Which mathematics for mathematics teachers and how should it be taught ?

Nicole Bopp
IUFM (Institut Universitaire de Formation des Maîtres) d'Alsace
IRMA (Institut de Recherche Mathématique Avancée)
Strasbourg

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Introduction

Teacher training in France

⟶ : one year

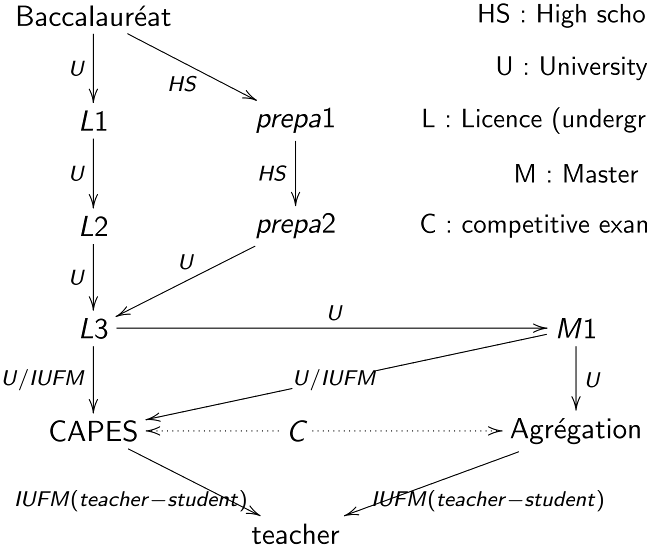
HS : High school

U : University

L : Licence (undergraduate)

M : Master

C : competitive examination



Introduction

Aim of the talk

To give a description of standard training from baccalauréat to CAPES of mathematics teachers (left column).

- To highlight some of the difficulties in mathematics for students and (young ?) teachers
- Are university courses in mathematics relevant for future teachers ?
- To convince my colleagues that teacher training is not just a subset of future researchers training.

Some difficulties

I will give some examples of questions

- to which most students don't know the answer,
- which they never ask themselves.

I will speak about

- students who have graduated in mathematics,
- mathematics taught in junior high school (from 12 to 16 years old).

(Very) elementary examples
Numbers

- Justify $\dfrac{a}{b} = \dfrac{c}{d} \Longrightarrow \dfrac{a}{b} = \dfrac{a+c}{b+d}$?
- Justify the existence of $\sqrt{2}$.
- Why is it the same constant π which occurs in the formulas for the area and perimeter of a circle ?
- Give (a lot of) examples of real numbers.
- Why do we need more than $\mathbb{Q}$, $\mathbb{Q}[\sqrt{p} \mid p \text{ prime}]$ or even $\mathbb{Q}[\pi, e, \sqrt{p} \mid p \text{ prime}]$?

Thanks to Aline Robert for some of these examples

Elementary examples

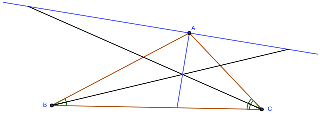
High school geometry

- Give a definition of orthogonal lines.
  - *Using the scalar product*. In junior high school ?
  - *The lines form a right angle* . What is a right angle ?
  - *An angle equal to* 90°. What is an angle ? a degree ?
- Explain the equality $\widehat{AOB} = \widehat{A'OB'}$?
  - *The angle measurements are the same*. Is the result given by a protractor ?
  - *I prefer to speak of angles between vectors*. OK, explain $(\overrightarrow{OA}, \overrightarrow{OB}) = (\overrightarrow{OA'}, \overrightarrow{OB'})$?
  - *There is a rotation with center* O *which transforms* $[OA)$ *in* $[OA')$ *and* $[OB)$ *in* $[OB')$. What is a rotation ?
  - *It is defined by a center and an angle. Oops ! !*

Elementary examples

High school geometry

- The angle bisectors of a triangle are concurrent. Why ?



  - *An angle bisector is the set of points equidistant from two sides*. Which bisector (inside or outside the triangle) ?
  - *The union of two bisectors*. Where is the intersection of two inside bisectors ?
  - *It is inside the triangle*. Why ?
  - *... I should use barycentric coordinates*.

Other examples of this type : position, convexity, ...

How should this be dealt with in the classroom ?
Observe the result, say that it is too difficult to be proved, say nothing ?
The teacher has to be aware of the situation.

Notions not understood

Our students accept the necessity for *local* logic :

proofs to solve exercises ;

using theorems proved in courses ;

verifying proofs of theorems (line after line).

But they don't feel

- the need for *global* coherence

  that's the job of the professor ;
  he *chooses* the order in which results are presented ;
  for students, the results just pile up ;

- the need to ask *why* a particular notion is introduced. For example, why is it interesting

  to recognize a vector space or ring structure ?
  to complete a space ?

Therefore, there exists no link between our "beautiful" courses in linear algebra, real analysis, topology or affine geometry and what they already know .

Notions not understood

An example : beginnings of analysis

Why do we have to go to so much trouble to rigorously prove results used in highschool ?

- Each increasing and bounded sequence is convergent. Why do we introduce a sup ?
- A function with positive derivative is increasing. Why do we use a difficult theorem to prove it ?
- Each continuous function is the derivative of some function. Why do we construct the Riemann integral ? The relationship between area and primitive should not be obvious ?

Then we teach more subtle notions but after 3 years mathematical studies the students

have never seen a construction of $\mathbb{R}$,

have no idea that such a thing exists.

Notions not understood

An other example : quotient structure

The first example explicitely given is $\mathbb{Z}/n\mathbb{Z}$ (most frequently in L3).

How the students will be able to recognize such structure in the following equalities

- $\frac{a}{b} = \frac{a'}{b'}$
- $\widehat{ABC} = \widehat{A'B'C'}$
- $\overrightarrow{AB} = \overrightarrow{A'B'}$

and in the notion of "grandeur" (lenght, area, volume, angles, etc..) ?

Some official texts on this subjects are too difficult to read for most of the teachers.

Some explanations

- not enough personal work
  A french speciality : the best scientific students are not in the university for the first 2 years !
- students aim

  get a degree rather than
  understand and be happy to search (and find !)
- a strategy for success

  "learn" text of theorems
  copy solutions of exercices from previous examinations
- induced by

  the breaking up of our courses
  the type of examinations (only one written examination)
  the type of exercices in examinations
- We encourage not much our students to

  ask (themselves) questions
  ask why some notion is introduced
  use writing to be sure of his own argument

Which offer from universities ?

My informations are coming from systematic inspection of web sites of

- (french) universities (training cursus),
- mathematics departments,
- IUFM.

First observation : CAPES preparations (4th year) are very similar

- about ten written problems (5 hours composition)
- a few summary courses and exercices
- many speachs of students followed by teachers comments (+ use of pocket computers)
- a short training course in highschool

Reason : final examination is national and competitive (800/4000)

- written part : 2 problems (5 hours) ;
- oral part : 2 presentations (1/2 hour).

Which offer from universities ?

Specific training in L3 (for futures teachers)

Second observation : big differences between universities

- ↓ same training for all students in maths
- ↓ from 1 (geometry) to 2 or 3 optional courses (analysis or/and probability without measure theory)
- ↓ a totally independant training : metric spaces (not general topology), differential calculus on $\mathbb{R}^n$, elementary algebra..)

Rarely some courses called « d'ouverture »

- history of mathematics (Lille, Nantes, Nice, Paris 6)
- epistemology (Besançon, Lyon)
- mathematics teaching (Versailles)
- mechanic of solar system (Lille)

Which offer from universities ?

Two explanations

1. The size of the university

A lot of different (mathematics) training in a big one (6 at Paris Sud, 5 at Paris 6,...).

In particular, mutual courses for applied maths and teachers trainings.

2. The distance between IUFM and math department

More great it is, less specific courses exist.

A way to measure this distance is to look if

- there exist some links on the math department web site toward a CAPES page
- courses for oral examination are cutted in two parts : one teached in IUFM, the second one in math department.

Remark : the existence of a team in didactic does not imply specific teacher training (see Bordeaux, Marseille, Lyon)

Which offer from universities ?

Encourage students to work differently

A few examples appearing in universities cursus

- Oral examinations (colles) Paris 7, Grenoble.
- 12 written home works in Toulouse (called "projet")
- are mathematics contained in the other "projets" ?
- work with computer (interactive exercices) : WIMS in Nice, Paris Sud, Caen,..

Some other examples

- Isolated experiments on "débat scientifique" (see Rogalski talk) .
- Methodology courses were introduced in 2000'

  Which content ? logic, structure of a mathematical text, searching and reading references on a subject ?
  Experiment(s) in Strasbourg is not really satisfying.
  Elsewhere ? litterature is poor (see publimath).
- Commission inter-IREM universités ?

Which offer from universities ?

Mathematics from today

Our students don't know

what are doing mathematicians,
applications of mathematics they have studied.

Some attempt to repare this ignorance :

- work on a specific subject with student-teacher
- researchers conference for teachers (IREM)
- journals for teachers

but ..... only a minority of teachers is reached by these actions.

Why not introduce a course on scientific knowledge, for students, for example

- Fourier series, sounds and images
- Radon trasform and scanner
- linear algebra and correcting codes
- scientific calculus and simulation...

Conclusion

- Give us and ask for resources to encourage students to work from the first university year (oral examinations, written home works..)
- Dissertation (on maths) and/or talks as soon as possible
- Encourage students to *search* in groups
- Think cursus in L also for students who will not enter in master
- Introduce scientific knowledge in L.

Obstacle : ideological dislocation among mathematicians

We have to convince some of our colleagues that

- most of our students are able to understand maths,
- it is important to reach high school teachers (give some of their precious time to IREM).

Conclusion

Nevertheless..

In spite of all the deficiencies of our teachers training, there exist great school teachers. I met some of them in IREM.