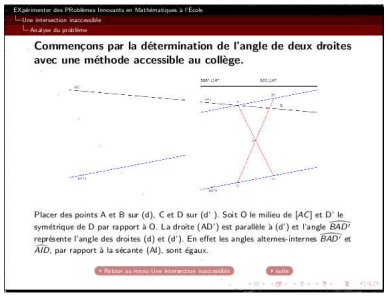
(*two clicks and two screens in one second*)

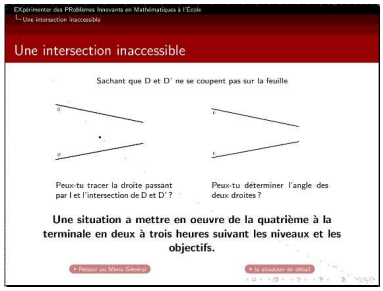
The mathematical situation… (*they read*)

Possible for our pupils (*click, click*)

I would like to see that (*the mouse turn over the menu "possible maths objects…"*)







The second important observation, linked to the concept of acceptability can be seen by the feeling of trust in the authors of the resource; at the beginning of the exploration, the two teachers click on the menu: theoretical framework, and after some seconds says:

"We are not going to read the whole text…"

And, later, in front of a situation, one of the teachers said:

"We are going to read what they say…"

These two brief sentences show us the growing of the confidence during the use of the resource and can be considered as a clue of the acceptability of the resource. The other observations and particularly the use of the resource to construct a real lesson confirm us in the feeling of the acceptability of the resource.

Realisation

In order to go on in the evaluation of the resource, a second experimentation has been built with the goal of testing the utility and the acceptability of the resource; we observed a research problem lesson focusing more particularly on the interactions between pupils and teacher during the situation of action; later we interviewed the teacher.

The chosen mathematical situation was the trapezoidal numbers and the question given to the pupils was:

What are the whole numbers which are sum of at least two consecutive integers?

Interrogating the two theoretical frameworks, the interview with the teacher allows us to bring to light utility and acceptability of the resource, but also to understand the position of this resource in the teacher's milieu.

Utility of the resource is in this case obvious, the teacher having prepared the lesson with the resource:

"Yes, yes I use it… I read all you wrote about this problem. Oh, yes, without the resource, I think I should not give this problem to my pupils, because I would have spent too much time to do this work… I would not do that!"

Regarding acceptability, a lot of clues allow us to consider, for this teacher and in this experimentation, the resource as acceptable, for example the feeling that the lesson created using the resource brought a new dimension to her course:

"I think I'll do that earlier next year, to create something in the class, precisely, this dynamic which makes the pupils actors, as I said to you, a pupil was speaking from the board, and I was at the back of the class, and the other pupils ask questions… I think it's a good way to involve pupils in the maths lessons, to put a lot of them in maths… For me, it's very confident, visibly they enjoy this time, and I think it's something important to insert pleasure in maths lessons, it's something which questions me, because it's so easy to do maths without pleasure!"

Moreover, the interview confirms the position of the resource in the objective milieu of the teacher in a posture of preparation of a lesson including a research problem.

In an other hand the observation of a group of pupils gives us interesting feedback about the mathematical objects students deal with and shows that the *a-priori* analysis of the resource corresponds to the reality of the class; for example, one of the mathematical object which was highlighted by the authors of the resource related to this problem was the powers of two. In other words, the hypothesis was that powers of two belong to the objective milieu of the pupils and, consequently are a field of experiencing; the confrontation of pupils with these objects allows them to change their position in the milieu and to bring with the help of the institutionalisation these objects in the reference milieu of the pupils:

F2: (using her calculator) two to the power five gives thirty two… Yes ; two to the power seven gives one hundred twenty eight

G: two hundred and fifty six, five hundred and twelve, thousand and twenty four, two thousands and twenty eight …

F2: how do you calculate to obtain the results so quickly?

G: you multiply by two

F2: Ah yeah right!

In this small excerpt, the two definitions of the powers of two as an iterative or recursive process are called up and the link between these definitions is made by F2; it is possible to think that the recursive definition belongs now to her objective milieu and a necessary work must be done to institutionalize it in her reference milieu. The fact that this object was present in the resource allows the teacher to pay attention to this dialogue and to use it in her lesson:

CONCLUSION

The described engineering and the results of observations and interviews show the place of the resource in the milieu of the particular teachers involved in this experiment, and clearly show the utility, usability and acceptability of this resource. Regarding the didactical theory of situations, this experimentation shows that the resource emplaced in the material milieu of the teachers can be mobilised in their objective milieu and used in the setting up of research problem lessons in the classroom. The resource also allows teachers to launch themselves in the different milieu of the students and to understand the position of mathematical objects in these milieu.

However, new questions appear, in particular linked to the genesis of this resource and its transformation from an external resource possibly used by a teacher to a document available in his/her environment.

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