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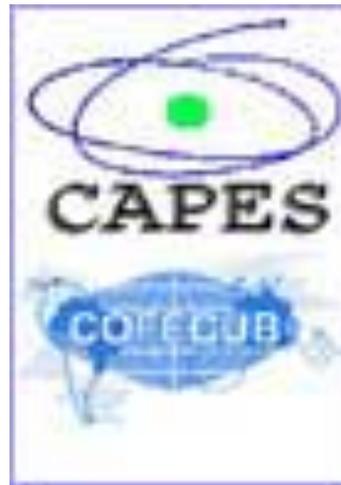
A COMPARATIVE STUDY OF THE SECONDARY-TERTIARY TRANSITION

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Plan

- I. Origin of this work and focus of the research report.

- II. Expectations and rationale

- III. Theoretical Framework

- IV. Research Methodology

- V. Typicality of the tasks and More Global Perspective

- VI. Conclusion

Origin of this work

- This research work situates within a project Capes-Cofecub involving Brasilian and French universities, and entitled:

- **"Study of the professor and his mathematical activity: conceptions and sources in and to his professional acting"**

  whose axis 4 is more specifically devoted to transition issues between secondary and tertiary mathematics education.

Focus of the research report

- In the project, transition issues are approached through complementary perspectives:
  - Curricular comparison
  - Analysis and comparison of students' and teachers' representation and practices
- The research report focuses on curricular comparison, considering more especially the functional world in the two countries at this secondary-tertiary transition.

Expectations and rationale

- **What is expected from this comparison?**

  - a better understanding of the transition issues in these two countries

  - a clearer view of the potential and limits of didactic actions carried out and of educational resources developed in one context to address transition issues in the other one.

  - a better differentiation between the general and the contextual in transition issues, in order to limit the risk of over-generalizations so frequent in the educational area.

Expectations and rationale

- **Why focusing on the functional world?**

  - Because of its fundamental role in mathematics and in mathematics education.

  - Because existing research shows that important changes in the relationship with functions generally occur in the transition

Theoretical Framework

- The main theoretical framework is ATD (Anthropological Theory of Didactics), and especially two main constructs of this theory :
    - the notion of praxeology
    - the notion of hierarchy of levels of co-determination
- ATD seems especially adapted to our study as it considers that praxeologies depend on the institutions they live in, and as it offer tools for analysing the complex system of conditions and constraints conditioning the ecology of knowledge.

◆ Primitive terms to describe mathematical knowledge

PRAXEOLOGY

PRACTICAL BLOCK:
'know-how'

THEORETICAL BLOCK:
'knowledge'

Types of problems or problematic tasks

Techniques

Any task is performed by a technique

'Technology'
(technè + logos)

Theory

Any practice needs to be described, explained, justified
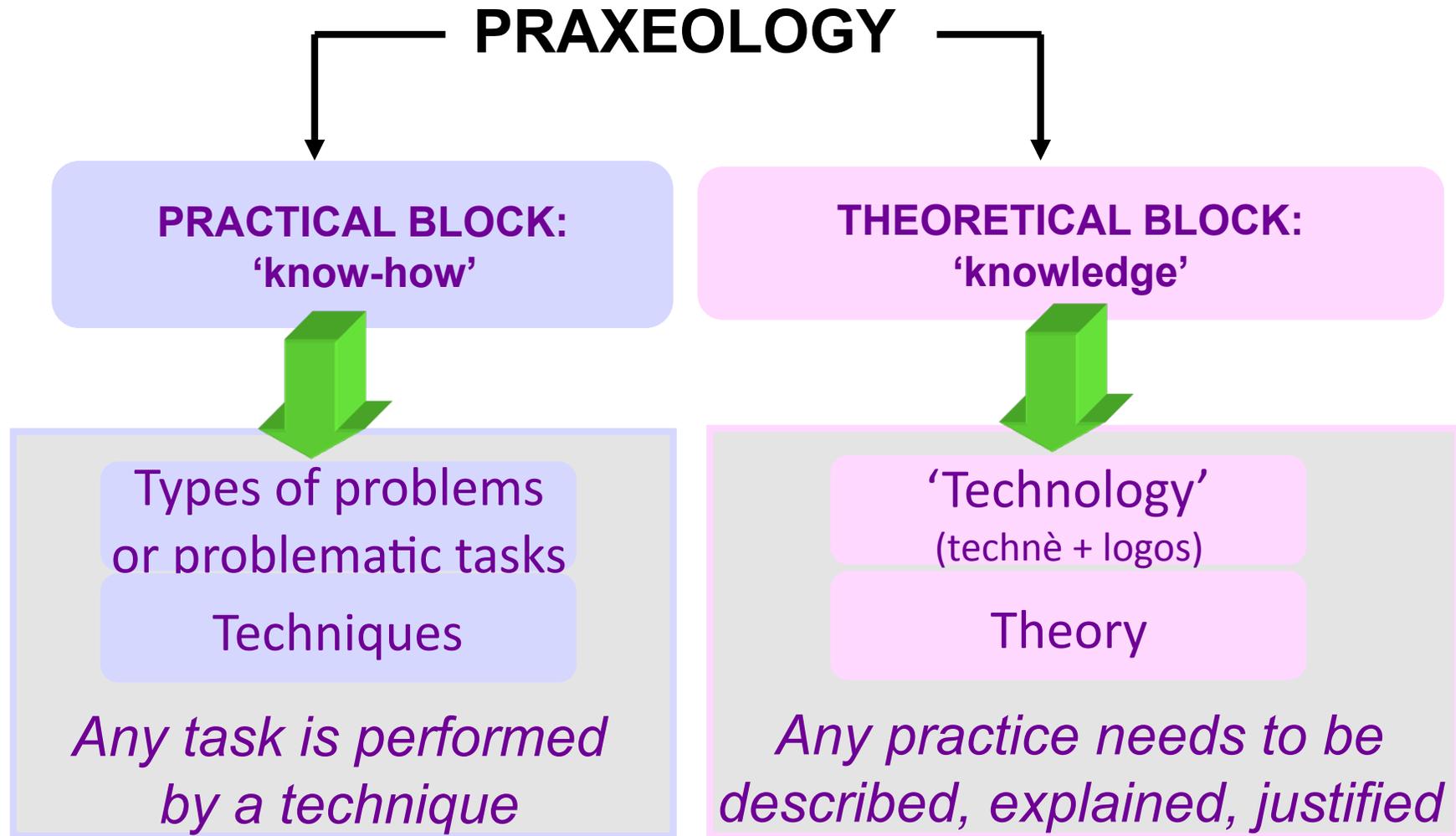
GENERAL MODEL OF HUMAN PRACTICES

LEVELS OF CO-DETERMINATION

Civilisation

Society

School

Pedagogy

Discipline

Domain

Sector

Theme

Subject

Work with numerical data

GRAPHS, IMAGES

Opinions: "look and see"

Competences

Experience and observation

Creativity

Working in groups

Computer assisted

MATHS

Measure of quantities

Functions

Geometry

Exponential function

PEDAGOGICAL

MATHEMTIC - DIDACTIC
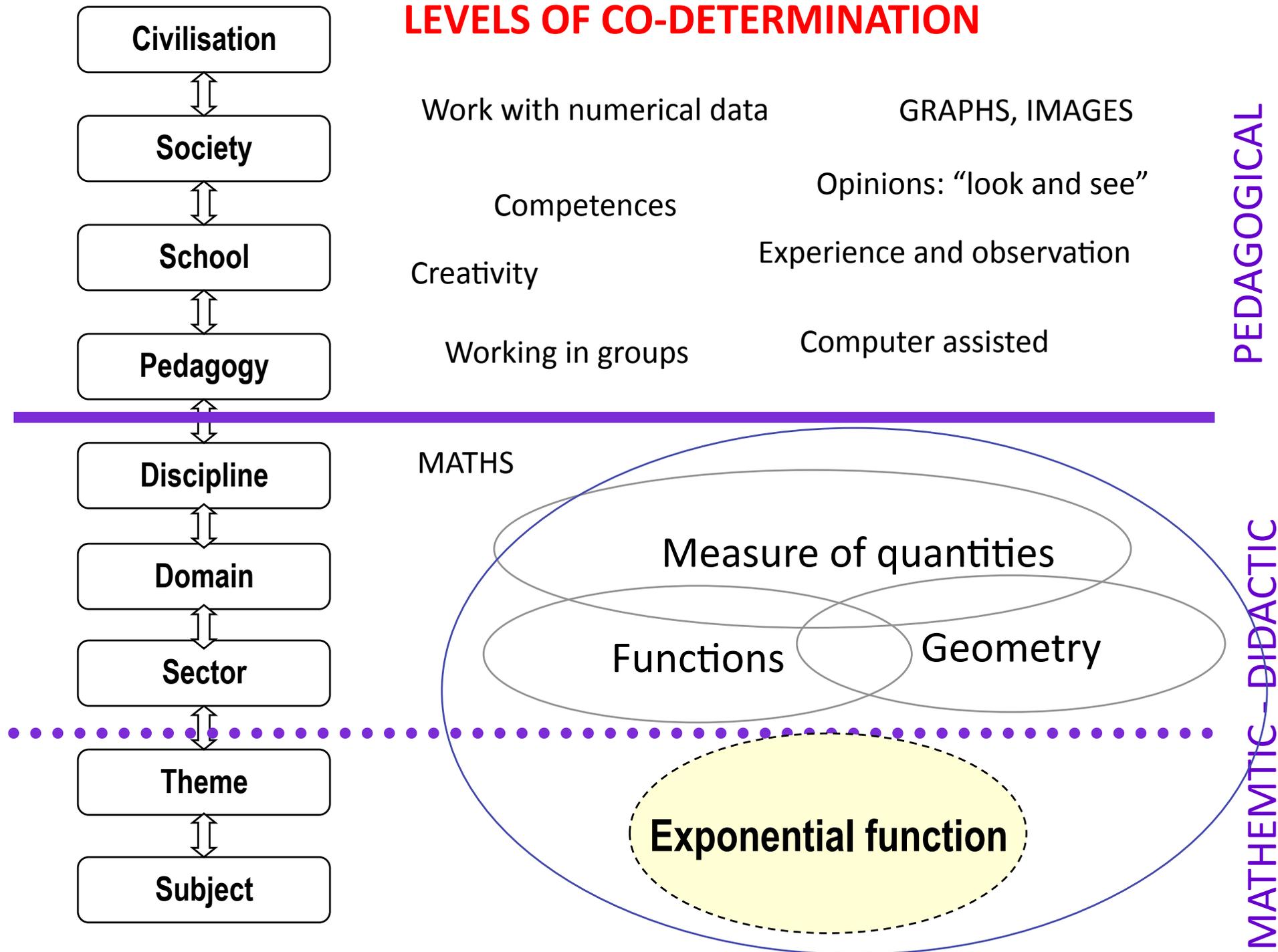
Research Methodology

. The project methodology combines different approaches, organized around:

- an institutional approach structured around the analysis of curricular structures and resources in the two countries with particular focus on the last years of secondary education and the beginning of tertiary education, and on the study of institutional expectations regarding functions at the end of high school and entrance to university through national and regional assessment tools;

- an approach of the personal relationships developed by the students with the concept of function, and also more globally of their representation of mathematics, its teaching and learning, and their perception of the secondary-tertiary transition, through common questionnaires and interviews;

- an approach of continuities and discontinuities between teaching practices in secondary and tertiary institutions in the two countries.

The specific sources used for this research report

- Standard curricular documents,

- two complementary sources of data:

  - for Brazil, the evaluations used for the selection of students at university entrance, and more especially ENEM, FUVEST;

  - for France, baccalauréat proofs for the scientific section.

- In the two cases, data have been collected and analysed over the last ten years, in order to allow the identification of regularities.

THE BRAZILIAN AND FRENCH EDUCATIONAL SYSTEMS

- **Similarities**;
- the global structure for education: 5 ("fundamental 1" , 4("fundamental 2"), 3 (secondary)
- **Differences**:
- no differentiation in secondary education in Brazil / differentiation from grade 11 in France
- national proposal for curricula In Brazil / national curricula in France
- national secondary examination giving access to university in France / national examination ENEM and specific examination organised by universities ("vestibular")

Contrasting three typical tasks

Task 1: FUVEST 2004. Two functions f and g are defined by their algebraic expression: $f(x)=x^2-2x+1$ and $g(x)=mx+2m$, m being a real positive number. Students are asked to draw the graphical representation of f and g for m=1/4 and m=1, and then to find the roots of the equation $f(x)=g(x)$ for m=1/2, and finally to determine the number of roots of this equation according to the value of m.

What is typical in this task?

- The functions involved are linear, quadratic and absolute value.

- One of the functions depends on a parameter.

- Students are asked to draw a graphical representation from the algebraic expression.

- There is a progression in the questions, based on the use of particular values of the parameter.

- The technical requirements in terms of algebraic work are not obvious, in particular due to the presence of absolute values and parameters.

- **Task 2:** *Bac S* **in 2009**. A function on $[0,+\infty[$ is given by its algebraic expression $f(x)=\ln(1+xe^{-x})$ and a graphical representation. In part I, students are asked to prove that its limit in $+\infty$ is 0, that its derivative has the sign of $1-x$, and to study its variation. In part II, the integral of f between 0 and a real positive number λ is introduced and denoted $A(\lambda)$. Two methods are then proposed to find an upper-bound of $A(\lambda)$. In the first one, students are asked to represent the area corresponding to $A(\lambda)$ on the graphical representation given to them and prove that $A(\lambda)\leq\lambda.f(1)$. In the second method, students are asked to calculate the integral between 0 and λ of xe^{-x}, integrating by parts, then, admitting that $\ln(1+u)\leq u$ for u positive, to prove that $A(\lambda)\leq-\lambda e^{-\lambda}-e^{-\lambda}+1$. They are finally asked to use the two methods for finding upper-bounds of $A(5)$ with a precision of 10^{-2} and say which is the best approximation of $A(5)$ between these limits.

What is typical in this task?

- The function proposed involves logarithms and exponentials.

- The first part is oriented towards the study of its variations, using derivatives and calculating limits.

- The second part involves an integral. It asks for an estimation of it based on the use of functional overestimations and the manipulation of inequalities.

- The exercise ends by a calculation requiring the use of a calculator.

- Many intermediary results are given and students are guided by several intermediate questions.

- The level of technical expertise required is limited.

ENEM

- Suppose that the exponential function $y = 363e^{0,03x}$, where $x = 0$ corresponds to the year 2000, $x = 1$ corresponds to the year 2001, and so forth, and y is the population in million inhabitants in the year x, is used to estimate the population aged 60 or older in developing countries between 2010 and 2050. So taking $e^{0,3} = 1.35$, it is estimated that the population aged 60 years or more will in 2030 between
  A 490 and 510 million.B 550 and 620 million.
  C 780 and 800 million.D 810 and 860 million.
  E 870 and 910 million.

- In ENEM functions are always linked to some "real" context and applications, and the same occurs in most textbooks.

More Global Perspective

- Our study shows that :

  - the functions are essentially considered of one real variable, but the habitat of functions differs between France and Brazil. Analysis in France and algebra in Brazil.

- This leads to different organizations between subjects, themes, sectors and domains.

More Global Perspective

- In Brazilian secondary education:

    - **sector level** - functions

    - **domain level** - algebra

    - **theme level** - polynomial functions, and
        especially those with degree 2

    - **subject level** - the algebraic resolution of
        equations involving such functions

    - introduction of parameters plays an essential
        role in the complexification of students'
    activity   for a given praxeology and in access to
        generalization.

More Global Perspective

- In the French context :

  - **sector level** - functions

  - **domain level** - analysis

  - **theme level** - exponential and trigonometric
    functions beyond polynomial functions

  - **subject level** - the study of the variation of
    functions including asymptotic behavior
  and the   search for extrema

More Global Perspective

The influence of higher levels of co-determination:

- In **France**, differential and integral calculus was part of the secondary curriculum since the 1902 reform whose ambition was to give scientific humanities a similar position to classical humanities.
- The focus on approximation appears in the sixties, the balance between quantitative and qualitative approaches reinforces from the eighties
- In **Brazil**, the observed characteristics also reflect a tradition of algebraic teaching in secondary schools whose origin can be traced up to the turn of the nineteen century. Analysis becomes a domain in tertiary education.

Coherence between functions and sequences

- In the Brazilian context, for the central theme of recurrent sequences a typical task is the determination of terms and reason for arithmetic and geometric sequences, on the basis of some partial information. The associated praxeologies are based on algebraic techniques and technology.

- In France, a typical task is the study of the convergence of such sequences both qualitatively and quantitatively. The associated praxeologies use analytic techniques and technology.

Conclusion

- We have tried to show how :

  - TAD can be used, in a comparative perspective, for investigating similarities and differences between two educational cultures regarding functions at the transition between secondary and tertiary education

  - Contextual characteristics situated at different levels of co-determination contribute to these .differences.

- Becoming aware of such contextual influences which tend to remain invisible to those who stay inside a given educational system seems to us crucial for envisaging productive collaborative work, and also for envisaging evolutions inside a given system.