Innovations and didactical studies about mathematical teaching at the first university year in France

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I. Some innovations between 1984 and 2000

II. Didactical studies linked to these innovations

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I. Some innovations

Between 1984 and 2000, due to strong incitement from the government, many innovations happened at the first university year in France. We give some examples only.

1/ Before this period, pedagogical links between teaching maths and physics, in particular about the uses of differential procedures, were tested in the University Paris 7 in 1979-80 and 1980-81, all along the first year.

2/ The scientific debate by students in lecture hall was used in the university of Grenoble ; it was the heart of an yearly teaching. This type of teaching was repeated during several years. It was especially effective in teaching integral.

3/ Teaching differential equations as graphical and qualitative objects took place in the university of Lille 1, each year from 1987 to 1996.

4/ A new way of teaching linear algebra was introduced in 1984 in the university of Lille 1, and improved each year until 1996. It was based on the fact that linear algebra formalizes, unifies and generalizes several types of mathematical activities.

5/ Projects by small groups of students took place in several universities : on mathematical subjects (Lille 1), on relations between maths and physics or other scientific domains (Lille 1, Paris 7), about scientific occupations (Lyon 1), …

6/ Different sorts of experimental teaching about mathematical and logical reasoning took place in the universities of Valence and of Grenoble.

7/ Teaching about mathematical games and reasoning in combinatorics is a regular option in Grenoble since ten years.

II. Didactical studies

A first analysis of some innovations is made in 1990 in a report of the "*Commission Inter-Irems Université*", entittled : "Teaching mathematics in another way in the first university year". In this report a more didactical approach is developed on some aspects. More than 3 000 copies was bought by mathematical teachers in the universities !

More recently, papers of A. Robert (RDM 1998) and M. Artigue (Notices of the AMS 1999) make surveys of some didactical questions about teaching and learning mathematics at the university level.

Here, we will emphasize only some particular research. It is to be noted that sometimes didactical studies were at the origin of innovations, and sometimes it was innovations which justified didactical studies.

1/ Questions about mathematical and logical reasoning

For this question, we refer to the lecture of V. Durand-Guerrier.

2/ Links between mathematics and physics about the differential and integral procedures.

Researchers in didactics (of maths and physics) in universities of Paris 7 and Grenoble made in 1987-89 an epistemological and didactical analysis of the difficulties of students to use differential or integral procedures to solve physical problems (in particular with methods for modelling). A substantial report on this subject, from a CNRS group, was published in 1989 by the Irem of Paris 7.

This sort of research is now expanding, due to a governmental decision to improve links between maths and physics in the last years of secondary teaching. Several didactical studies, experimental or theorical, on teaching and learning of differential equations in maths and physics, are developped (Grenoble, Didirem in Paris 7, Lille 1).

Epistemological analysis about teaching the notion of integral is extending the first work initiated in Grenoble university.

3/ Teaching qualitative and graphical theory of differential equations

An experimental cursus started in 1987 in the university of Lille 1. It was based on a project of M. Artigue. Her analysis of the effects of the teaching on the students led to modifications of the cursus.

In particular, the effectiveness of the introduction by fields of slopes and of the use of computer graphics was pointed out. M. Artigue also noticed that students encountered difficulties for using sharp theorems of analysis (involving the least upper bound, for ex.) for predicting the behavior of solutions. Giving some geometrical theorems about curves solutions of a differential equation appeared as quite more efficient.

It is to be noticed that this cursus required to train students in using changes of registers between graphics and formulas for functions.

4/ Studies generated by the teaching of linear algebra

(a) They were initiated by a work of A. Robert and J. Robinet. It led to a deeper analysis of the epistemological nature of the different sorts of mathematical concepts, and to studies about consequences of these differences for teaching. For example, the following properties for a notion are to be distinguish.

* Notions are formalizing, unifying and generalizing (FUG) various previous knowledge (it is the case of linear algebra) ; for these notions, it is very difficult to find initial problems ("fundamental situations" in Brousseau theory). It often seems only possible to implement in teaching a convergence of several mathematical domains in order to create a problematic leading to the FUG notions, with the aid of a "meta" discourse.

* Notions are first a tool for solving a problem ("réponse à un problème" : RAP) ; in this case, the "tool-object dialectics" of R. Douady seems an useful approach for teaching (it is the case for the notion of integral), and there often exist fundamental situations.

* Notions appear as extensions of other notions, or of previous domains of operations (uniform continuity, ring structure of integers and of real and complex numbers).

(b) Both the work of J.-L. Dorier in his thesis and the analysis on the teaching of linear algebra in the university Lille 1 reinforced this point of view about linear algebra. The book "L'enseignement de l'algèbre linéaire en questions" exposed these epistemological and didactical analysis, and compared this French approach to other in Canada and US.

Among other studies in France about linear algebra, we may mention also those developped in the university of Strasbourg (with an accent on the change of registers) and in university Paris 7 (with an accent on the necessity of flexibility in settings, registers and points of view).

5/ Studies about scientific debate and work in small groups (workshops)

Studies on scientific debate took place in the university of Grenoble (see for example "Teaching mathematics in another way in the first university year"). These studies stressed on the role of doubt in the construction of mathematical concepts, and on the role of social discussions in a scientific community.

In the same spirit (and in the same report) a didactical analysis of “work in small groups” of students (workshops) pointed out the conditions of effectiveness of such teaching. The crucial point is that enough time is allocated to students for really searching in problems solving. The determination of which kind of activities are favorable to work in small groups was studied : introduction of new notion, practice about various uses of knoledge, working with methods, modelling physical problems…

6/ Analysis of conceptions, knowledge and flexibilities of beginner students, and on gaps or differences between secondary level and first university year.

Several studies on these subjects are presented in papers published by Didirem in university Paris 7. Some results appeared in "Teaching mathematics in another way in the first university year". We will only emphazise two points.

(a) An analysis in terms of change of settings (numerical, symbolic, graphical…) led to what was called the "hypothesis of blocks". A "block" groups scores in a given setting of different items of some tests. Better chances of success are predicted for a student if he has no empty “block” (even if all his blocks are low) than if he has one empty block. This hypothesis has been confirmed for mathematical analysis (A. Robert) and for linear algebra (J.-L Dorier).

(b) F. Praslon made a sharp study on the gaps between secondary level and first university year, about the beginning of mathematical analysis. In particular, his work pointed out a great number of "micro-gaps", and proposed activities for the transition secondary-university.

7/ “Working levels” of mathematical knowledge in resolving exercices

Studies in this domain was initiated by a work of A. Robert, both on secondary and universitary levels. She pointed out three “working levels”.

* "Technical" level : the uses of mathematical knowledge are isolated and simple.

* "Mobilizable" level : it is necessary to adapt knowledge to the problem, or to change a little of point of view, or to use an intermediate step, but with an indication in the terms of the problem.

* "Available" level : students must think themselves to the knowledge or the changes of setting useful for solving the problem.

Observations in classrooms or analysis of lists of exercises given to students show that a great part (sometimes all !) of the proposed exercises are at a technical level (and often with few time for their resolution).

Effects on learning of exercices of these three types were studied in a work of J. Pian for students of the fourth university year. First, students took a test with items of these three levels, then several monthes later, they took a new but analogous test.

If N_1 and N_2 are their global notes (on 100), the "normalized progress" is defined by

$$P = (N_1/73)^2(N_2 - N_1) \quad \text{(73 is the better note).}$$

Let be t the number of items of technical level correctly solved by a student, and m the number of mobilizable or available items correctly solved. With 50 students, the regression plane in coordinates (t, m, P) of the results is

$$P = - 0.06 + 0.02\, t + 1.32\, m\ .$$

The effectiveness for progress to be able to solve exercises at the mobilizable or available levels is obvious. Such results stress the importance to make students work on exercises which are not only technical, and this requires also to give students time enough in searching other types of exercises).

III. The present situation for innovations

Since 2000, in France, the cursus in universities is splitted into mini-units of short length, and so only innovations about short range of knowledge can happen or continue. It is the case for example of the teaching of history, epistemology and didactics of maths in the university of Lyon 1, or the option about mathematical games in Grenoble.

At the contrary, for example, the teaching on the linear algebra given in Lille 1 would no more be possible : it played on the long time, with convergence of different types of knowledge, progressive abstraction and learning of methodologies.

We know only one exception, with the teaching organised by F. Pham in 2001-2003 in the university of Nice, based on a geometrization of the multivariable analysis and a naïve and formal use of differentials with a variety of meanings (as Leibniz and Bernoulli).

IV. What impact on the reality of teaching mathematics at the university level ?

1/ A total failure

None innovation we presented in this lecture and none result in didactical studies were taken in account in the cursus at the university level, neither in pedagogical practice. As soon as a leader of an innovation leaves, it disappears immediately !

I do not think to be pessimistic. There are too many obstacles in the organization of the universities and in the occupation of universitary teachers to enable such changes.

2/ An alternative : the formation of universitary teachers ?

Since 1991, some students preparing a PhD-thesis must have a formation to universitary teaching and have to teach somewhat (they are "monitors"). So there exist more than 25 "Centres d'initiation à l'Enseignement Supérieur" (CIES), where students at the thesis level must follow this formation.

In fact, few CIES give a real pedagogical formation to their monitors. Sometimes, didactical researchers take part in the formation, and so some didactical knowledge can be passed on. It is the case of studies about mathematical and logical reasoning in the CIES of Lyon, for example.

In the CIES of Grenoble, the formation is based on the initiation to scientific debate and to the constructive approach in education.

Recently, it was given in the CIES of Paris a formation based on the presentation of results of didactical studies on the teaching at the university level. The main goals are the following :

* the monitors understand the importance to make their students search with sufficiently long time ;

* they become able to analyse statements of exercises for predicting the possible real mathematical activities of their students on these exercises ;

* they understand the notion of setting, register, point of view, and the importance to lead their students to use them.

So we use with monitors different types of exercises, in order to show them these didactical analyses. We give here only one example.

"Détermine the sign of $f(x) := x^2 \cos x - \sin x$ on $[0, \pi/2]$"

We discuss with the monitors some points :

* What students' activity if we give the indication "one can factorise by $\cos x$" ? And if we do not give this indication ?

* What will students learn if we let them derive 1, 2, 3… times the function ?

* What is the role of students' failure when using this last method ?

* How long must students search before the teacher give them the key indication ?

…

***In fine*, our hope is that if sufficiently many monitors have such a formation, perhaps it will be possible to change something in the teaching of mathematics at the university level…**