

Numeric tools for learning and teaching mathematics at tertiary level

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Overview

- Principles
- Example of realisations
- Results of use
- The future



1. Principles

Motivations to build/to use such tools :

- Pragmatic motivations
 - To explore new tools and new potentialities
 - To train students to use tools they will have to use in their professional life
 - To link University and technology advancement
- Didactic or pedagogic motivations
 - To use these tools to support theoretical (specific) goals



Six goals

- To rely on picture and more generally on semiotic representations of concepts.
- To favour Constructivist learning approach.
- To promote non standard exercises
- To develop students' autonomy and self regulated learning.
- To allow individualization in the teaching process.
- To support weak students or students not present in the University.

2. Example of realisations

A decorative graphic consisting of several horizontal lines of varying lengths and colors (teal, light blue, and white) extending from the left side of the slide towards the right, positioned below the section header.

UoL (University on line)

A set of learning objects containing multimedia resources

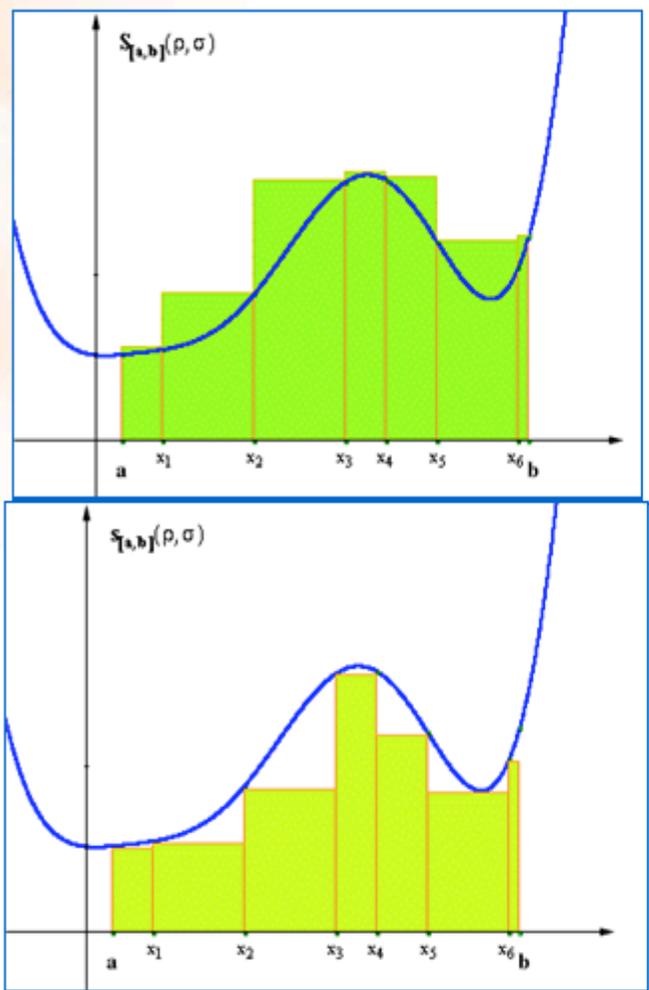
- Developed since 1998
- Through a partnership between 13 French universities
- 4 topics (Mathematics, physics, chemistry and biology)
- Content of the two first years of University
- Structured in sections: to simulate, to observe, to learn, to train and to assess.

<http://www.uel.education.fr/>

- apprendre
- exercer
- évaluer

b. somme de Darboux supérieure associée à f et le nombre $S_{n,\sigma}^+(f, \sigma) = \sum_{i=1}^n M_i(x_{i-1}, x_i)$

- Introduction
- Intégrale de Riemann
 - Définition:
 - Préliminaires
 - Subdivisions d'un intervalle
 - Sommes de Darboux
 - Fonction intégrable au sens de Riemann
 - Exemples de fonctions intégrables
 - Fonctions monotones
 - Fonctions continues
 - Autres exemples
 - Propriétés de l'intégrale de Riemann
 - La relation de Chasles
 - Linéarité de l'intégrale
 - Le théorème de la moyenne
 - Les inégalités
 - Conséquences
 - Primitive et intégrale définie
 - Primitives usuelles
- Calcul d'intégrales : méthodes générales
- Calcul pratique d'intégrales
- Approximations numériques



Pour voir la vidéo (2.8 Mo) cliquez sur le lien ou sur l'image





Wims <http://wims.auto.u-psud.fr/wims/>

- A library of on-line interactive mathematics resources which includes exercises for all levels: from primary to tertiary education.
- A collaborative project available in six languages.
- Teachers open a class and build exercises sheet by choosing exercises in the library or programming new ones.
- Exercises are with random parameters so student may search the same exercise several times.



WWW Interactive Multipurpose Server

(WIMS) at wims.auto.u-psud.fr

[what's new](#) [forums](#) [mirrors](#) [preferences](#) [help](#)



[Virtual classes](#) [students' area](#) [teachers' area](#) [example classes](#) [help](#)

Search among [WIMS activities](#) [browse](#)

[All the exercises](#)

On this site, you may find

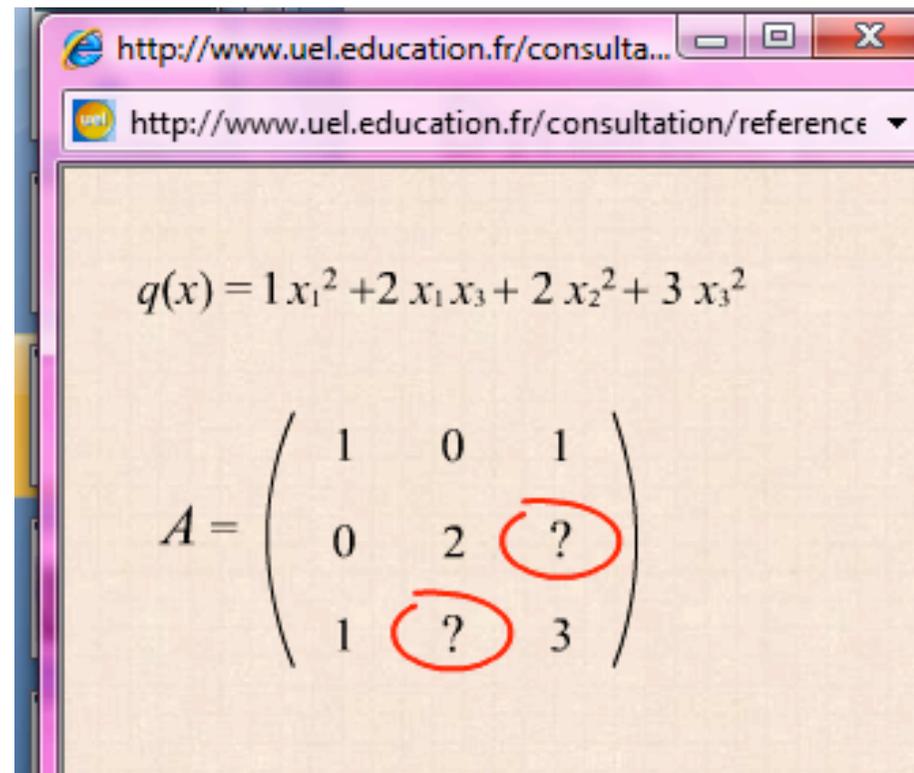
- [Lessons and references](#) on various topics.
- [Online calculators and plotters](#) : numbers, functions, matrices, curves, surfaces, etc.
- [Interactive exercises](#) of various styles and levels.
- [Mathematical recreations](#) : puzzles and games.
- [Virtual classes and portals](#) to manage scored student works.
- [Misc. interactive documents](#).

You may also [browse the site](#).

Create my own [simple interactive exercises](#) or [full-power modules](#)
[tech doc](#) [download](#) [backward links](#) [usage statistics](#)

Dynamic pictures

- quadratic form



A screenshot of a web browser window. The address bar shows the URL <http://www.uel.education.fr/consultation/reference>. The main content area displays the quadratic form $q(x) = 1x_1^2 + 2x_1x_3 + 2x_2^2 + 3x_3^2$ and its corresponding symmetric matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & ? \\ 1 & ? & 3 \end{pmatrix}$. The question marks in the matrix are circled in red.

$$q(x) = 1x_1^2 + 2x_1x_3 + 2x_2^2 + 3x_3^2$$
$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & ? \\ 1 & ? & 3 \end{pmatrix}$$

Dynamic picture

- [ellipse](#)

Dessiner une ellipse avec une ficelle

Un crayon C, guidé par une ficelle fixée en deux points F et F', dessine une ellipse

Cette méthode respecte la propriété $MF + MF' = \text{constante}$

Déplacer F pour agrandir la ficelle

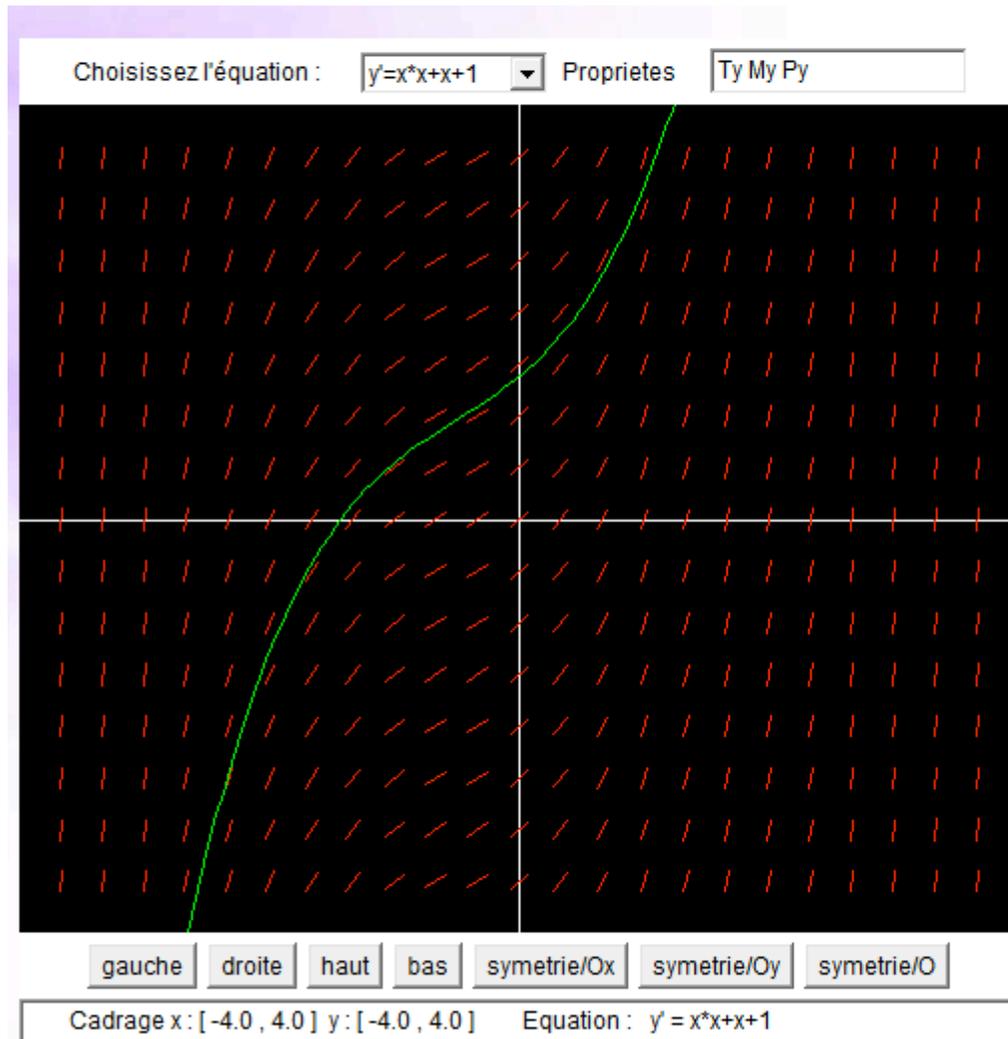
Déplacer F

G.T.
01/02

Utilisation des applets

XiTi

- Interactive picture



Solution of differential equations

- This shows *the existence of a unique solution* through every point,
- It provides a skeletal cognitive schema for the solution process *before* it needs to be filled out with the specific methods of constructing solutions.

Simulation; Buffon's needle

Buffon's Needle Experiment

Copyright (C) 2002, Kyle Siegrist, Dawn Duehring.
 Modified by Dushyanth Krishnamurthy, Jeff Ma and Ivo Dinov

Buffon's needle experiment consists of dropping a needle on hardwood floor, with floorboards of width 1. The experiment is shown graphically in the first graph box. The angle X of the needle relative to the floorboard cracks and the distance Y from the center of the needle to the lower crack are recorded in the first table on each update. Each point (X, Y) is shown as a red dot in the scatterplot. The variable I indicates the event that the needle crosses a crack. The density of I is shown in blue in the distribution graph and is recorded in the distribution table. On each update, the empirical density of I is shown in red in the distribution graph and is also recorded in the distribution table. Finally, π is shown in blue in the last graph and is recorded in the last table, and on each update, Buffon's estimate of π is shown in red in the last graph and is recorded in the last table. The parameter is the needle length L , which can be varied with a scrollbar.

OK

Show Model Distribution: ON OFF

Run	X	Y	I	Distrib...	Data	pi Estimate	Data
1	1,773	0,371	0	0	0,681...	Value	3,27
2	0,156	0,128	0	1	0,318...	Error	0,128

To developp students' autonomy

Exemples de fonctions injectives et non monotones - Wi...

http://www.uel.education.fr/consultation/reference/mathematiques/ani...

Exemples de fonctions injectives et non monotones

On peut trouver des fonctions injectives et non monotones:

1) Soit/la fonction numérique définie sur l'intervalle $[0,2]$ par

$$f(x) = \begin{cases} x & \text{si } x \in [0,1] \\ 3-x & \text{si } x \in [1,2] \end{cases}$$

à propos **gf** accueil

on montre que $\forall x \in I, (u < x < v) \Rightarrow (f(v) < f(x) < f(u))$,
ment identique au précédent, en intervertissant les rôles joués par $f(u)$ et

[Cliquez moi pour voir cette démonstration !](#)

que soient les éléments a et b de I vérifiant $a < b$, la fonction f est sur $[a,b]$.

[Cliquez moi pour voir cette démonstration !](#)

tion f est strictement monotone sur I .

[Cliquez moi pour voir cette démonstration !](#)

Remarque :
L'hypothèse de la continuité de f n'intervient pas dans la démonstration de " f strictement monotone $\Rightarrow f$ est injective".
Une fonction strictement monotone est toujours injective, qu'elle soit continue ou non ; par contre il est essentiel de supposer que f est continue sur un intervalle pour démontrer la deuxième partie de la proposition : si on enlève la contrainte de la continuité de la fonction, on peut trouver des fonctions injectives et non monotones.

[Cliquez moi pour des exemples de fonctions injectives et non monotones](#)

Internet | Mode protégé : activé 75%

Internet | Mode protégé : activé 100%

4 Micro... 2 Explor... Université... Exemples ... préSENTA... lien - Mic... FR 19:11

Individualization in the feedback

[WIMS Home](#)

[Intro/Config](#)

[References](#)

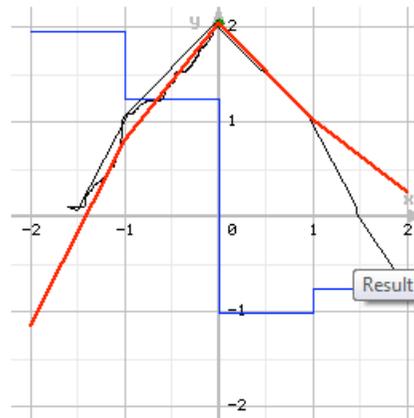
[About](#)

[WIMS Help](#)

Primitive draw

Exercise. You see below [the graph](#) of a function $f(x)$. With the mouse, please draw that of its anti-derivative passing through the [green point](#), for x between -2 and 2.

Here is the result. (The [blinking curve](#) represents the anti-derivative. Deviation = 64.3 pixels. Score: 0/10.)



ATTENTION Your curve does not even represent a function! There are values of x who would have several images.

[A new function.](#)

Non standard exercise: Open task

(<http://www.stack.bham.ac.uk/>).

Question 1

[Focus](#) [Top](#) [1](#) [Bottom](#) [Validate](#) [Mark this question](#) [Help](#)

Give an example of a function $f(x)$ with a stationary point at $x=2$ and which is continuous but not differentiable at $x=0$.

Your last answer was interpreted as:

$$(x-4)x$$

Your answer is partially correct.

Your answer is differentiable at $x=0$ but should not be. Consider using $|x|$, which is entered as `abs(x)`, somewhere in your answer.

Your mark for this attempt is 0.67. With penalties, and previous attempts, this gives 0.57 out of 1

Answer:

Non standard exercise: Learning step by step

[WIMS Home](#)

[Intro/Config](#)

[References](#)

[Help](#)

[About](#)

[WIMS Help](#)

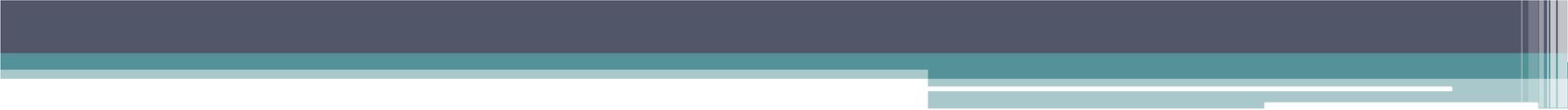
Transfinite arithmetic

Exercise. We have a subset $\{2,6,8\}$ in $\mathbb{Z}/13\mathbb{Z}$. Try to transform this subset into $\{0,8,9\}$ by successively applying additions and multiplications by elements of $\mathbb{Z}/13\mathbb{Z}$. [\[Help\]](#)

Step	0	1	2	3	4	5	6	7	8	9	10	11	12	Method
Start			●				●		●					--
1							●		●			●		$\times 4$
2	●								●		●			$+ 2$
3	●	●					●							$\times 4$
4	●								●	●				$\times 8$
Goal	●								●	●				--

Bravo! You have reached the goal after 4 steps. Score: 10/10.

[Once again. Introduction / reconfiguration.](#)



A priori limits

- Some limits depend on the resource :
 - Students may be logged or not.
 - Exercises may be conceived with random parameters or not.
 - Communication tools may be available or not for students or teachers.
- Some limits are general (for the time being)
 - Feedbacks and answers' analysis are crude
 - The result is assessed, not the process.

Exercice 1

Soit la suite de terme général

$$u_n = \left[2 \sin \frac{1}{n} + \frac{3}{4} \cos n \right]^n$$

Vous avez répondu que la suite est convergente

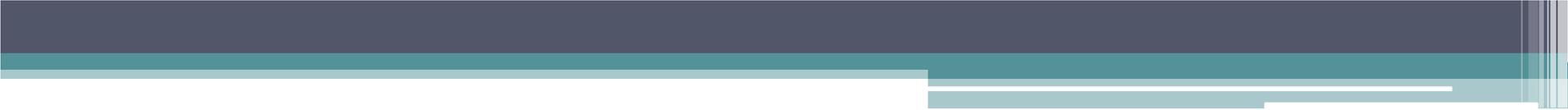
Pourquoi?

Quelle méthode employez-vous?

- Comparaison à une suite de référence
- Utilisation des théorèmes algébriques sur les limites
- Utilisation du théorème sur les suites monotones bornées
- Utilisation du théorème d'encadrement (ou théorème des 'gendarmes')
- Utilisation du théorème sur les suites adjacentes
- Démonstration directe utilisant la définition de la limite
- Utilisation du théorème de Cauchy
- Autre

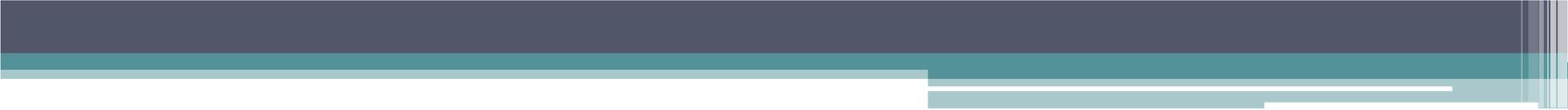
3. Results of use

- Macro level results
- Micro level results



Macro level results (case of UoL)

- Few use, in spite of an institutional pressure (Uol sponsored by the State)
- It seems that only the teachers who built the system use it
- Students use it only if their teachers strongly insist.



WHY?

- Material obstacles.
- Lack of flexibility of the resource, teachers prefer to use micro applets and they seldom endorse the whole content.
- Students are mainly motivated by their exam, and Uol is far from it.



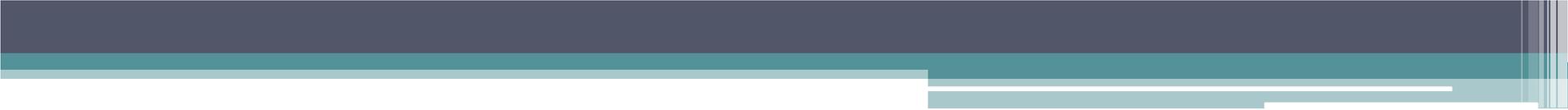
But...innovative schemes

- UoL as teaching material for :
 - distance education
 - some courses after adaptations by the teacher.
- UoL as a reference in Annals.
- UoL in self training lab sessions with a guideline or a tutor (autonomous use seems too difficult).



Macro level results (case of Wims)

- Quite a success.
- Wims is used for home work (assessed or not), for exams, and for training.
- A community of users is active with forum and meetings.



Why?

- Resources are flexible : teachers
 - build their own exercises sheet,
 - choose parameters and difficulty level,
 - may program new exercises,
 - may add guidelines or course material.
- Wims's use is part of the assessment so students are very concerned.
- It takes time to build exercises sheets but the correction is automatic so it saves time too.



Micro level results

- Observation from :
 - An experimentation : teaching with UoL
 - Several classes using Wims.
- Results
 - The importance of students engagement
 - Student autonomy? Not clear , the teacher is needed in all cases (in lab sessions as in home work) to make the scenario and to help
 - Individualization? perhaps

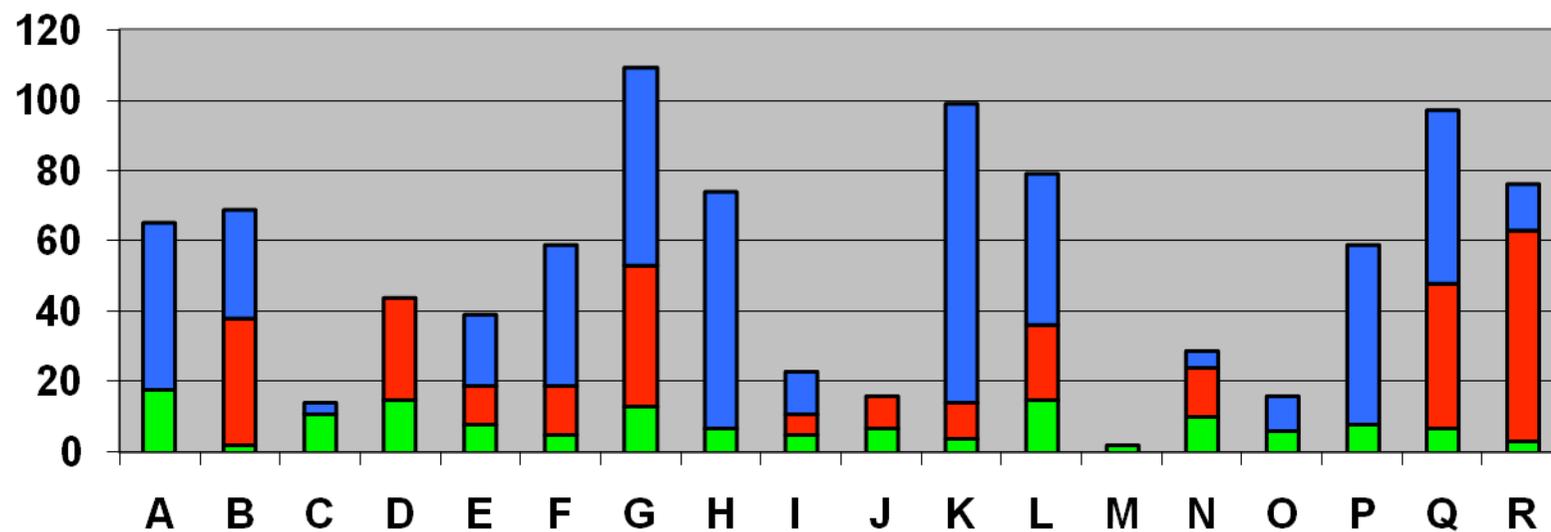
Individualization

	Classic sessions	Lab sessions
Teacher only speaks to a limited number of students	39%	72%
Student is working when teacher supports him/her	67%	always

Observation of the same teacher 4h lab session 8h classic session in an experimentation : teaching with UeL.

Individualization

For a Wims exercice : different students' paths



■ Temps à la première rencontre ■ Temps pour avoir 10 ■ Temps supplémentaire

Qualitative results

- More time than anticipated by teachers for direct applications and efficiency of the system.

(ex average of 17 mn to compute correctly the partial derivatives of $f(x,y)$)

- Students exchange on the solving process not on its result, due to the random parameters of each exercise.

An example of log file analysis

- Two exercises in a lab self training session

<i>Direct application</i>		<i>Difficult exercise</i>		
23	02:22:05	19	02:40:28	Total time spent on the exercise
23	00:06:11	19	00:08:27	Average time spent on the exercise
22	00:01:19	0		Average time to get 10
23	462	4	8	Number of answers submitted
23	8,19264225	4	3,875	Average mark
14	5,71428571	1	1	Average mark of the three first marks
14	8,33333333	1	1	Average mark of the three first marks

Coming back to our six goals

Goals

- To rely on the picture's role
- To favour constructivism
- To promote non standard tasks

- To develop students autonomy

- To allow individualization
- To support weak students

Results

- Possible (sometimes)
- Maybe ...
- Sometimes but far from summative assessment
- More a prerequisite than a result
- Possible
- Unclear



Consistency with other researches

- Results regarding autonomy are consistent with similar research in secondary level.
- Results are consistent with research on innovation:
 - Positive opinions of actors already convinced but difficulty in expanding the system
 - Unexpected positive outcomes (importance of direct application)
 - Too ambitious goals.

The future

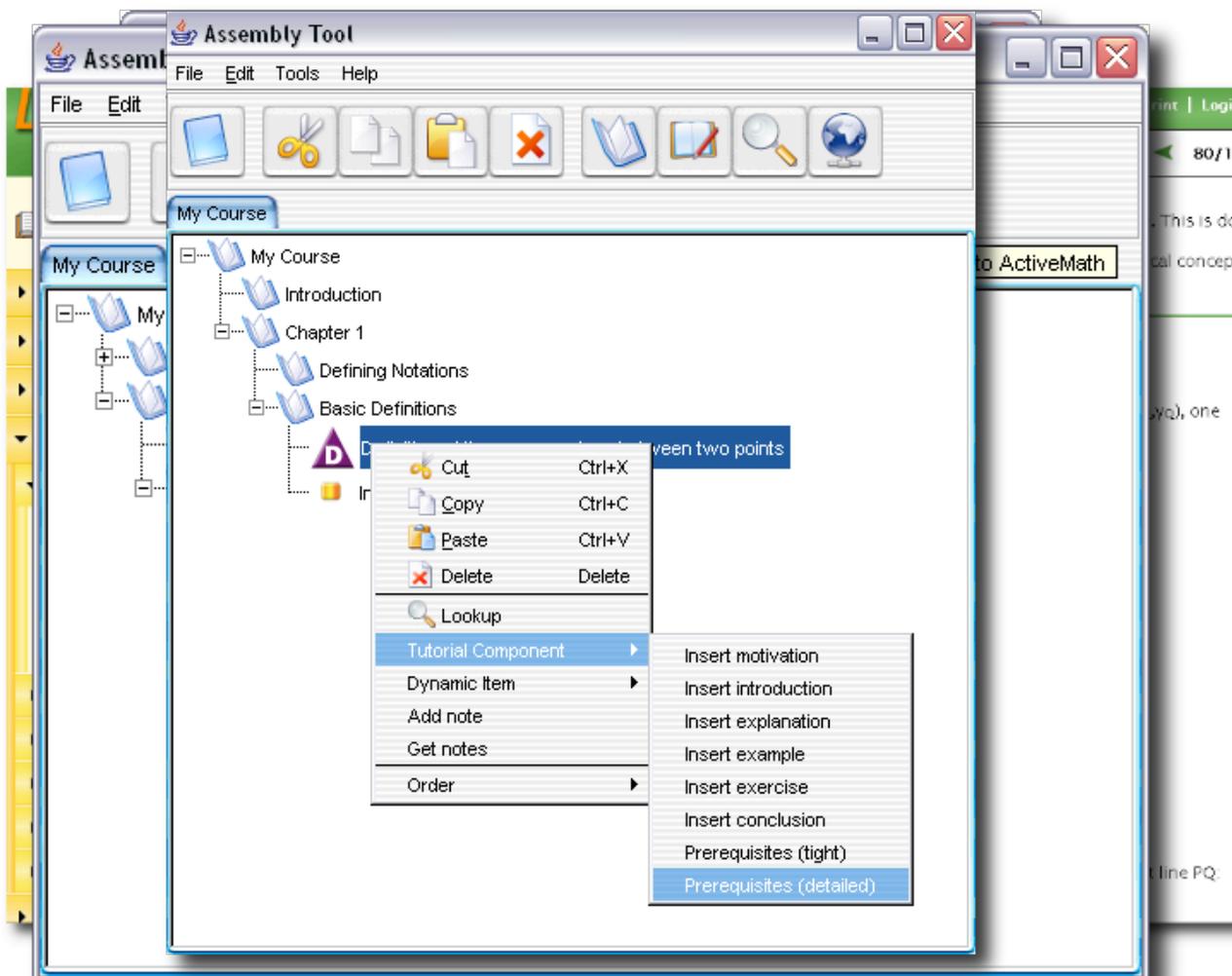




The future

- Technology improvements allow for:
 - Flexibility (ex projet european activmath)
 - Assembly tool
 - Authoring tool for content and for exercise
 - Tools communication
 - International community

Tools for teacher: assembly tool



Tools for teacher: authoring exercises

The screenshot displays the 'Le Math Active' software interface for authoring exercises. The interface is divided into several sections:

- Top Bar:** Shows 'Le Math Active' on the left, 'Exercise (created by Eva) Editing exercise step: step1' in the center, and 'Exercise: chocolate' on the right.
- Left Panel (Elements and Properties):**
 - Elements:** Includes buttons for 'Text', 'Blank', and 'Selection'. Below these are radio buttons for 'Type' with options: 'item reference', 'literal', and 'formula' (selected).
 - Properties:** Includes a 'Metadata' button, a 'Lang: en' dropdown, and a large text area for input.
 - Transition to:** Includes buttons for 'New Step' and 'Existing'.
 - Type of transition:** Includes radio buttons for 'Conditional', 'Default', 'Hint', and 'Unconditional' (selected).
 - Help:** A button at the bottom.
- Central Editor:** A window titled 'http://localhost:8080 - Extasy: fill in blank - Mozilla Firefox' is open, showing a 'Blank' exercise editor. It has tabs for 'Exercise', 'Basic', 'Arithmetic', 'Relations', 'Analysis', 'Functions', 'Logic', and 'Linalg'. Below the tabs are input fields for 'b' and 'a blank', and an 'OK' button.
- Right Panel (Diagram):** A diagram showing a 'problem' node connected to a 'step1' node by a diagonal arrow.
- Bottom Bar:** Includes buttons for 'Save', 'Metadata', 'Variables', and 'Description'.

Teacher: Overall Group Performance

The screenshot shows the 'Student Inspector' application window. The 'Performance' tab is active, displaying settings for 'calculus course 2'. The interface includes a 'Selection' section (1) with radio buttons for 'All students', 'The first 17 student(s) (first' according to the ording)', and 'Only students with scores below average' (0.75). The 'Time Constraint' section (2) has 'From' (2004, 11, 1) and 'To' (2004, 1, 1) fields. The 'Ordering' section (3) has a dropdown set to 'from the worst to the best'. The 'Min num exercises' section (4) has a field set to 4. A 'Show' button (5) is located to the right. Below these settings is a bar chart (6) titled 'Overall performance of the worst 17 students (students with more than 4 ex)'. The chart shows performance rates for 17 students, with a horizontal yellow line at 0.6. The data is as follows:

Student	Performance Rate
George	0.14
911er	0.18
Christian	0.24
Hans	0.25
Marguerite	0.33
Anne	0.35
albr07	0.41
Maria	0.5
Zileon	0.5
alex1	0.5
Dory	0.51
Dino	0.52
Mateo	0.53
Crga	0.55
Lily	0.57
Steffen	0.58
Frogghen	0.62

To the right of the chart is a list (7) titled 'Students with less than 4 ex' containing names like alex3, Kichik, 4321, Balmazzar, Bisserlzubloed, Carolin, Lurch, 1234, 25TeTe, Sven, Terry, Tsigler, alex, alexandra, Isolde, and Teresa.

Group performance

- 1 – Students
- 2 – Time interval
- 3 – Ordering
- 4 – Min number of exercises required
- 5 – Request data
- 6 – Performance data
- 7 - Students with not enough ex

Teacher: Misconceptions per Group

Student Inspector

Menu

Browser Admin Analyzer

Groups, Students and

All students
calculus course 1
calculus course 2

Performance Misconceptions Content coverage

Misconception overview

Group: calculus course 2

Time constraint

From 2004 1 1 To 2004 1 1 Show

Most frequent misconceptions - Distribution

All misconceptions

Rank	Misconception/Error type	%
1	difference quotient is not $f(x)/x$	46.6
2	forgot denominator of difference quotient	9.0
3	exponent raised, not lowered	8.1
4	forgot to lower the exponent	6.7
5	fraction not simplified	6.3

Misconception 'difference quotient is not $f(x)/x$ ' broken down according to students

Student	Count	in %
FlowerPower	32	13.2
Dory	22	9.1
Frogchen	18	7.4
Tarni	17	7.0
Dino	16	6.6

Changing to Browser mode!

Group Misconceptions

- 1 – Time interval
- 2 – Request data
- 3 – Most frequent errors
- 4 – Complete list of errors with relative frequencies
- 5 – Frequency distribution of selected error (per student)

International community

The screenshot shows a Windows Internet Explorer browser window displaying the Purdue University Department of Mathematics Problem of the Week website. The browser's address bar shows the URL <http://www.math.purdue.edu/pow/>. The website header includes the Purdue University logo, the text "College of Science Department of Mathematics", and navigation links for "Purdue Home", "Purdue Search", "Purdue Visit", and "Purdue Giving". A search bar is visible on the right side of the header. The main content area is titled "Problem of the Week" and "Current Problem". It features a link to "Download current problem as Adobe Acrobat document" and a section titled "About the Problem of the Week...". This section describes the program's goal to stimulate interest in mathematics and provides submission details, including the address: "Problem of the Week, Department of Mathematics, Purdue University, 150 N. University Street, West Lafayette, IN 47907-2067". It also mentions a prize fund of \$300.00. The bottom of the browser window shows the Windows taskbar with various application icons and the system clock displaying 17:47.

español

Purdue Home Purdue Search Purdue Visit Purdue Giving

PURDUE
UNIVERSITY
College of Science
Department of Mathematics

College of Science Portals >> For Future Students For Current Students For Alumni & Friends For Faculty/Staff

Department Home
About Us
Alumni & Friends
People
Courses
Academic Programs
Research
Calendar
K-12 Outreach
Resources
Jobs
Problem of the Week
Search

Problem of the Week 
Current Problem
[Download current problem as Adobe Acrobat document](#)

About the Problem of the Week...

A panel in the Mathematics Department publishes a challenging problem once a week and invites college & pre-college students, faculty, and staff to submit solutions. The objective of this is to stimulate and cultivate interest in good mathematics, especially among younger students. Solutions are due within two weeks from the date of publication and should be sent by campus or U.S. mail to:

Problem of the Week
Department of Mathematics, Purdue University
150 N. University Street, West Lafayette, IN 47907-2067

Solutions may also be faxed to 765-494-0548.
Every Purdue student who submits three or more correct solutions will receive a Certificate of Merit. Each semester, a prize fund of \$300.00 will be distributed among the Purdue undergraduates who have contributed at least six correct solutions for that semester series.

Problem History:

Internet | Mode protégé : activé 100%

Courrier ent... Montréal - ... Disque amo... Microsoft P... Department ... FR 17:47

Problem of the week

<http://www.math.purdue.edu/pow/>

Problem No. 14 (Spring 2008 Series)

Suppose a convex hexagon has vertices A_1, A_2, \dots, A_6 in clockwise order and that no side is larger than 1. Show that at least one of the major diagonals is no larger than 2. Here the major diagonals are A_1A_4 , A_2A_5 , and A_3A_6 .

- Thank you

References

- Cazes, C., Gueudet, G., Hersant, M., and Vandebrouck, F., 2006. 'Using E-Exercise Bases in mathematics: case studies at university', *International Journal of Computers for Mathematical Learning*, 11(3): 327-350, Kluwer Academic Publishers
- Ruthven, K. and Hennessy, S., 2002. A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics*, 49(2-3): 47-86.
- Sangwin, C. J., 2005. On Building Polynomials. *The Mathematical Gazette*, 89(516), 441-451.
- Vandebrouck, F. and Cazes, C., 2005. Analyse de fichiers de traces d'étudiants : aspects didactiques, *Revue STICEF*, Vol 12, ISSN : 1764-7223
- Xiao G., 2000. *Interactive Mathematics Server Journal of online Mathematics and its applications*.
<http://www.jama.org/articles/xiao/xiaotop/html>
-