Numeric tools for learning and teaching mathematics at tertiary level

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Overview

- Principles
- Example of realisations
- Results of use
- The future

1. Principles

Motivations to build/to use such tools :

- Pragmatic motivations
  - To explore new tools and new potentialities
  - To train students to use tools they will have to use in their professional life
  - To link University and technology advancement
- Didactic or pedagogic motivations
  - To use these tools to support theoretical (specific) goals

Six goals

- To rely on picture and more generally on semiotic representations of concepts.
- To favour Constructivist learning approach.
- To promote non standard exercises
- To develop students' autonomy and self regulated learning.
- To allow individualization in the teaching process.
- To support weak students or students not present in the University.

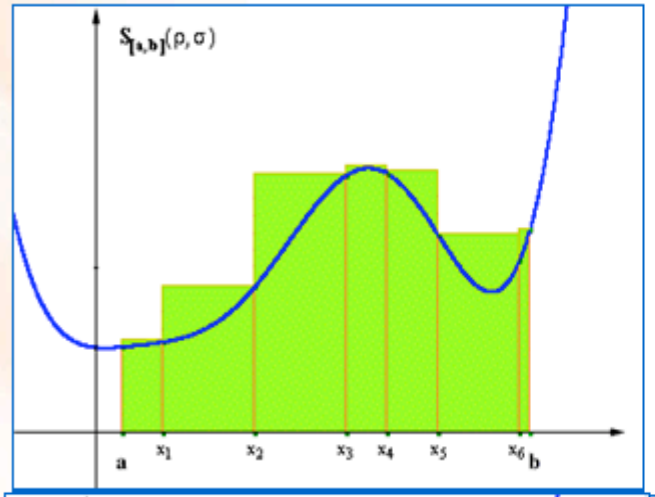
2. Example of realisations

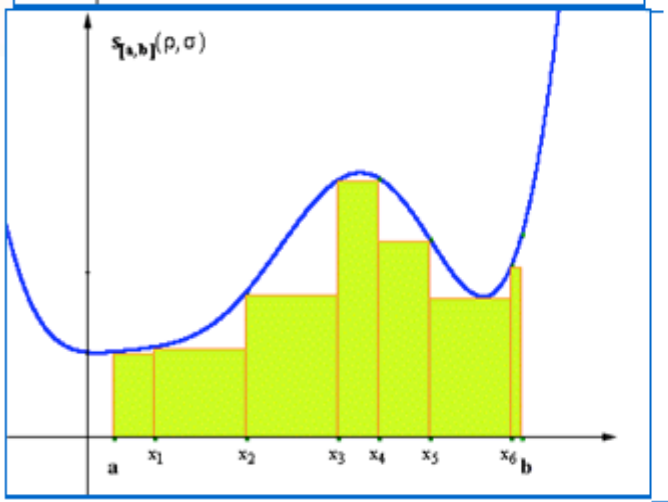
UoL (University on line)

A set of learning objects containing multimedia resources

- Developed since 1998
- Through a partnership between 13 French universities
- 4 topics (Mathematics, physics, chemistry and biology)
- Content of the two first years of University
- Structured in sections: to simulate, to observe, to learn, to train and to assess.
  http://www.uel.education.fr/

b. *somme de Darboux supérieure associée à f et le nombre* $S_{\sigma,\rho}(f,\sigma) = \sum_{i=1}^{n} M_i(x_i - x_{i-1})$

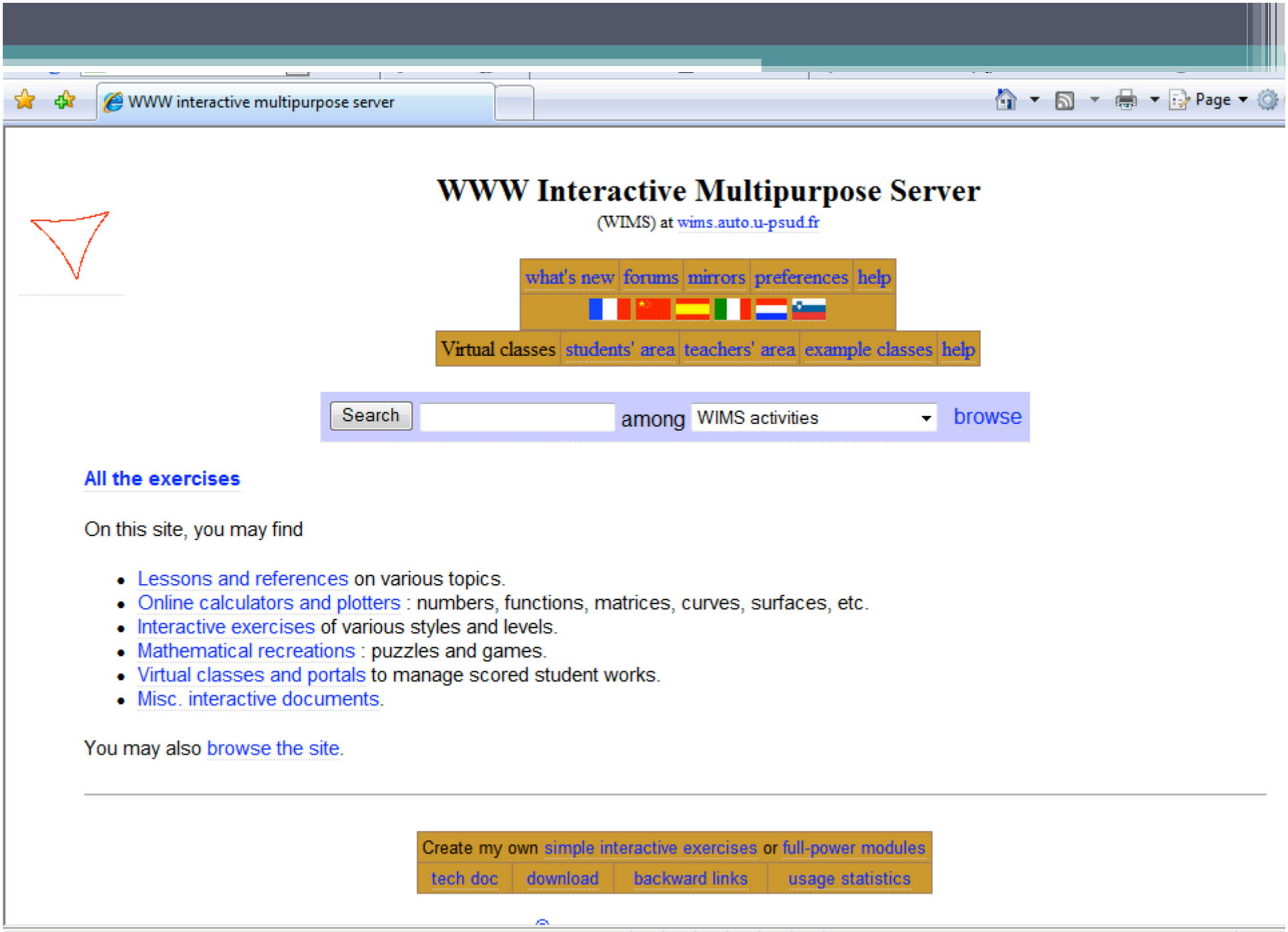




Pour voir la vidéo (2.8 Mo) cliquez sur le lien ou sur l'image

Wims http://wims.auto.u-psud.fr/wims/

- A library of on-line interactive mathematics resources which includes exercises for all levels: from primary to tertiary education.
- A collaborative project available in six languages.
- Teachers open a class and build exercises sheet by choosing exercises in the library or programming new ones.
- Exercises are with random parameters so student may search the same exercise several times.


WWW interactive multipurpose server
Page
WWW Interactive Multipurpose Server
(WIMS) at wims.auto.u-psud.fr
what's new
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mirrors
preferences
help
Virtual classes
students' area
teachers' area
example classes
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Search
among
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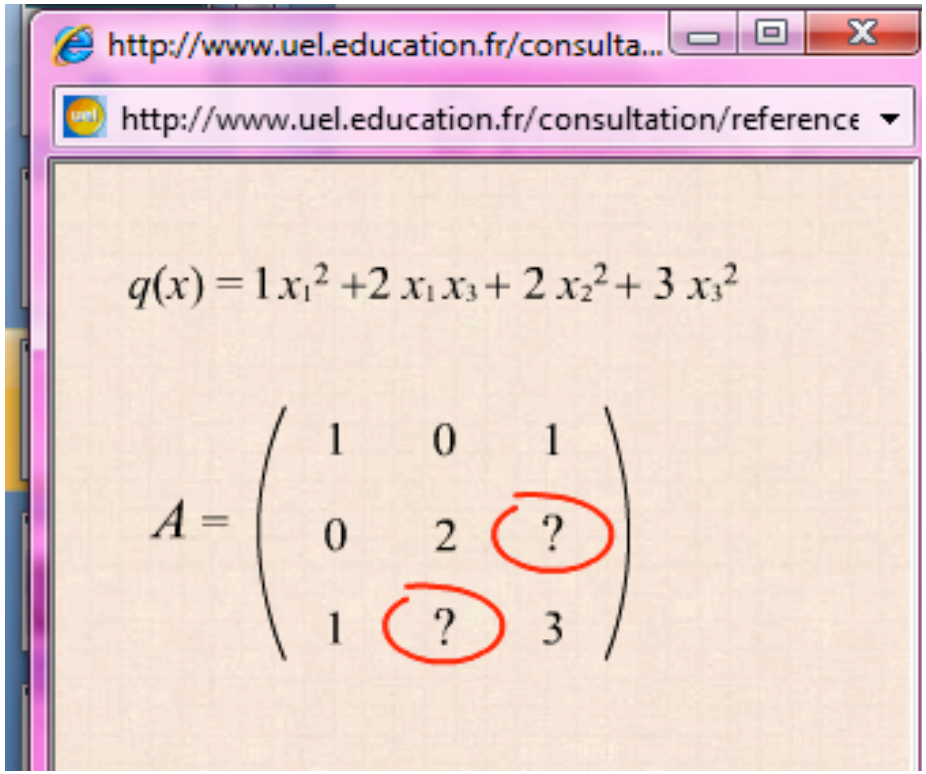
browse
All the exercises
On this site, you may find
Lessons and references on various topics.
Online calculators and plotters : numbers, functions, matrices, curves, surfaces, etc.
Interactive exercises of various styles and levels.
Mathematical recreations : puzzles and games.
Virtual classes and portals to manage scored student works.
Misc. interactive documents.
You may also browse the site.
Create my own simple interactive exercises or full-power modules
tech doc
download
backward links
usage statistics

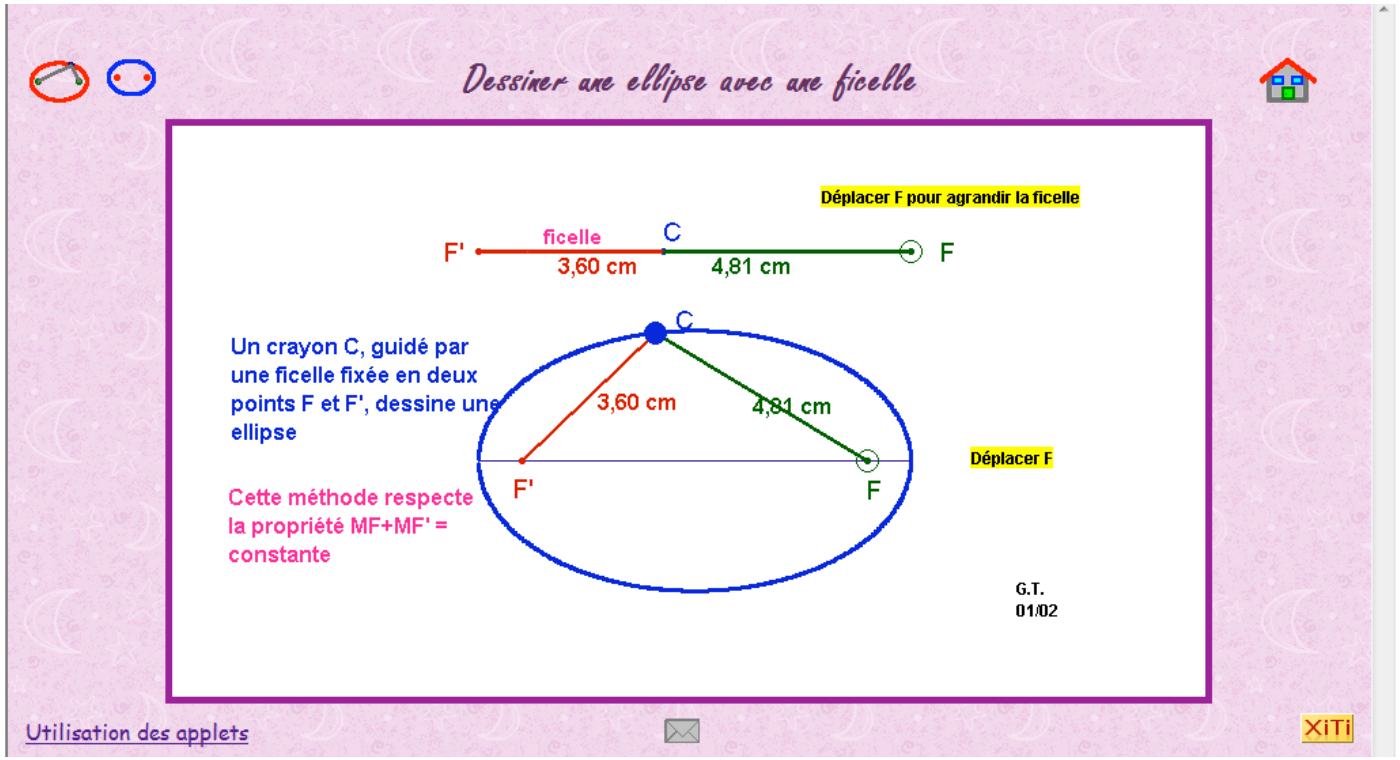
Dynamic pictures

- quadratic form



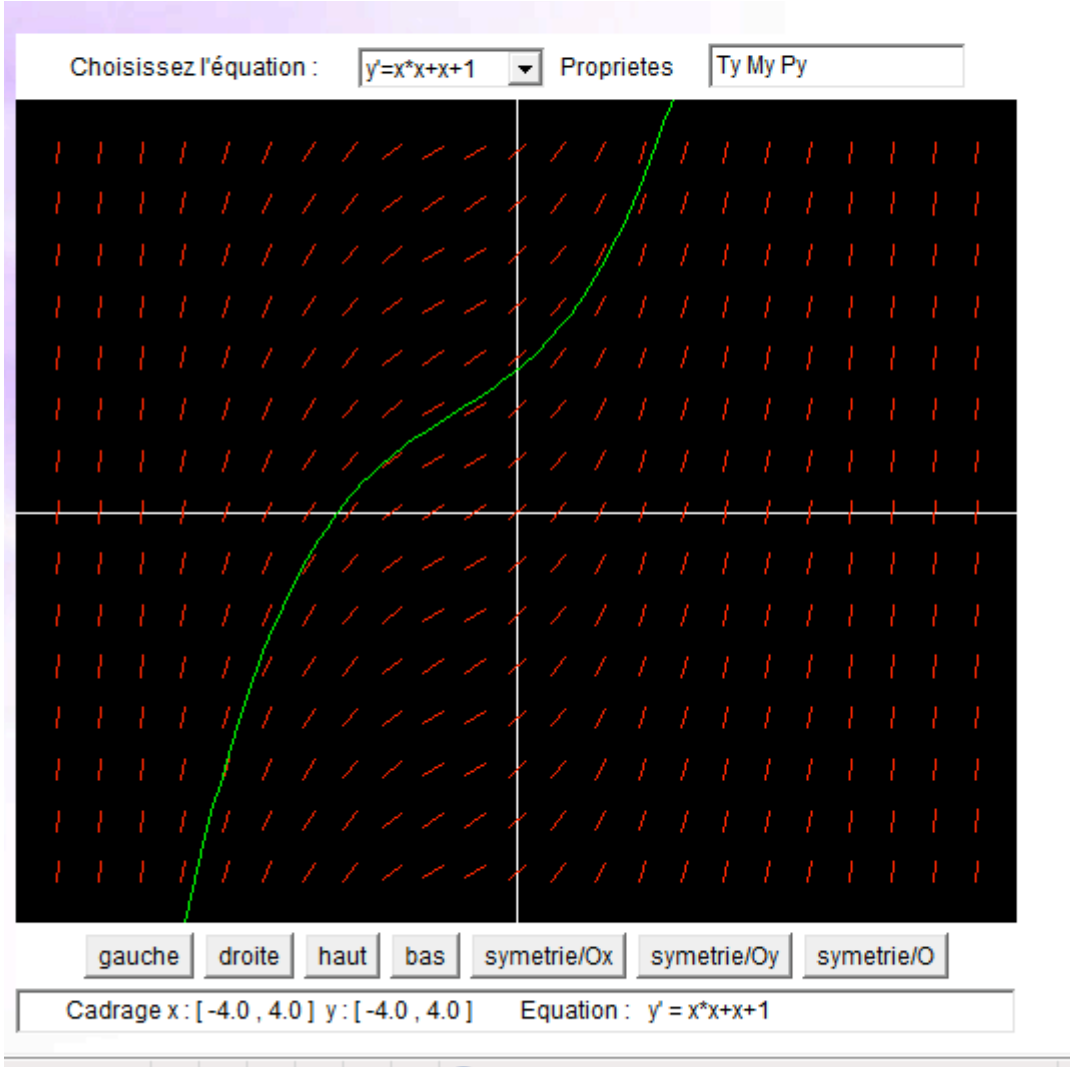
Dynamic picture

- ellipse



Solution of differential equations

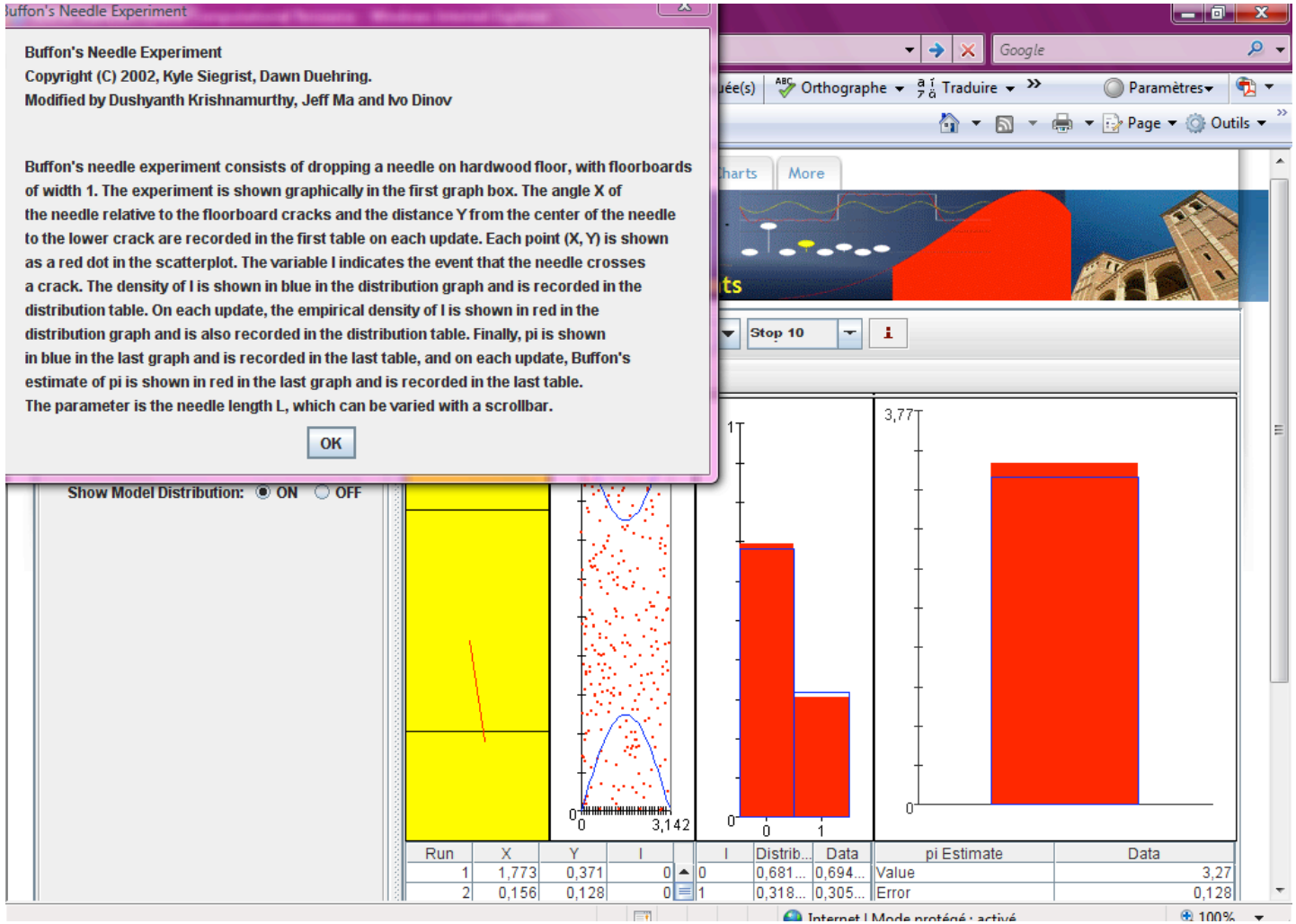
- Interactive picture



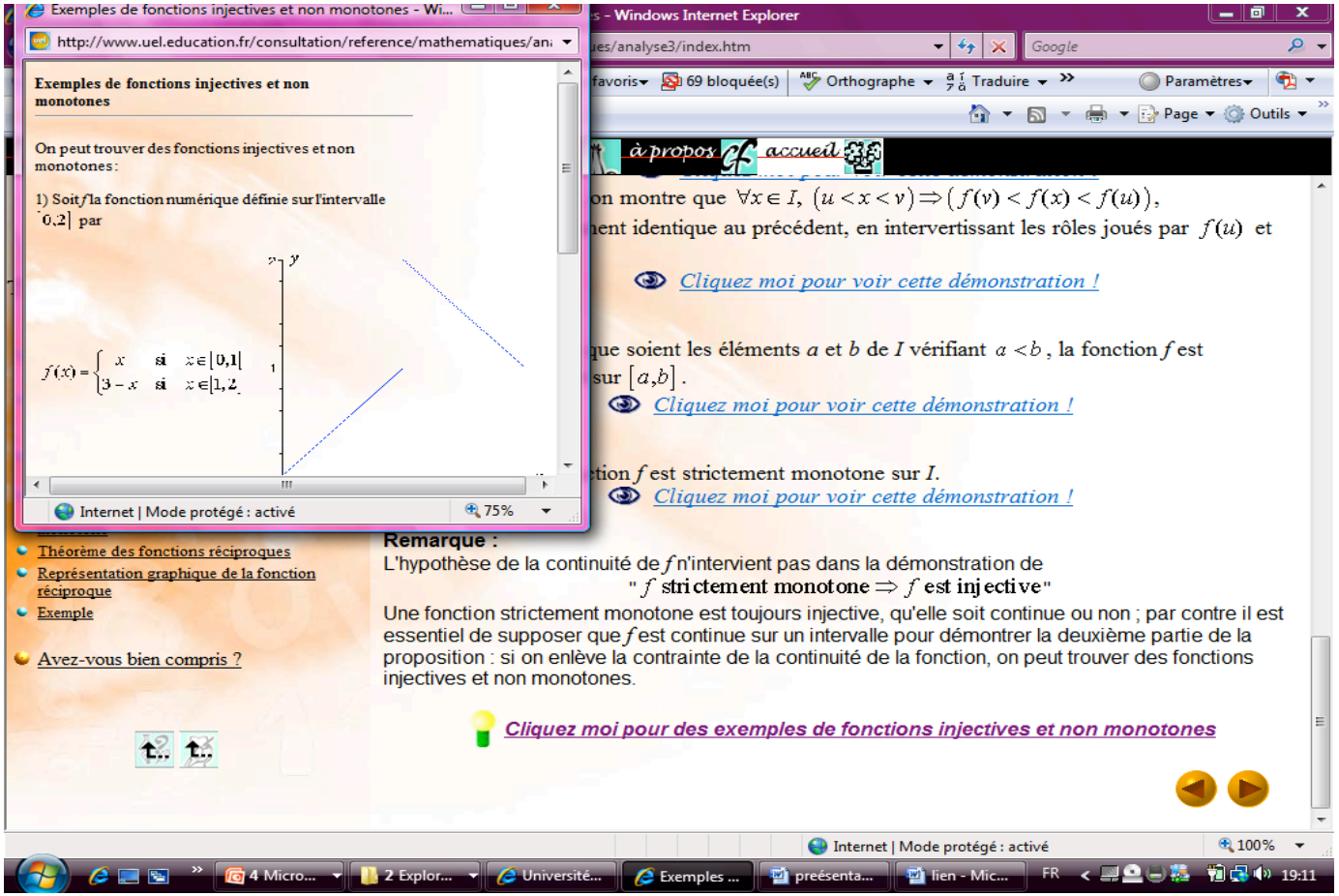
- This shows ***the existence of a unique solution*** through every point,

- It provides a skeletal cognitive schema for the solution process *before* it needs to be filled out with the specific methods of constructing solutions.

Simulation;Buffon's needle



To developp students' autonomy



Individualization in the feedback

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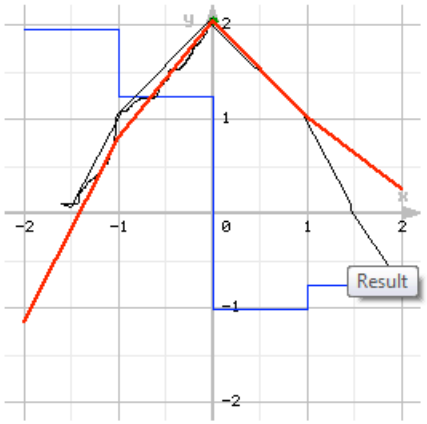
Primitive draw

Exercise. You see below the graph of a function f(x). With the mouse, please draw that of its anti-derivative passing through the green point, for x between -2 and 2.

Here is the result. (The blinking curve represents the anti-derivative. Deviation = 64.3 pixels. Score: 0/10.



ATTENTION Your curve does not even represent a function! There are values of x who would have several images.

A new function.

Non standard exercise: Open task

(http://www.stack.bham.ac.uk/).

Question 1 Focus Top 1 Bottom Validate Mark this question Help

Give an example of a function f(x) with a stationary point at x=2 and which is continuous but not differentiable at x=0.

Your last answer was interpreted as:

$$(x-4)\,x$$

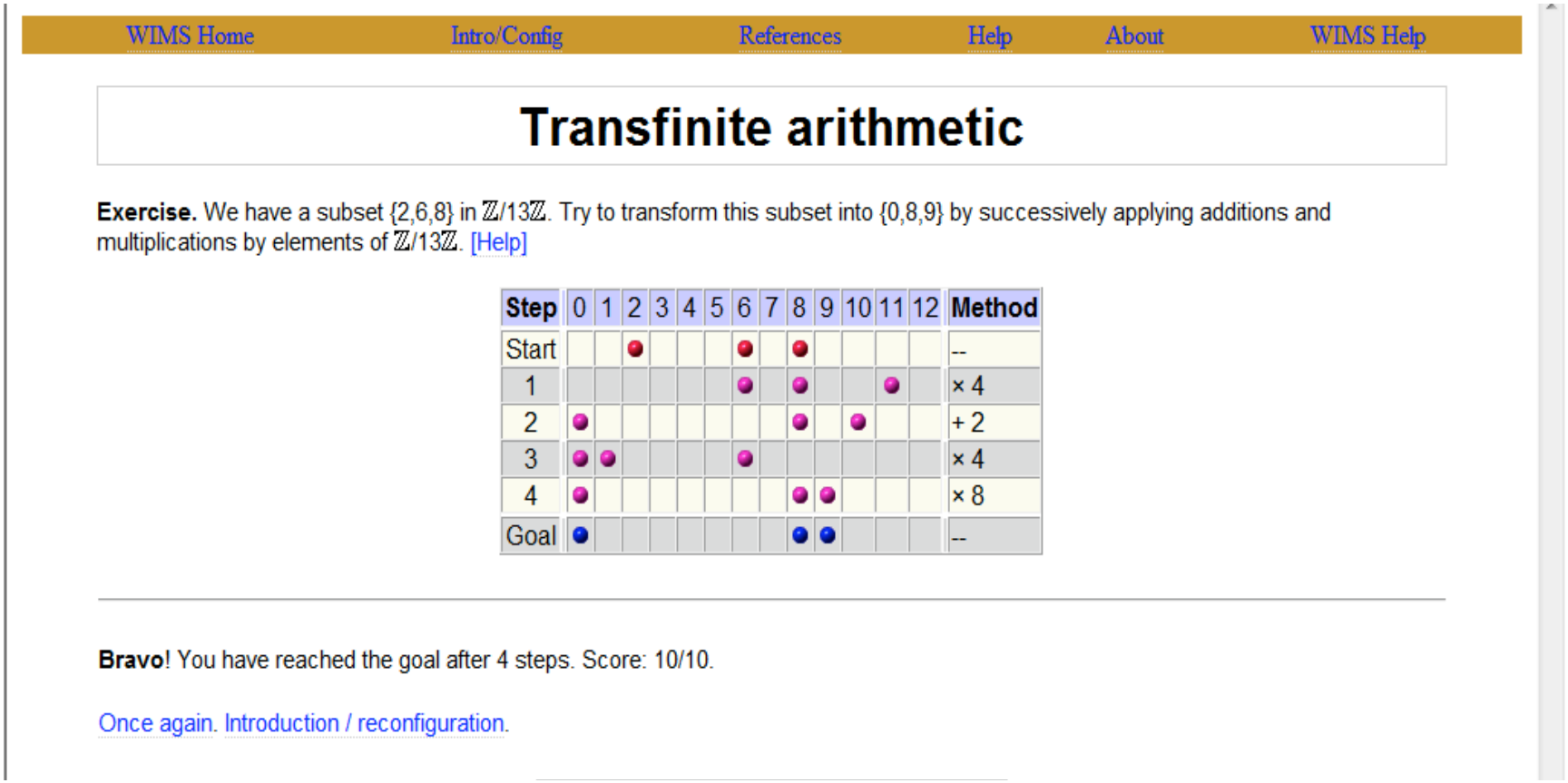
Your answer is partially correct.

Your answer is differentiable at x=0 but should not be. Consider using |x|, which is entered as `abs(x)`, somewhere in your answer.

Your mark for this attempt is 0.67. With penalties, and previous attempts, this gives 0.57 out of 1

Answer: (x-4)*x

Non standard exercise: Learning step by step



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Transfinite arithmetic

Exercise. We have a subset {2,6,8} in $\mathbb{Z}/13\mathbb{Z}$. Try to transform this subset into {0,8,9} by successively applying additions and multiplications by elements of $\mathbb{Z}/13\mathbb{Z}$. [Help]

Step	0	1	2	3	4	5	6	7	8	9	10	11	12	Method
Start			●				●		●					--
1							●		●			●		× 4
2	●								●		●			+ 2
3	●	●					●							× 4
4	●								●	●				× 8
Goal	●								●	●				--

Bravo! You have reached the goal after 4 steps. Score: 10/10.

Once again. Introduction / reconfiguration.

A priori limits

- Some limits depend on the resource :
  - Students may be logged or not.
  - Exercises may be conceived with random parameters or not.
  - Communication tools may be available or not for students or teachers.

- Some limits are general (for the time being)
  - Feedbacks and answers' analysis are crude
  - The result is assessed, not the process.

Exercice 1

Soit la suite de terme général

$$u_n = \left[2 \sin \frac{1}{n} + \frac{3}{4} \cos n\right]^n$$

Vous avez répondu que la suite est convergente
Pourquoi?
Quelle méthode employez-vous?

- ○ Comparaison à une suite de référence
- ○ Utilisation des théorèmes algébriques sur les limites
- ○ Utilisation du théorème sur les suites monotones bornées
- ○ Utilisation du théorème d'encadrement (ou théorème des 'gendarmes')
- ○ Utilisation du théorème sur les suites adjacentes
- ○ Démonstration directe utilisant la définition de la limite
- ○ Utilisation du théorème de Cauchy
- ○ Autre

3. Results of use

- Macro level results
- Micro level results

Macro level results (case of UoL)

- Few use, in spite of an institutional pressure (Uol sponsored by the State)
- It seems that only the teachers who built the system use it
- Students use it only if their teachers strongly insist.

WHY?

- Material obstacles.
- Lack of flexibility of the resource, teachers prefer to use micro applets and they seldom endorse the whole content.
- Students are mainly motivated by their exam, and Uol is far from it.

But…innovative schemes

- UoL as teaching material for :
  - distance education
  - some courses after adaptations by the teacher.
- UoL as a reference in Annals.
- Uol in self training lab sessions with a guideline or a tutor (autonomous use seems too difficult).

Macro level results (case of Wims)

- Quite a success.
- Wims is used for home work (assessed or not), for exams, and for training.
- A community of users is active with forum and meetings.

Why?

- Resources are flexible : teachers
  - build their own exercises sheet,
  - choose parameters and difficulty level,
  - may program new exercices,
  - may add guidelines or course material.
- Wims's use is part of the assessment so students are very concerned.
- It takes time to build exercises sheets but the correction is automatic so it saves time too.

Micro level results

- Observation from :
  - An experimentation : teaching with UoL
  - Several classes using Wims.
- Results
  - The importance of students engagement
  - Student autonomy? Not clear , the teacher is needed in all cases (in lab sessions as in home work) to make the scenario and to help
  - Individualization? perhaps

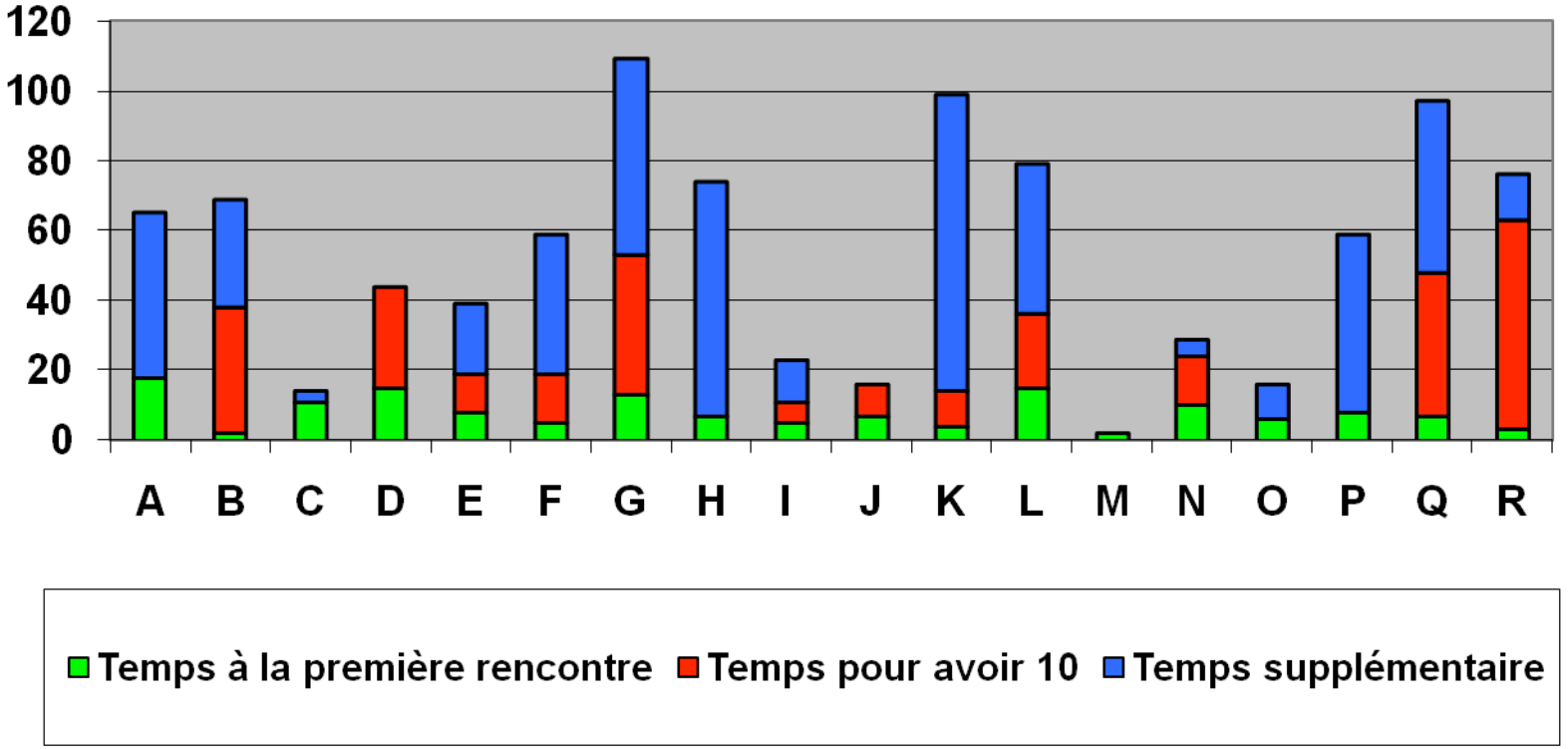
Individualization

	Classic sessions	Lab sessions
Teacher only speaks to a limited number of students	**39%**	**72%**
Student is working when teacher supports him/her	**67%**	**always**

Observation of the same teacher 4h lab session 8h classic session in an experimentation : teaching with UeL.

Individualization

For a Wims exercice :different students' paths



Qualitative results

- More time than anticipated by teachers for direct applications and efficiency of the system.

(*ex average of 17 mn to compute correctly the partial derivatives of f(x,y))*

- Students exchange on the solving process not on its result, due to the random parameters of each exercise.

An example of log file analysis

- Two exercises in a lab self training session

Direct application		*Difficult exercice*		
23	02:22:05	19	02:40:28	Total time spent on the exercise
23	00:06:11	19	00:08:27	Average time spent on the exercise
22	00:01:19	0		Average time to get 10
23	462	4	8	Number of answers submitted
23	8,19264225	4	3,875	Average mark
14	5,71428571	1	1	Average mark of the three first marks
14	8,33333333	1	1	Average mark of the three first marks

Coming back to our six goals

Goals	Results
• To rely on the picture's role	• Possible (sometimes)
• To favour constructivism	• Maybe …
• To promote non standard tasks	• Sometimes but far from summative assessment
• To develop students autonomy	• More a prerequisit than a result
• To allow individualization	• Possible
• To support weak students	• Unclear

Consistency with other researches

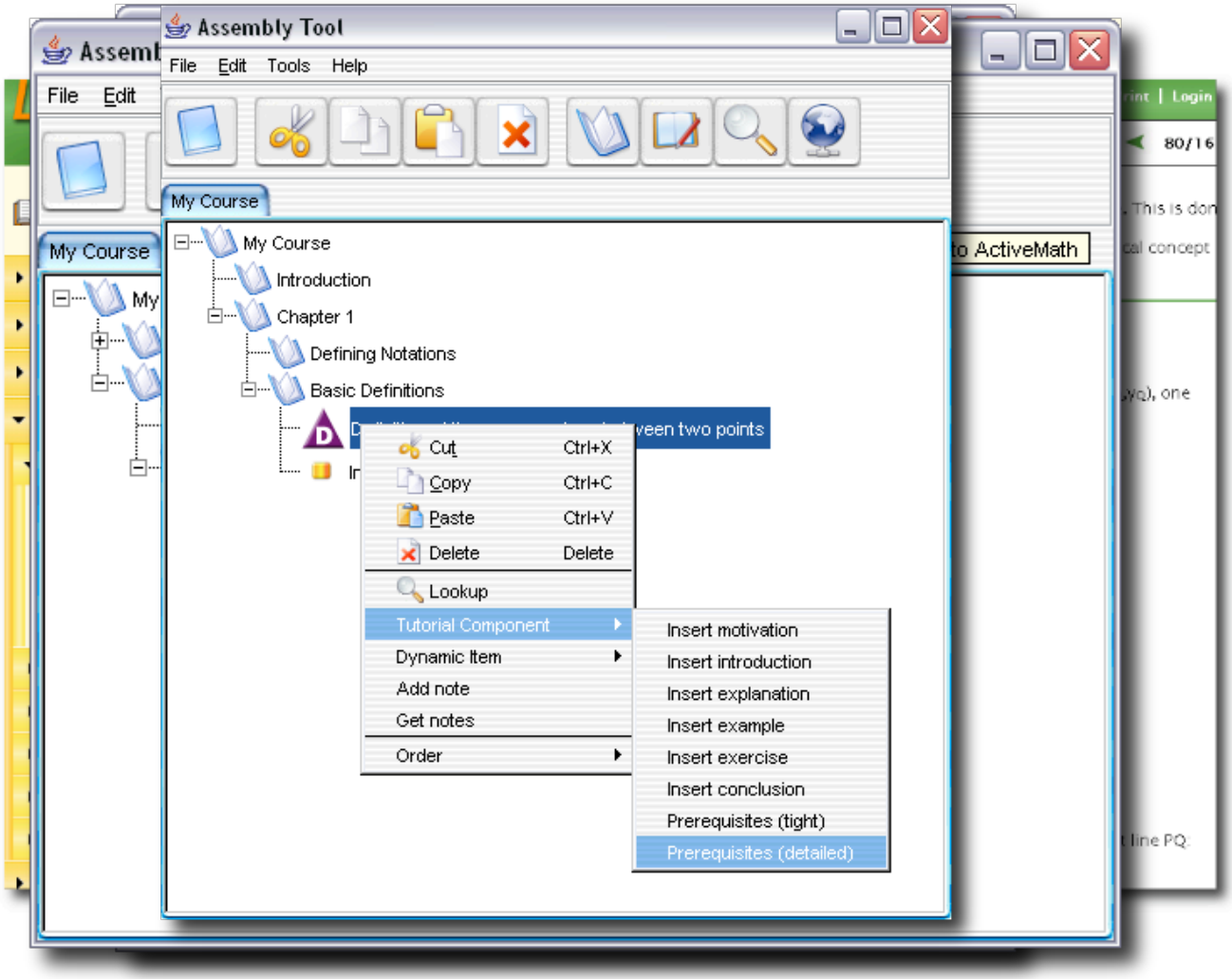
- Results regarding autonomy are consistent with similar research in secondary level.
- Results are consistent with research on innovation:
  - Positive opinions of actors already convinced but difficulty in expanding the system
  - Unexpected positive outcomes (importance of direct application)
  - Too ambitious goals.

The future

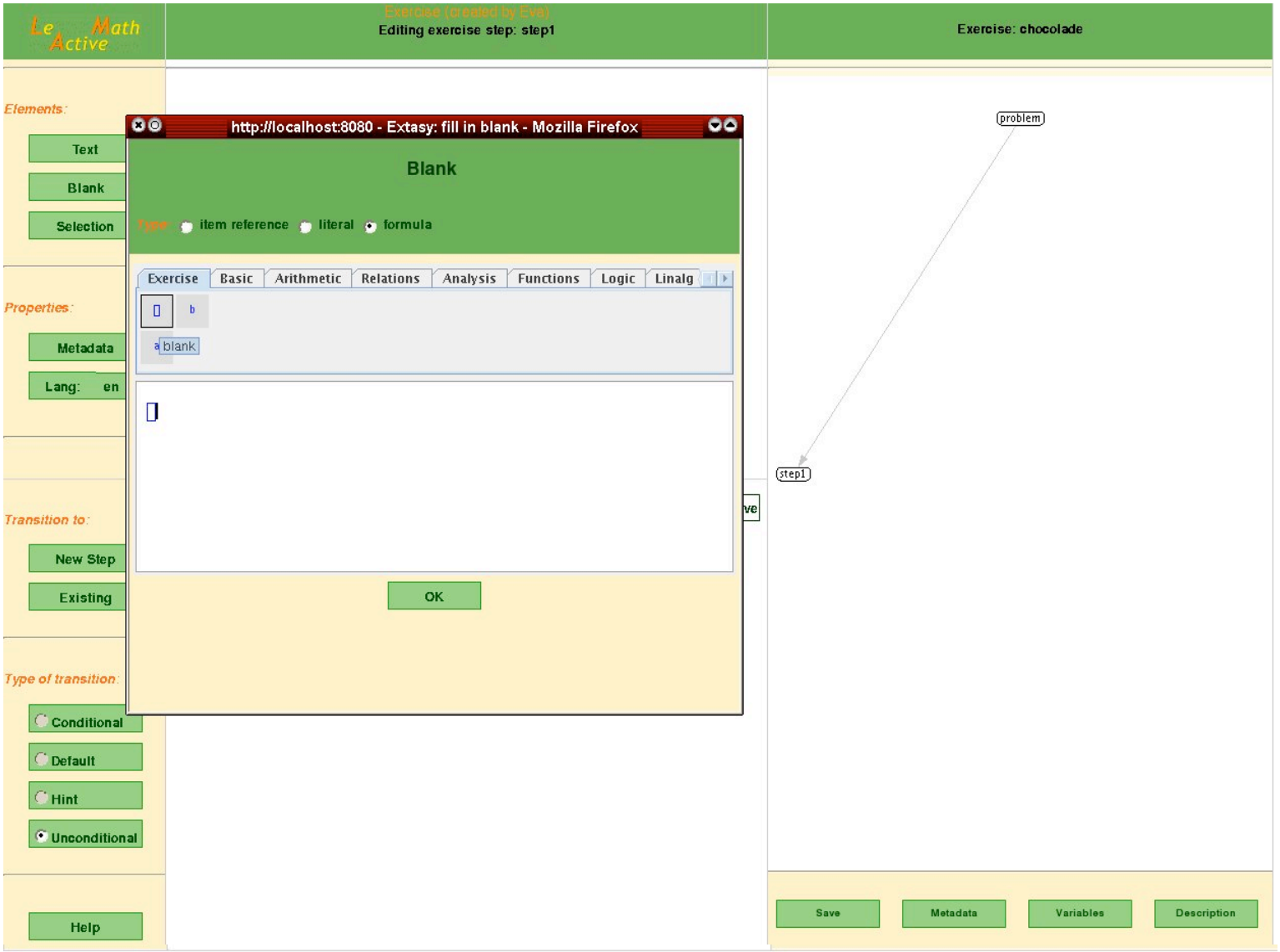
The future

- Technology improvements allow for:
  - Flexibility (ex projet european activmath
    - Assembly tool
    - Authoring tool for content and for exercise
  - Tools communication
    - International community

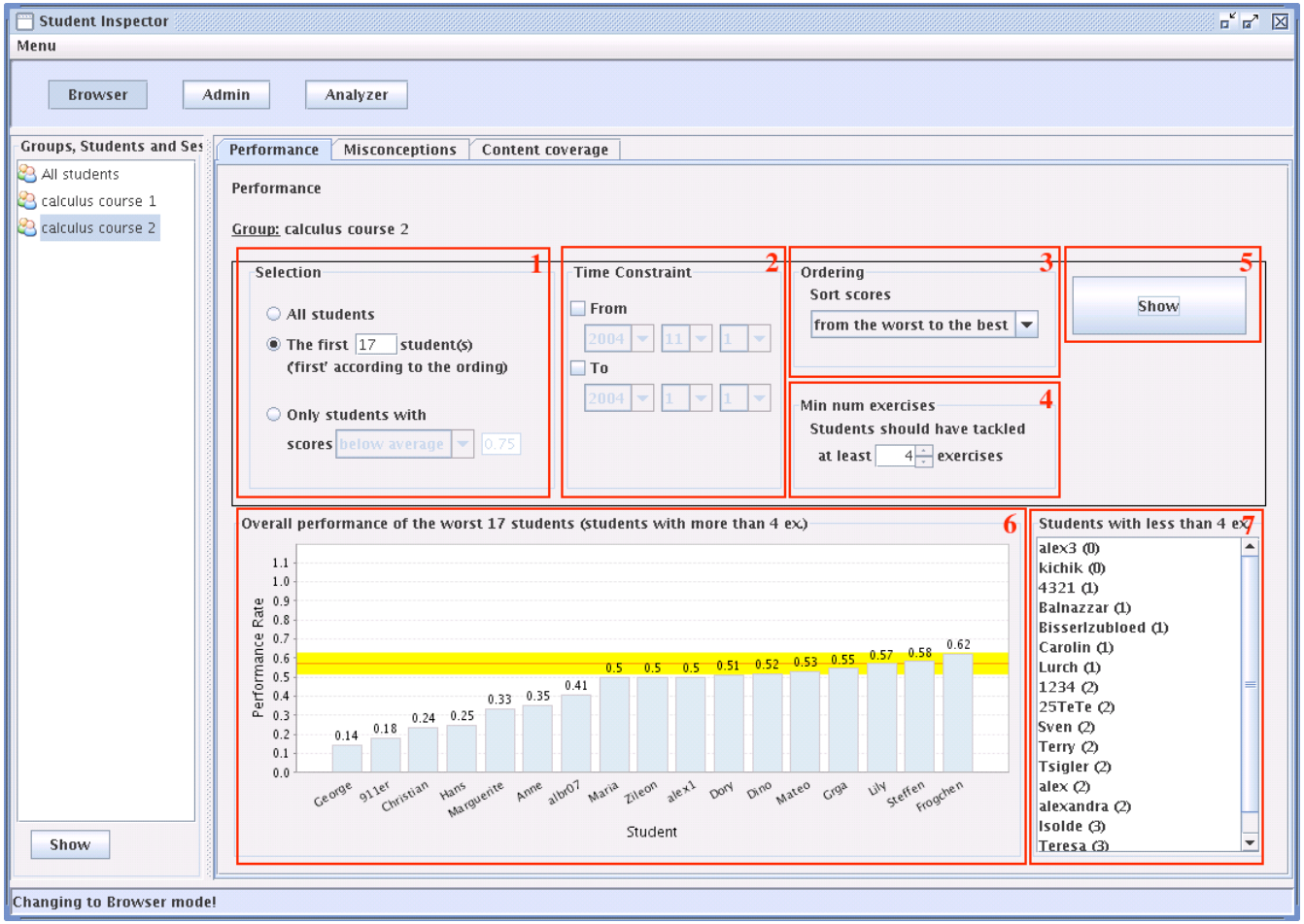
Tools for teacher: assembly tool



Tools for teacher: authoring exercises



Teacher: Overall Group Performance



Group performance

1 – Students

2 – Time interval

3 – Ordering

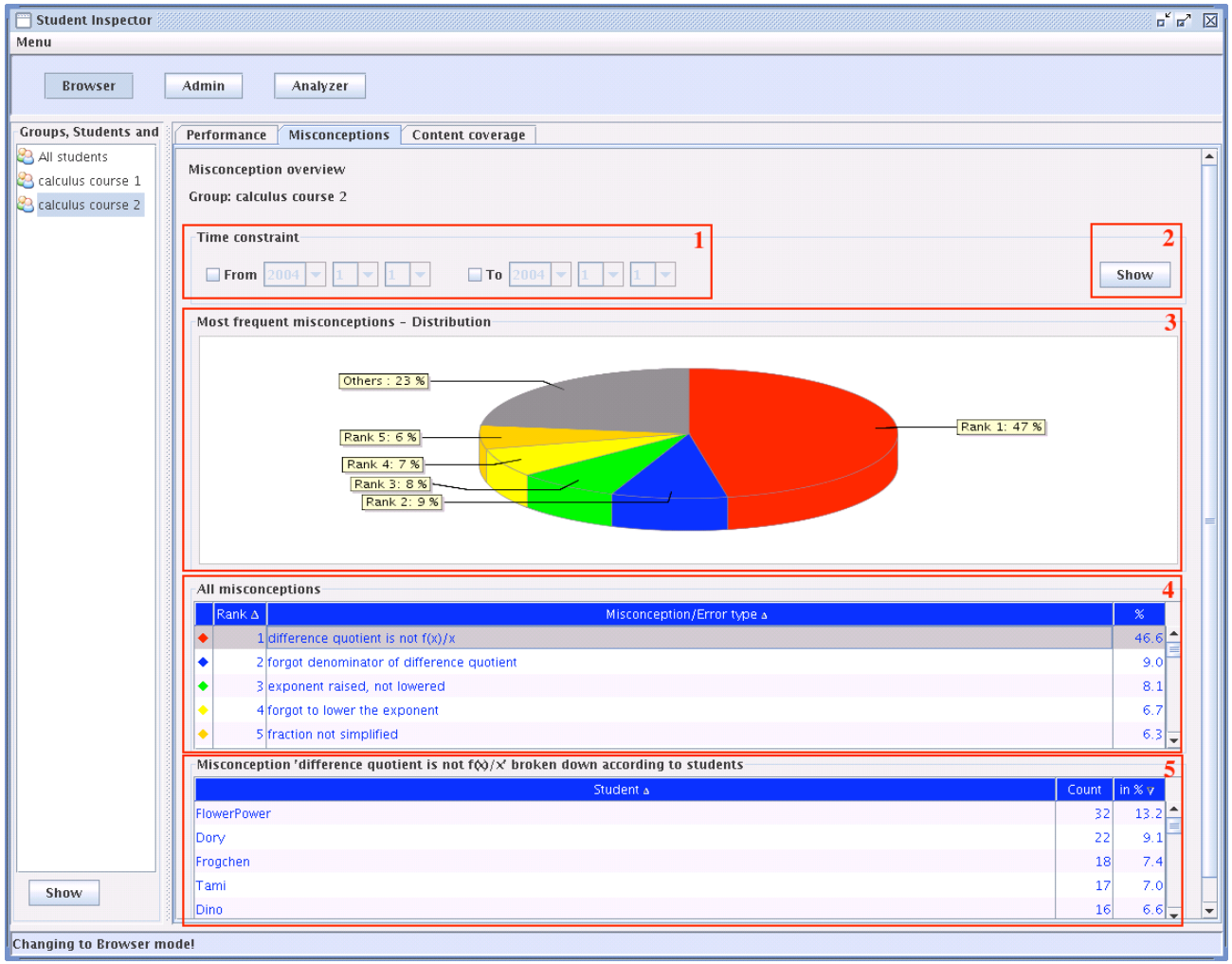
4 – Min number of exercises required

5 – Request data

6 – Performance data

7 - Students with not enough ex

Teacher: Misconceptions per Group



Group Misconceptions

1 – Time interval

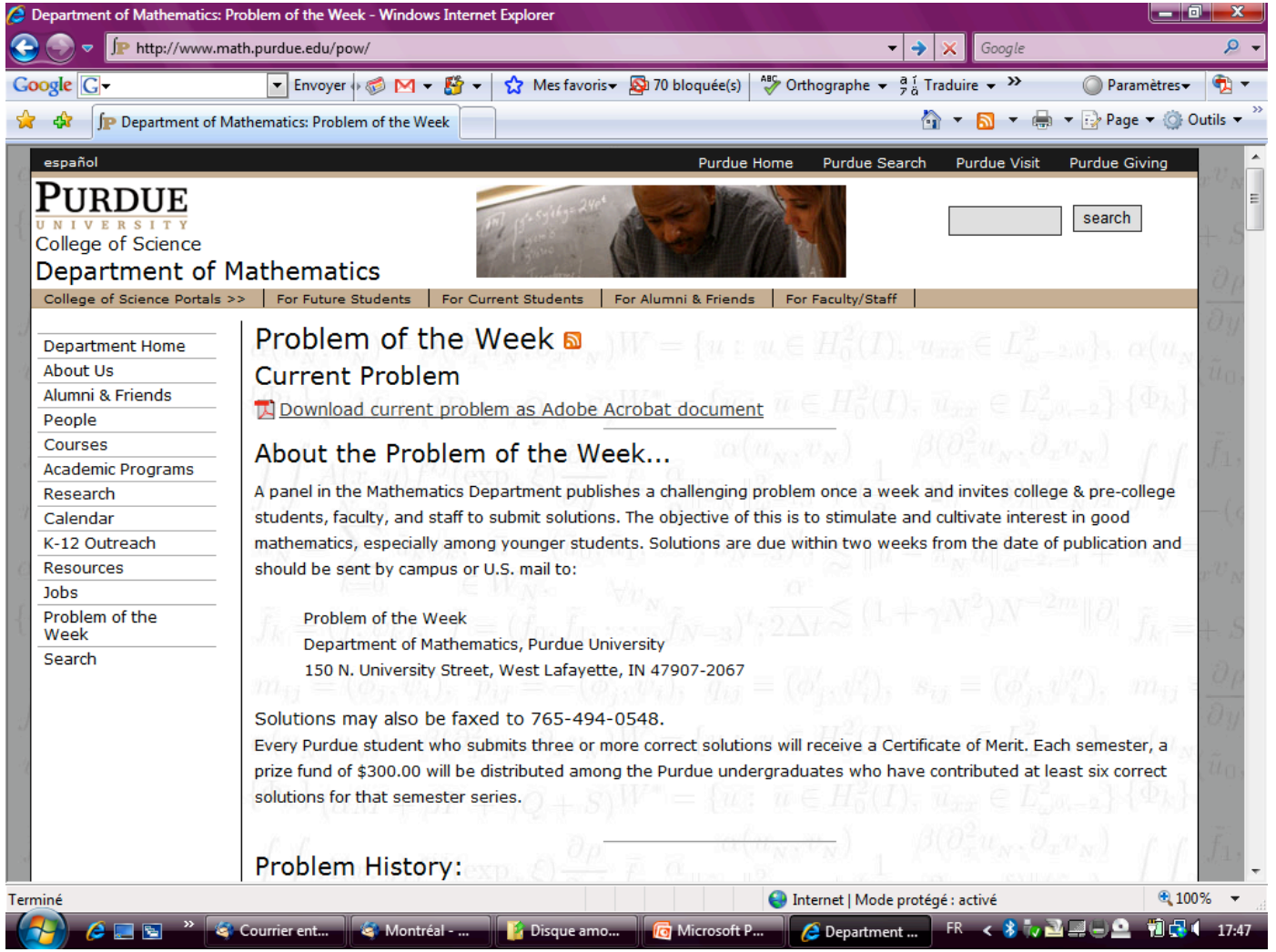
2 – Request data

3 – Most frequent errors

4 – Complete list of errors with relative frequencies

5 – Frequency distribution of selected error (per student)

International community



Problem of the week

http://www.math.purdue.edu/pow/

Problem No. 14 (Spring 2008 Series)

Suppose a convex hexagon has vertices $A_1, A_2, \ldots, A_6$ in clockwise order and that no side is larger than 1. Show that at least one of the major diagonals is no larger than 2. Here the major diagonals are A_1A_4, A_2A_5, and A_3A_6.

- Thank you

References

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