WORKING GROUP 1

AFFECT AND MATHEMATICAL THINKING

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Index of papers to be presented:

THE EFFECTS OF CHANGES IN THE PERCEIVED CLASSROOM SOCIAL CULTURE ON MOTIVATION IN MATHEMATICS ACROSS TRANSITIONS

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This study investigates the effects of changes in the perceived classroom social environment on students' motivation in mathematics across the transition from primary to secondary school and during the transition from one grade level to the next within the same school (elementary or secondary school). The comparisons of students who perceived an increase, decrease or no change in the classroom social environment across the transition to middle school indicated that students' who reported a decline in their classroom social dimensions also reported a decline in social aspects of motivation and an incline in negative self-esteem. Furthermore, the effect of the changes in the classroom social dimensions on motivation were found to be larger across the transition to middle school than across the transition within elementary school, whereas they were mirrored in the secondary school transition.

BACKGROUND AND AIMS OF STUDY

The period surrounding the transition from primary to secondary school has been found to result in a decline in students' motivation in mathematics (e.g. Athanasiou & Philippou, 2007, MacCallum, 1997). This decline was found to be related to certain dimensions of the school and classroom culture (e.g. Eccles et al., 1993, Urdan & Midgley, 2003). It has been suggested that the two types of schools are very different organizations with respect to "ethos" as well as to practices and that this discrepancy influences students' motivation and performance. Most children move from a relatively small, more personalized and task-focused elementary school to a larger, more impersonal and performance-oriented middle school where they face differences in grading and teaching practices and expectations (Midgley et al, 1995).

The focus of the above studies has been on the academic aspect of motivation and of the school environment. However, students' social perceptions and goals were found to influence their motivation within a new school setting and thus are a significant part of motivation. The importance of attending to the social aspects of students' transition experiences in order to gain a fuller understanding of young adolescents' motivation in school was reinforced by the study of Anderman & Anderman (1999), in which students' social perceptions made significant, unique contributions to their achievement goal orientations. Furthermore, many longitudinal studies documented that the discontinuity in the social environment students' face across the transition to secondary school has an effect on motivation in mathematics (e.g. Eccles et al., 1993). Social discontinuities include changes in the diversity of the student population, relations with teachers and classmates and sense of school belonging.

In these studies middle school classrooms were characterized by less positive teacher-student relationships than elementary school classrooms (Midgley et al., 1995). The study of Eccles et al. (1993), revealed that the students who moved from the mathematics classroom of a high–support teacher (with respect to fairness and friendliness) to a classroom of a low-support teacher showed a decrease in their ratings of the intrinsic value and the perceived usefulness and importance of mathematics, whereas students who experienced a change from low-to-high-support teacher showed an increase in their ratings of intrinsic value. Furthermore, Anderman & Anderman (1999) found that the feeling of belonging in one's school and the endorsement of social responsibility goals were associated with an increased focus on academic tasks and predicted an increased task goal orientation, whereas endorsement of social goals for forming peer relationships and maintaining social status were associated with an increased focus on the self and predicted an increased ability goal orientation.

All the above longitudinal research shed some light on the nature of motivational change and the influence that social classroom and school environmental factors have on this process during the transition from primary to secondary school. These studies however examined motivational change for students as a whole group assuming and inferring that the transition affects all students the same way. This is not necessarily the case; recent research in the area of students' perceptions of their classroom environments supports the view that students perceive the same environment in variable ways at least on some of its dimensions (Urdan & Midgley, 2003). If there are differences in students' perceptions of their classroom environment across the transition which should really be expected, then it is possible that students perceive the transition differentially.

Despite the above theoretical considerations we are aware of only one study, by Urdan & Midgley (2003), which examined the effect of moving from a classroom perceived to emphasize a mastery goal in elementary school to a performance goal structure in secondary school (i.e. that the purpose of engaging in academic work is to develop competence or to demonstrate competence respectively). These researchers compared students who perceived an increase, decrease and no change in the mastery and performance goal structures of their classrooms during the transition to middle school and across two grades within middle school. The results of their study indicated that changes in the mastery goal structure were more strongly related to changes in cognition, affect and performance that were changes in the performance goal structure, whereas the most negative pattern of change was associated with a perceived decrease in the mastery goal structure of classrooms across the transition to middle school.

The aim of the present research is twofold. Firstly, to examine the effects of changes in the perceived classroom social environment on students' motivation in mathematics across the transition from primary to secondary school (grade 6 to 7). To this end the classroom social environment was operationalized focusing on three dimensions: (a) teacher fairness and friendliness (**FAI/FRI**), (b) cooperation and interaction (**COOP/INTE**), and (c) competition (**COMPET**), whereas students' motivation was conceptualized involving social cognitive (orientations and goals) and affective dimensions (self-esteem). Secondly, to investigate whether the changes observed in students' perceptions of classroom social environment and the related motivation across the transition to middle school are mirrored across the transition from one grade level to the next within the same school context. More specifically, the research questions are formulated as follows:

(1) What are the effects of the direction of change in the perceived classroom social environment on students' motivation in mathematics across the transition from primary to secondary school?

(2) Are the changes observed in students' perceptions of the classroom social environment and the related changes in motivation across the transition from primary to secondary school mirrored across the transition from grade 5 to 6 in elementary school and across grade 7 to 8 in secondary school?

METHOD

Participants in this study were 331 students who were followed over a period of two consecutive school years. The students were divided in three Cohorts. The 220 students in Cohort T (CT) experienced the transition from primary to secondary school (grade 6 to 7); the 42 students in Cohort E (CE) were followed over the last two years of elementary school (grade 5 to 6), and the 69 students in Cohort S (CS) were followed over the first two years in secondary school (grade 7 to 8).

Data were collected through a self-report questionnaire in the spring semester of each school year, since by that time of the year students' motivation and their perceptions of the classroom social environment are well developed and established. The questionnaire was comprised of 42 items measuring four dimensions referring to students': (a) social motivational goals (students' social reasons for engaging in math work with 14 items tapping three specific motivational goals such as competition/social power, social concern and affiliation e.g. for affiliation "In mathematics I try to work with friends as much as possible"); (b) social motivational goal orientation (4 items tapping students' perceptions of how socially oriented they are e.g. "I am most motivated when I am showing concern for others in mathematics"); (c) self-esteem in mathematics (students' perceptions of their competence in doing mathematics with 8 items tapping two dimensions such as positive and negative self-esteem e.g. for negative self-esteem "I often make mistakes in mathematics"); and (d) classroom social dimensions (16 items measuring three dimensions referring to teacher fairness/friendliness, cooperation/interaction and competition e.g. for cooperation/interaction "We get to work with each other in small groups when we do math"). The items referring to the first three dimensions were adapted from the Inventory of School Motivation Questionnaire (McInerney, Yeung & McInerney, 2000), whereas the items for the latter were adapted from the Student Classroom Environment Measure (Eccles et al., 1993). All statements were presented at a five-point Likert-type format (1=Strongly Disagree, 5=Strongly Agree). The reliability estimates were found to be quite high for all the scales ranging from α=.69 to α=.88.

Data processing was carried out using the SPSS software. The statistical procedure used in this study was Repeated Measures ANCOVA. Change group (CG-3 levels) was the independent, between-groups factor and time of measurement (TM-2 levels) was the within-groups repeated measures component. For all the analyses, gender was included as a covariate to control for any differences by gender.

In order to provide answers to the two research questions, three groups of students for each of the classroom environment variables were created. To create the three groups, students' classroom environment scores were firstly standardized. Next, the change score was calculated by subtracting students' scores on the first measurement from the respective scores on the second measurement, in each classroom dimension. The change scores for each dimension were then divided into three groups: (i) increase; (ii) no change; and (iii) decrease in classroom environment variable. The groups were created by using .50 standard deviations as the cut-off such that students in the "increase" groups scored at least half a standard deviation above the mean change score, those in the "decrease" groups scored at least half a standard deviation below the average change score, and those in the "no change" groups were within .50 standard deviations either above or below the mean change score. Half standard deviation was selected as the cut-off point to make sure that the groups created would be different from one another and yet maintain a large number of participants in order to allow comparisons across groups.

RESULTS

To answer the first research question, CT students' responses were analysed using Repeated Measures ANCOVAs. Table 1 presents the means, standard deviations and the F ratios for the Change Group x Time of Measurement interactions (CG x TM) for each of the three social dimensions change groups on each of the dependent variables. The alphabetical superscript '^{a}' within each classroom social dimension change group indicates that the means in grades 6 and 7 are significantly different from one another. Similar numeric superscripts indicate non significant differences between group means on variables measured in 7^{th} grade using univariate post hoc tests. The .05 level of significance was adopted for these comparisons.

The analyses indicated that the CG x TM effect was significant for social goal orientation, social concern and affiliation goals and negative self esteem for the **FAI/FRI** and the **COOP/INTE** change groups. Examining the results from the 6^{th} to 7^{th} grade transition, it appears that the most negative pattern of change in motivation was associated with a perceived decline in **FAI/FRI** and **COOP/INTE** classroom social dimensions. Specifically, the tests of simple effects within groups indicated that students' social goal orientation, social concern and affiliation goals were significantly lower in 7^{th} grade than in 6^{th} grade within the group that perceived a decrease in **FAI/FRI** and in **COOP/INTE** across the transition to middle school. No significant differences were found between the 6^{th} and 7^{th} grade means for either the perceived "no change" or "increase" groups. The opposite pattern was observed for negative self-esteem, i.e., students' mean ratings were significantly higher in 7^{th} grade than in 6^{th} grade within the group that perceived a decrease in **FAI/FRI** and in **COOP/INTE** across the transition. The univariate post hoc tests of 7^{th} grade means revealed that the mean ratings of students in the **FAI/FRI** and in the **COOP/INTE** "decrease" change groups on social goal orientation, social concern and affiliation goals were significantly lower than the mean ratings of students in the "no change" or "increase" groups, whereas their negative self-esteem was significantly higher. Also, the analysis of TM effect revealed a significant decline from 6^{th} to 7^{th} grade in social goal orientation ($F=3.341$, $p<0.05$), social concern ($F=8.656$, $p<0.01$) and affiliation goals ($F=2.946$, $p<0.05$) and a significant incline in negative self-esteem ($F=3.038$, $p<0.05$). Since no statistically significant differences were found between the means of students in the **FAI/FRI** and in the **COOP/INTE** "no change" or "increase" groups from primary to secondary school for social orientation, goals and negative self-esteem, these declines in orientation and goals and the incline in negative self-esteem were not evident for students who perceived no change or an increase in both the above classroom social dimensions.

The ANCOVA analyses for **COMPET** change groups indicated that the CG x TM effect was significant for social goal orientation, competition/social power, social concern and affiliation goals and negative self-esteem. The largest differences were associated with a perceived incline in **COMPET** classroom social dimension. Specifically, the tests of simple effects within groups indicated that students' social goal orientation, social concern and affiliation goals were significantly lower in 7^{th} grade than in 6^{th} grade within the group that perceived an increase in **COMPET** classroom environment across the transition from primary to secondary school. In both the perceived "no change" and "decrease" groups there weren't any significant differences between the 6^{th} and 7^{th} grade means. For competition/social power goal and negative self-esteem the opposite pattern was observed since students' mean ratings were significantly higher in 7^{th} grade than in 6^{th} grade within the group that perceived an incline in **COMPET** environment across the transition. The univariate post hoc analyses of 7^{th} grade means revealed that the mean ratings of students in the **COMPET** "increase" change group on social goal orientation, social concern and affiliation goals were significantly lower than the mean ratings of students in the "no change" or "decrease" groups, whereas their competition/social power goal and negative self-esteem were significantly higher. Also, the analysis of TM effect revealed a significant decline in social goal orientation ($F=3.427$, $p<0.05$), social concern (F=9.507, p<0.01) and affiliation goals (F=3.105, p<0.05) from 6th to 7th grade and a significant incline in competition/social power goal (F=9.144, p<0.01) and negative self-esteem (F=3.247, p<0.05). Since there were no statistically significant differences between the means of students in the **COMPET** "no change" or "decrease" groups from primary to secondary school for social orientation, competition/social power, social concern and affiliation goals and negative self-esteem, these declines in orientation and goals and the incline in competition/social power goal and negative self-esteem were not evident for students who perceived no change or a decrease in the **COMPET** classroom social environment.

Dependent Variables	Teacher fairness and friendliness change groups (FAI/FRI)						F Interaction: CG by TM
	Decrease (N = 62)		No change (N = 89)		Increase (N = 69)		
	6th grade	7th grade	6th grade	7th grade	6th grade	7th grade	
Social goal orientation	3.23[a] (.81)	2.60[1] (.93)	3.09 (.97)	2.90[2] (.95)	3.14 (.84)	2.99[2] (1.03)	4.873***
Compet/social power	2.03 (.95)	2.47 (1.14)	2.24 (1.00)	2.72 (1.12)	2.25 (.89)	2.46 (1.12)	ns
Social concern goal	3.91[a] (.87)	3.30[1] (89)	3.95 (.91)	3.79[2] (.99)	3.90 (.93)	3.72[2] (1.10)	3.987***
Affiliation goal	3.17[a] (.89)	2.73[1] (1.05)	3.40 (.96)	3.27[2] (.95)	3.32 (.93)	3.11[2] (.91)	4.268***
Positive self-esteem	3.70 (.75)	3.06 (1.07)	3.81 (.69)	3.42 (1.10)	3.79 (.74)	3.21 (1.19)	ns
Negative self-esteem	3.21[a] (.88)	3.97[1] (.86)	3.27 (.86)	3.49[2] (.91)	3.39 (.77)	3.48[2] (.88)	5.488***
Dependent Variables	Classroom cooperation and interaction change groups (COOP/INTE)						F Interaction: CG by TM
	Decrease (N = 78)		No change (N = 63)		Increase (N = 79)		
	6th grade	7th grade	6th grade	7th grade	6th grade	7th grade	
Social goal orientation	3.27[a] (.81)	2.56[1] (.84)	3.23 (.80)	2.98[2] (1.00)	2.96 (.99)	3.03[2](1.05)	6.581*
Compet/social power	1.95 (.84)	2.43 (1.15)	2.24 (.95)	2.46 (.97)	2.37 (1.02)	2.79 (1.20)	ns
Social concern goal	4.02[a] (.88)	3.33[1] (.97)	4.05 (.73)	3.83[2] (1.00)	3.73 (1.01)	3.73[2](1.01	5.912**
Affiliation goal	3.37[a] (.98)	2.86[1] (.99)	3.35 (.83)	3.18[2] (.89)	3.22 (.95)	3.19[2] (.99)	4.259***
Positive self-esteem	3.75 (.76)	3.07 (1.15)	3.80 (.73)	3.18 (1.07)	3.77 (.69)	3.49 (1.12)	ns
Negative self-esteem	3.30[a].(86)	3.92[1] (.95)	3.32 (.72)	3.51[2] (.82)	3.26 (.90)	3.42[2] (.88)	5.018**
Dependent Variables	Classroom competition change groups (COMPET)						F Interaction: CG by TM
	Decrease (N = 76)		No change (N = 64)		Increase (N = 80)		
	6th grade	7th grade	6th grade	7th grade	6th grade	7th grade	
Social goal orientation	3.24 (.80)	3.08[1] (.96)	3.09 (.92)	2.92[1] (.91)	3.11[a] (.93)	2.59[2] (1.03)	4.785***
Compet/social power	2.35 (1.06)	2.48[1] (1.16)	2.22 (.97)	2.40[1] (1.12)	2.00[a] (.79)	2.79[2] (1.08)	4.955***
Social concern goal	3.88 (.94)	3.72[1] (.95)	3.92 (.88)	3.74[1] (.89)	3.97[a] (.88)	3.42[2] (1.10)	3.877***
Affiliation goal	3.40 (1.00)	3.36[1] (1.04)	3.24 (.87)	3.17[1] (.88)	3.29[a] (.91)	2.71[2] (.95)	3.744***.
Positive self-esteem	3.91 (.70)	3.21 (1.07)	3.71 (.65)	3.25 (1.10)	3.69 (.78)	3.30 (1.21)	ns
Negative self-esteem	3.39 (.89)	3.55[1] (.90)	3.21 (.75)	3.41[1] (.87)	3.27[a] (.86)	3.85[2] (.89)	4.057***

Table 1: Means, Standard Deviations and Summary of Repeated Measures ANCOVAs on motivational variables by changes in classroom social dimensions

***p<0.001 **p<0.01 ***p<0.05**

To answer the second research question, the same set of analyses were conducted as students moved from 5th to 6th grade in elementary school (CE) and from 7th to 8th grade in secondary school (CS). Table 2 presents the means and the F interaction (CG x TM) for all the classroom social dimension change groups for students in CE and CS. Standard deviations are not presented due to space limits.

Regarding the comparability of results involving the direction of changes in classroom social dimensions between the elementary to secondary school transition (grade 6 to 7) and the elementary school transition (grade 5 to 6), the patterns of results involving all the classroom social dimensions change groups across the transition from primary to secondary school were not replicated during the elementary school transition. There were no significant interactions for **COMPET** change groups, whereas for **FAI/FRI** and **COOP/INTE** only one significant interaction was observed involving social goal orientation with students' perceptions across the transition within elementary school changing the same way as the perceptions of students across the transition from primary to secondary school.

	Teacher fairness/friendliness (FAI/FRI)				Cooperation/interaction(COOP/INTE)				Competition (COMPET)			
	CE		CS		CE		CS		CE		CS	
	5th	6th	7th	8th	5th	6th	7th	8th	5th	6th	7th	8th
Social orientation												
Decrease	3.44^{a}	3.12^{1}	3.36^{a}	2.62^{1}	3.47^{a}	3.02^{1}	3.50^{a}	2.57^{1}	3.33	3.37	3.39	3.25^{1}
No change	3.75	3.60^{2}	3.12	2.95^{2}	3.37	3.48^{2}	3.25	3.17^{2}	3.40	3.36	3.14	3.08^{1}
Increase	3.31	3.57^{2}	3.12^{a}	3.43^{3}	3.62	3.71^{2}	2.85^{a}	3.09^{2}	3.66	3.55	2.96^{a}	2.57^{2}
F Interaction: CG by TM	3.181***		6.145**		3.560***		5.562**		ns		2.991***	
Compet/social power goal												
Decrease	2.15	2.17	2.63	2.27	2.43	2.25	2.73	2.51	2.20	2.23	2.63^{a}	2.24^{1}
No change	2.16	2.19	2.65	2.56	2.06	2.07	2.58	2.39	2.28	2.07	2.59	2.44^{1}
Increase	2.17	2.13	2.65	2.72	2.05	2.20	2.66	2.77	2.05	2.16	2.73^{a}	3.04^{2}
F Interaction: CG by TM	ns		ns		ns		ns		ns		4.777***	
Social concern goal												
Decrease	3.86	3.71	4.00^{a}	3.66^{1}	3.18	2.89	3.59^{a}	3.05^{1}	3.66	3.30	4.01	3.89^{1}
No change	3.80	3.50	3.71	3.62^{1}	3.96	3.90	3.94	3.80^{2}	3.79	3.93	3.73	3.77^{1}
Increase	3.78	3.75	3.72	3.73^{1}	4.17	4.03	3.71^{a}	3.94^{2}	3.95	3.77	3.55^{a}	3.19^{2}
F Interaction: CG by TM	ns		2.998***		ns		3.840***		ns		3.241***	
Affiliation goal												
Decrease	3.38	3.05	3.68^{a}	3.27^{1}	3.41	2.89	3.46^{a}	2.71^{1}	3.26	3.25	3.29	3.05^{1}
No change	3.61	3.26	3.12	2.92^{2}	3.18	2.87	3.39	3.28^{2}	3.29	2.88	3.11	3.16^{1}
Increase	3.00	2.70	2.75	2.89^{2}	3.35	3.19	2.57^{a}	2.80^{1}	3.35	2.83	3.03^{a}	2.73^{2}
F Interaction: CG by TM	ns		3.125***		ns		4.553***		ns		3.310***	
Positive self-esteem												
Decrease	4.01	3.97	3.34	3.23	3.31	3.49	3.34	3.21	3.51	3.44	3.47	3.27
No change	3.73	3.67	3.48	3.32	4.12	3.91	3.51	3.35	4.13	4.15	3.42	3.26
Increase	3.87	3.87	3.73	3.52	4.05	4.08	3.66	3.46	3.98	3.97	3.71	3.59
F Interaction: CG by TM	ns		ns		ns		ns		ns		ns	
Negative self-esteem												
Decrease	3.40	3.42	3.12^{a}	3.60^{1}	3.06	3.16	3.59^{a}	3.88^{1}	2.94	3.00	3.30	3.20^{1}
No change	3.34	3.36	3.29	3.16^{2}	3.43	3.42	3.09	3.08^{2}	3.72	3.76	3.26	3.19^{1}
Increase	3.01	3.01	3.41	3.34^{2}	3.16	3.16	3.33	3.23^{2}	3.16	3.14	3.27^{a}	3.63^{2}
F Interaction: CG by TM	ns		3.565***		ns		3.243***		ns		2.987***	

Table 2: Means and Summary of Repeated Measures ANCOVAs on motivational variables by changes in classroom social dimensions for students in CE and CS

***p<0.001 **p<0.01 ***p<0.05**

On the contrary, the patterns of changes in classroom social dimensions change groups for students across the transition from primary to secondary school were mirrored for students across the transition within secondary school, with some notable exceptions. Firstly, social goal orientation increased significantly from 7th to 8th grade among those students who perceived an increase in **FAI/FRI** and **COOP/INTE** classroom social environment but decreased significantly for those students who perceived a decrease in **FAI/FRI** and **COOP/INTE** social environment over time. A similar pattern was observed for the analysis regarding social concern and affiliation goals as the dependent variable for the **COOP/INTE** social dimension. In addition, the comparison of the differences found across the transition to secondary school (6th to 7th grade) with those found during middle school (7th to 8th grade) among the **COMPET** social dimension change groups revealed similar directions of change for social orientation, social concern and affiliation goals and negative self-esteem. However, a significant difference over time was found for the competition/social power goal. The students who moved from 6th to 7th grade and perceived an increase in the **COMPET** social dimension of their classroom reported endorsing competition/social power goals significantly more, whereas students in the no change or decrease groups did not change significantly in their adoption of competition/social power goal. But when students moved from 7th to 8th grade, the endorsement of competition/social power goal decreased significantly among those students who perceived a decrease in the **COMPET** social environment over time.

DISCUSSION

The results of the study suggest that when students make the transition to middle level schools they are likely to move into classrooms that are characterized by less teacher-student relations, less cooperation and interaction whereas competitiveness is emphasized. Despite those general trends, there are students who perceive no difference in their classroom social environment before and after the transition and other students who perceive an increase in their classroom social orientation. Recent studies have contributed to our understanding of what occurs within classrooms, but nothing is known about the effects of moving from one classroom social environment to another. Thus, while it has been documented that the classroom social environment changes after the transition from primary to secondary school, it remains unclear what effects these differences might have on students' motivation in mathematics. The present study shed some light on these issues.

More specifically, the results of the study revealed that students who reported a decline in their classroom social environment across the transition to middle school also reported a decline in the social aspects of their motivation and an increase in negative self-esteem. Also, it was found that among students who reported an increase in the social environment of their classrooms after the transition, the general negative pattern of change in motivation was not evident. These results suggest that whereas a perceived increase in classroom social dimensions has advantages, the disadvantages associated with a perceived decrease in the classroom social environment are even stronger. Perhaps social messages in the classroom are more evident to students when they are first removed than when they are perceived to be added. In other words, students may not notice the presence of social dimensions in the classroom as much as they notice their absence. This may be particularly true when students move from what has been described as the more nurturing elementary school environment to the more impersonal middle school classroom environment (Anderman & Anderman, 1999).

The changes in motivation associated with changes in the perceived classroom social dimensions were found to be larger during the transition to middle school than they were during the last two years in primary school. This finding is pretty logical taking into consideration the fact that the classroom environment in elementary school is almost the same across grades. On the contrary, the effect of changes in the perceived classroom social environment and changes in motivation that were found across the transition to middle school were replicated within the first two yeas of middle school. Therefore, the stress of moving to middle level schools does not enlarge the size of the effects of changes in the perceived classroom social dimensions on motivation, despite the fact that previous research has documented that the transition to middle level school can be a stressful time in students' lives (e.g. Eccles et al., 1993).

Although the size of the changes in motivation associated with changes in the perceived classroom social environment were quite similar across the transition to middle school and within the first two years in middle school, there were some interesting differences in the direction of the changes and in which change groups the largest differences were found. The changes in the means were largest among students in the decrease groups for **FAI/FRI** and **COOP/INTE** dimensions from 6th to 7th grade. For students in the 7th to 8th transition the differences within these groups remained whereas differences in the **FAI/FRI** and **COOP/INTE** increase groups were found since students' who perceived an increase in the above social dimensions reported higher social orientation and goals and lower negative self-esteem. It also appears that the pattern of change among the **COMPET** social dimension change groups differed across the two time periods of the study. For example, the **COMPET** increase group reported a decrease in motivation from 6th to 7th grade, whereas when students made the transition from one grade to the next within middle school the **COMPET** decrease group reported an increase in their motivation.

These shifting patterns of results are evident due to the fact that the transition to middle school influences the salience of the presence or absence of social messages in the classroom (Anderman & Anderman, 1999). When moving from a smaller and perhaps more social oriented elementary school environment to a middle school environment, students may be particularly aware of decreases in the emphasis on social orientations and goals in the classroom, creating stronger effects on motivation among those students who perceive a decrease in the classroom social environment.

Once familiar and comfortable with the middle school environment, however, increases in the classroom social environment become as salient as decreases and the effects of these two types of change become more even.

The findings of the present study highlight the effects of changes in the classroom social environment on students' motivation in mathematics during the transition from one school context to another or from one grade level to the next within the same context. Therefore, longitudinal studies examining these issues can assist in unravelling the complexity of motivational change across transitions. Such studies should examine different aspects of motivation (academic, social and affective) and various dimensions of the classroom or school environment. This multidimensional perspective is very important in order to understand not only the effects of what is more prevalent in classrooms but in determining what the most facilitative environments are, even if they are uncommon, in order to test the effects of these environments on the nature of change in students' motivation in mathematics. Such information will be useful for teachers, educators and policy makers in their planning to make systemic transitions easier so fewer students are lost.

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“MATHS AND ME”:

SOFTWARE ANALYSIS OF NARRATIVE DATA ABOUT ATTITUDE TOWARDS MATH

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Some years ago we undertook a research study aimed to obtain a ‘grounded’ characterization of attitude toward maths through the use of a narrative tool: we assigned to a large sample of Italian students the essay “Maths and me”, collecting more than 1600 texts. In this contribution we present some preliminary results, obtained using a piece of software for text analysis, regarding the way students of different grades describe their relationship with mathematics. In particular, we discuss the results from a comparative analysis between students of different school levels in order to find analogies and differences in the description of their own relationship with maths.

INTRODUCTION

Many research studies carried out in the last two decades in mathematics education highlight the relevance of affective factors to analyze and interpret students’ maths-related difficulties, and a specific field of research developed in recent years (for an overview see Zan R., Brown L., Evans J., Hannula M. 2006).

Among the affective factors, attitude toward mathematics is one of the most quoted constructs (by researchers in the field, teachers and educational institutions), but this “object” does not seem to have a well-defined and shared meaning. Among studies that explicitly give a definition, we can recognize three main different characterizations of attitude towards mathematics:

a) a “*simple*” definition, that describes attitude as the positive or negative degree of affect associated with mathematics (Haladyna, Shaughnessy J. & Shaughnessy M., 1983; McLeod, 1992);

b) a “*tridimensional*” definition, that recognizes three components in attitude: the degree of affect associated with mathematics, the beliefs regarding mathematics and the behaviour related to mathematics (Hart, 1989);

c) a “*bidimensional*” definition, that includes only emotions and beliefs and does not consider behaviour (Daskalogianni & Simpson, 2000).

Some critical issues are linked to the choice of a definition for attitude (Di Martino & Zan, 2001), in particular: the consistency between the chosen definition of attitude and the instruments to observe/measure it, the definition of *positive/negative* attitude in the case of multidimensional characterizations. To characterize students’ attitude toward mathematics *from the bottom,* we carried out a narrative study investigating which dimensions students use to describe their relationship with mathematics. After the characterization with the same data we could compare attitude of students belonging in different school levels.

In the field of mathematics education, narratives are more and more often used, especially in research about teachers' beliefs and teachers' practice (f.e. Da Ponte, 2001). Outside the field of teacher education, less numerous studies about *affect* make use of narratives: some have adults as their object (Karsenty & Vinner, 2000), others used narrative to report their own research (Hannula, 2003), others have students as their object (Ruffell et al.,1998). In this last case the studies are often carried out to criticize traditional instruments used to observe attitude rather than to carachterize from the bottom the construct itself.

We used students' narratives (autobiographic essay), confident that in this way students could have the possibility to talk about the aspects *they* considered relevant in their own experience with mathematics. The chosen instrument is consistent with an interpretive approach and allows many typologies of data analysis.

From a qualitative analysis of students' description of their relationship with mathematics (Di Martino & Zan, submitted), a multidimensional model for attitude toward mathematics emerges, characterized by three strictly interconnected dimensions: the emotional disposition toward mathematics, the view of mathematics, the perceived competence in mathematics. That suggests the need to overcome the dichotomy between positive/negative attitude, and move to the identification of different profiles of negative attitude.

In this contribution, we present a quantitative analysis of the same data carried out with the help of T-Lab [1], a powerful software for text analysis, giving some preliminary interpretations of these results: in particular comparing the attitude of students from different educational levels.

METHODOLOGY

We proposed the essay "Me and mathematics: my relationship with maths up to now" to students from different school levels. For the administration of the essays we gave the following guidelines: essays had to be anonymous, assigned and collected in the class not by the mathematics teacher. At the end, we collected 1662 essays [2] ranging from grade 1 to grade 13: 874 from 51 classes of 14 primary schools (grade 1-5); 368 from 24 classes of 8 middle schools (grade 6-8); 420 from 29 classes of 10 high schools (grade 9- 13).

In order to perform the statistical analysis with T-Lab we typed all data in a unique Corpus, respecting some specific guidelines, and we classified all essays with three control variables: identification number, grade and school level.

After this phase of data coding, we started to set the customized settings: selection of the lexical units to be included in the analysis, management of the lemmatization's phase, that is the reduction of the Corpus to their respective headwords called *lemmas* (for example general rules of lemmatization are: verbs' forms are taken back to the infinite tense, nouns to the singular form, and so on).

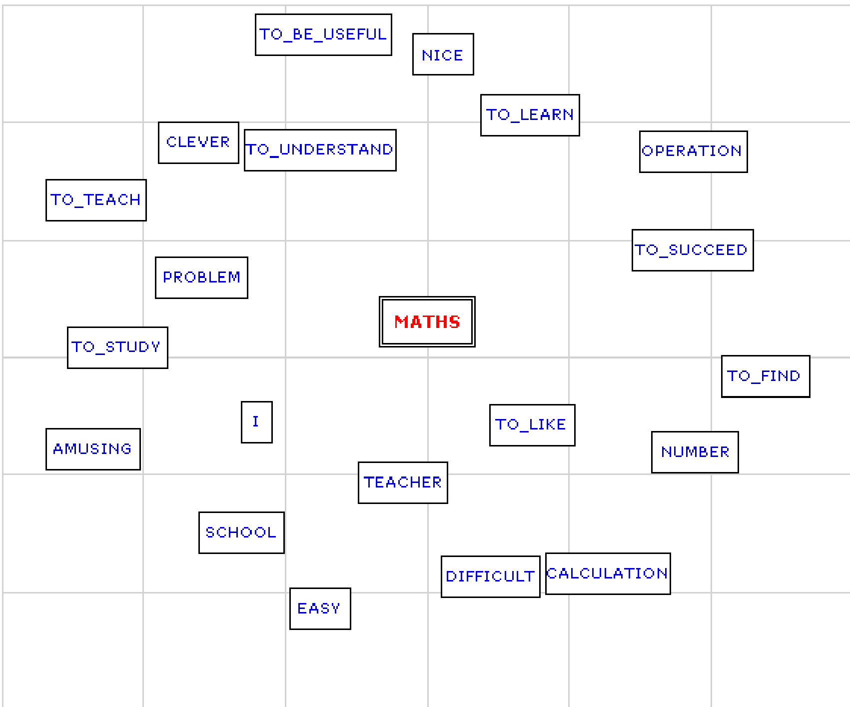
RESULTS AND DISCUSSION

Our attention will be focused on two typologies of analysis: co-occurrence and comparative analysis. The first one is finalized to find lexical units that more frequently are in co-occurrence [3] with some specific lemma, the latter is finalized to identify differences between texts from different subsets of the Corpus identified by some variables (in our case we selected the variable *school level*).

Co-occurrence analysis

Starting from the choice of the key-term '*maths*', the software calculates, in the whole Corpus, the lemmas with more co-occurrence with it through the association index of cosine [4]. This is a way to have a preliminary idea about the lexical units that students, in their autobiographical essays, more frequently associated with maths. Graph 1 is one of the outputs of the analysis: the nearness of each lemma to the central lemma '*maths*' is proportional to its degree of association.

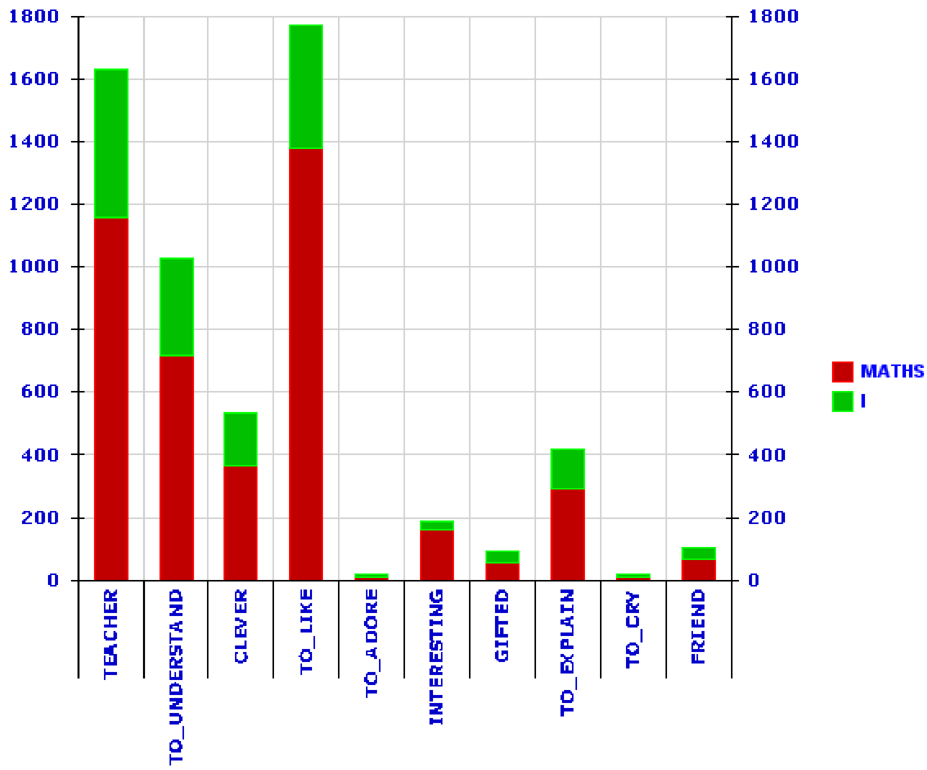
Graph 1: Lemmas associated with *maths*



This representation strikingly shows that the emotional disposition (concisely expressed by "*I like/do not like maths*") is very often in co-occurrence with maths: this is an indication that students tend to express their emotional disposition toward mathematics when they tell their relationship with mathematics itself. Moreover, the nearness of '*teacher*' can be interpreted in light of the fact that students recognize the teacher as a protagonist of their story with maths. For what concerns '*I*', it is obvious that, in an autobiographical essay regarding the writer's relationship with maths, the lemmas *I* and *maths* are in co-occurrence.

Another analysis enables us to find the lemmas that are more correlated to both terms: *I* and *maths*. In graph 2 the co-occurrence with the two terms is shown in decreasing order with respect to the chi square test [5].

Graph 2: co-occurrences with *I* and *maths*



The relevance of the *teacher* in students' building of their own relationship with maths seems to be confirmed. But other two dimensions emerge heavily: an affective one (linked to lemmas as *to_like*, *to_adore*, *to_cry* and also *friend*) and one correlated with the idea of success in maths (associated to lemmas as *to_understand*, *clever*, *gifted*).

Comparative analysis

As we said earlier, with this typology of analysis we try to underline the differences between the three groups of students, as identified by the variable '*school level*'.

The first analysis regards the specificities of each group: T-Lab compares the subset A of the Corpus with the rest of the Corpus, individualizing which lexical units are typical (by the Chi-square test) or exclusive of the subset A. In table 1, for each group (Primary, Middle, High) the ten lemmas with the biggest chi-square value are reported.

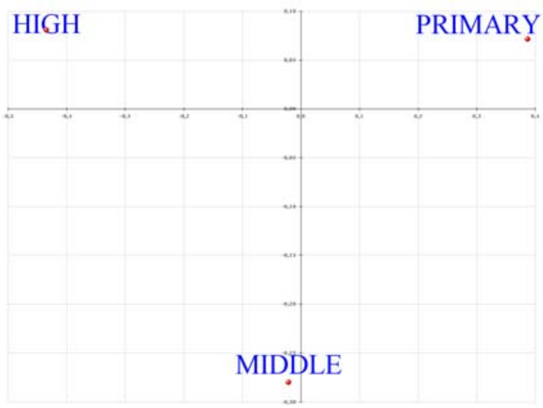
Table 1: Specificities of three school levels

Primary school				Middle school				High School			
WORD	CHI²	SUB	TOT	WORD	CHI²	SUB	TOT	WORD	CHI²	SUB	TOT
to_like	459,7	1560	2488	expression	63,7	114	282	to_succeed	254,5	645	1078
operation	222,2	397	539	rule	38,3	50	111	school	185,8	399	638
to_learn	166,0	535	837	to_study	31,2	211	714	to_study	87,9	382	714
nice	149,8	302	423	algebra	27,2	48	118	exam	72,2	66	80
amusing	141,9	284	397	complex	25,9	39	91	time	69,1	156	252
examination	135,4	179	222	arithmetic	21,6	56	154	task	65,9	201	349
number	111,3	410	658	complicated	20,1	56	157	method	60,3	64	82
geometry	89,0	376	619	Easy	17,9	119	401	teacher	52,8	955	2168
calculation	67,6	250	401	important	17,3	84	267	to_apply	52,2	56	72
error	63,5	151	220	maths	14,0	1288	5603	insufficient	49,9	49	61

Sub = number of word's occurrences in subset, Tot = number of word's occurrences in Corpus

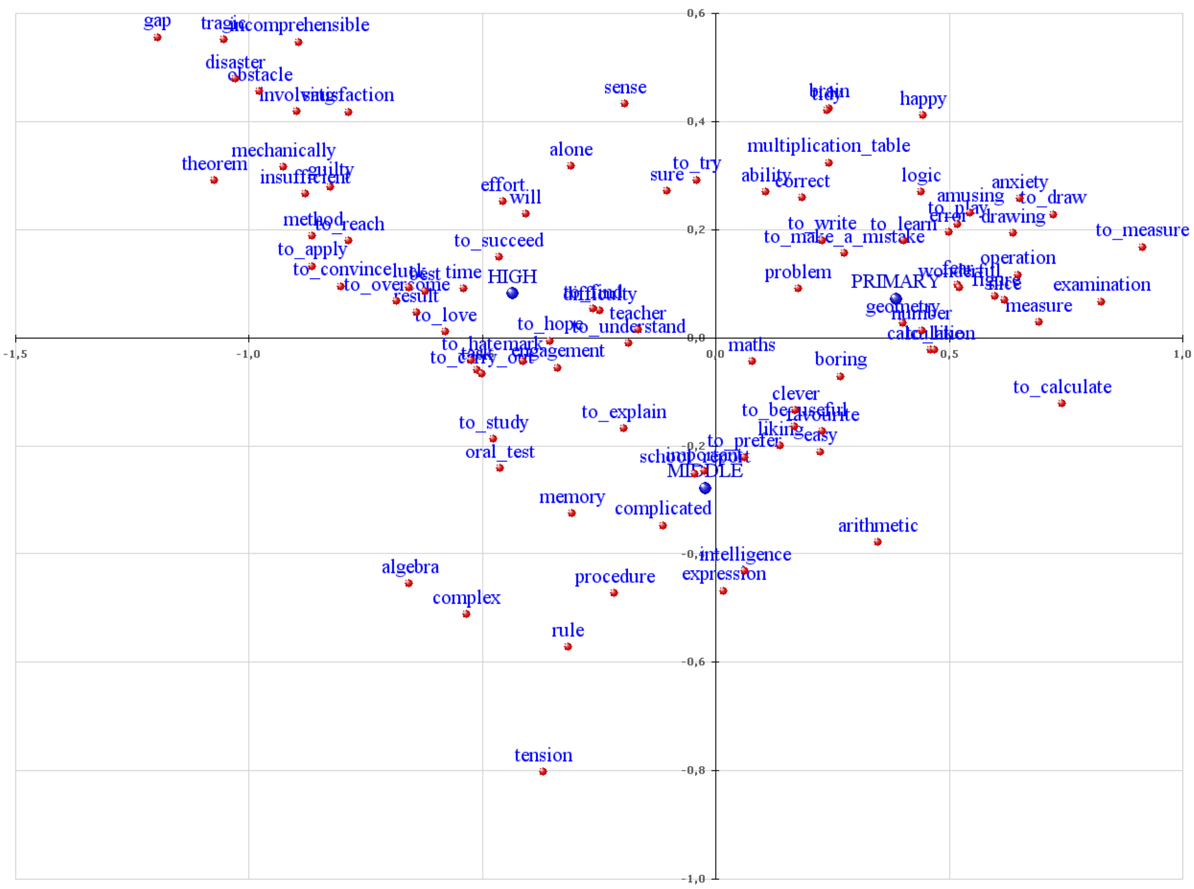
One interesting remark is about the strong characterization of the two extreme groups (Primary and High), testified by very high chi-square values. Moreover, looking at the first lemmas for each group, we can observe a shift from a *mastery-oriented* view (*to_learn*) of the relationship to a *performance-oriented* view (*to_succeed*, *exam*) and it is also interesting that the two first lemmas of the Middle group are related to an instrumental view of mathematics. According to our qualitative findings (Di Martino & Zan, submitted), this instrumental view is often combined with negative emotions towards mathematics and low perceived competence. This can be a possible explanation of the fact that in Italy the relationship with mathematics often becomes problematic just at middle school level. A factorial analysis allows us to characterize more precisely the specificity of the three groups: we can visualize their position in the factorial plane.

Graph 3: variables' position in factorial plane



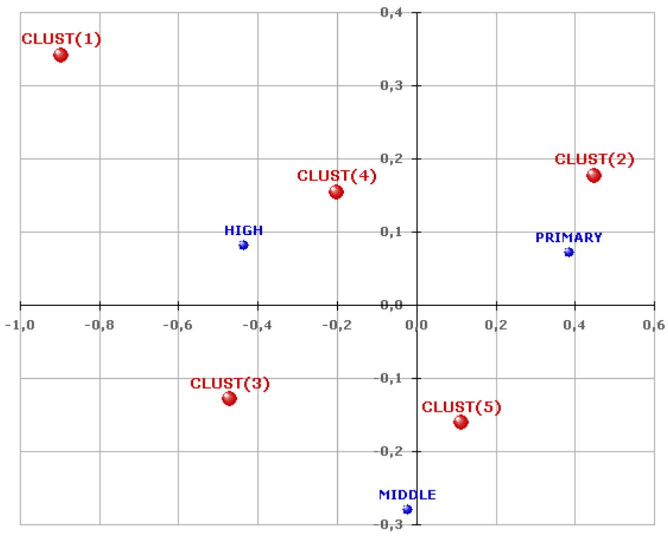
So we can observe that Primary and High groups are opposite poles in the X axis, while Middle group is characterized by its negative Y-component. In graph 4 all lemmas that define the factorial plane are reported: this allows us to interpret the meaning of the distance between the three groups:

Graph 4: lemmas in factorial plane

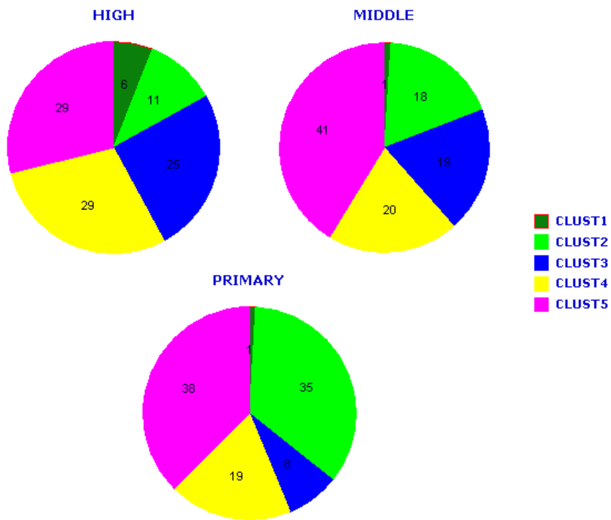


This analysis confirms the interpretations following the analysis of specificities. In particular, the Primary group seems to be characterized by descriptive – illustrative lemmas regarding mathematics (*geometry*, *number*, *calculation*, *measure*, *problem*) and by an often positive judgement of one's mathematical experience (*wonderful*, *nice*). The Middle group, strongly positioned at the negative pole of the Y-axis with respect to the other two groups, has many lemmas referring to an instrumental view of mathematics (*procedure*, *memory*, *rule*). Moreover, this group is in the 0 of the X-axis that is also characterized by emotional responses. Finally, the High group is characterized by very strong emotions (*to_love*, *to_hate*) and also by a particular attention to succeed (*to_succeed*). To summarize these results, it seems that at the beginning of the school experience with mathematics, curiosity prevails over other aspects and *novelty* is often appreciated. Besides, there is little stress related to assessment. After the move to the middle school level, students' attention seems to shift toward some procedural aspects of mathematics, so an instrumental view of mathematics emerges. This view rarely arouses a strong passion (negative or positive). In High school we find opposite lemmas for what concerns emotions (*love, hate*) but also perception of success; perhaps, this means that the relationship toward mathematics of these students becomes more radical than the relationship reported by their youngest colleagues. These interpretations are also reinforced by the cluster analysis that we performed with a partitioning method. We fixed to 5 the cluster numbers because with a smaller one we hadn't a clear distinction between groups identified by variables. We briefly report a table with the lemmas characterizing each cluster and the relationship between clusters and variables.

Graph 5: clusters and variables



Graph 6: Percentage groups subdivision in clusters



The percentages of cluster 1 are very small but it is present for any subdivision in clusters more than two. From an evolutionary point of view, we can observe that cluster 2 becomes less representative passing from 35% at Primary level to 11% at High level and cluster 5 is more or less stable from Primary to Middle level but becomes less representative at High level. While clusters 3 and 4 increase the number of their representatives. So it is very interesting to give a look to lemmas that characterize these four clusters in the following table (lemmas are in decreasing order of relevance):

Table 2: description of clusters 2, 3, 4 and 5

Cluster 2

to_like, to_learn, number, geometry, operation, nice, calculation, amusing, error, examination, multiplication_table, to_write, to_make_a_mistake, fear, figure, logic, to_calculate, measure, drawing, correct, to_discover, wonderful, get_angry, question, to_play, to_draw, ability, exercise_book, brain, to_read, happy, to_worry, to_measure, anxiety, to_reproach, tidy, heart, to_sweat_blood, to_cry, gaiety, punishment, to_bore, mysterious, angry, test

Cluster 3

to_study, school, to_explain, mark, task, engagement, time, to_hate, to_hope, to_improve, to_carry_out, algebra, to_comprehend, rule, explanation, complex, oral_test, course_book, luck, best, future, to_love, worsening, resolution, cause, gifted, sincere, memory, reasoning, patience, to_overcome, positive, passion, to_forget, fundamental, serious, set_theory, possible, negative, genius, unpleasant, to_attract, to_fascinate, to_repeat_year, competition, to_give_up, theory, able, procedure, nightmare, frightened, torment, unlucky, serene, unbearable, tension, surprise, to_persecute, suffering

Cluster 4

teacher, to_understand, to_succeed, to_find, to_think, difficulty, interesting, to_know, to_believe, to_talk, formula, to_try, attention, will, ugly, to_memorize, immediately, friend, truth, effort, blackboard, sure, alone, strange, to_appreciate, idea, quiet, pleasant, clear, to_reflect, confuse, to_upset, experience, impossible, to_imagine, sense, thought, reality, stupid, to_resign, terrible, dream, terror, to_make_curious, hateful, slow, pride, success, disgusting, sadness, horrible, shame

Cluster 5

maths, I, problem, difficult, clever, to_teach, easy, boring, exercise, to_be_useful, certainty, expression, important, to_solve, simple, liking, arithmetic, useful, complicated, to_reason, game, quickly, severe, exciting, happiness, school_report, mathematician, to_implement, fascinating, tiring, to_support, challenging, to_listen, intelligence, shout, dubious, to_confuse, tremble

Cluster 2 is centred on the description of the *objects* of mathematics as well as on related activities (*to_learn, number, geometry, operation, calculation, multiplication_table, to_write, figure, logic, to_calculate, measure, drawing, to_discover, to_play, to_draw, exercise_book, to_read, to_measure*). Cluster 3 centres on theories of success (*to_study, engagement, time, to_comprehend, rule, cause, gifted, memory, reasoning, patience,...*) like cluster 4 (*to_understand, to_succeed, to_find, to_think, to_know, to_believe, formula, to_try, attention, will, effort,...*), but whereas cluster 3 seems to be projected ahead (*to_hope, to_improve, future, to_overcome*), cluster 4 seems to be more *static* and centred on a definitive evaluation of what happened (*impossible, to_resign*,...), cluster 5 seems to be the cluster of balance between difficulties (*difficult, simple, complicated,…*) and usefulness (*to_be_useful, important,…*). Finally, all four clusters have some emotional components: surely clusters 3 and 4 are characterized by lemmas that evoke stronger emotions (*to_hate*, *to_love*, *nightmare*, *frightened*, *torment*, *tension*, *to_persecute*, *suffering* for cluster 3 and *terrible*, *terror*, *disgusting*, *hateful*, *pride*, *horrible*, *shame*) than cluster 2, which seems to be the one with the highest number of lemmas linked to *positive* emotions, and cluster 5.

CONCLUSIONS

An important aspect of the described research study is the combination of quantitative analysis with an interpretive approach. All the results we got led us to interpretive hypotheses, that become stronger if compared to, and interconnected with, the qualitative analysis performed on the same material (and partially described in Di Martino & Zan, submitted). We point out that if on the one hand, the obtained results offer extremely interesting stimuli, on the other hand they cannot provide certainties, due to the type of material we analyzed (*open* texts). In this case, we really ought to be cautious: the analysis of open texts based on lexical units only, without an analysis of the contexts within which these lexical units are used, might be problematic. To exemplify, the lemma *to_like* is not always referred to mathematics; the word *problem* might stand for a mathematical problem but also for a real life problem. Therefore, it was really important to compare results of this analysis with those of the qualitative one (described in Di Martino & Zan, ibidem): in particular, the results about the three dimensions characterizing attitude towards mathematics are confirmed.

The 'evolutionary' results that emerge from cluster analysis seem to be particularly interesting. A general deterioration of students' relationship with mathematics can be clearly detected but, most of all, as the school level increases, the lemmas used to describe one's relationship with mathematics suggest that the latter becomes more and more radical. Moreover, there seems to be a move from a phase of interest in the novelty of mathematics -the pleasure of discovery- to a phase in which succeeding prevails over the subject matter itself. One final remark: the fact that in this phase emotional aspects become more radical provides material for further reflection.

NOTES

1. The bibliography related to T-lab is available on-line: http://www.tlab.it/en/presentazione.asp

2. The collected essays constitute a convenient sample, obtained through a collaboration with teachers and heads of schools who accepted our requests. The schools are situated in six different area of Italy: from North to South.

3. Co-occurrence is when two or more lemmas are present together in the same text.

4. To calculate the cosine index between lemma X and lemma Y we have to consider a = # of essays with lemma X and Y, b = # of essays with lemma X and without lemma Y, c = # of essays with lemma Y and without lemma X. Cosine (lemma X, lemma Y) = a / square root of (a + b) x (a + c).

5. The Chi-square test is a well-known test used to check if the frequency values obtained by a survey are significantly different from the theoretical ones. T-Lab applies this test to 2x2 tables then the threshold values is 3.84 (df=1, p=0.05) or 6.64 (df=1, p=0.01).

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STUDENTS' BELIEFS ABOUT THE USE OF REPRESENTATIONS IN THE LEARNING OF FRACTIONS

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Cognitive development of any mathematical concept is related with affective development. The present study investigates students' beliefs about the use of different types of representations in understanding the concept of fractions and their self-efficacy beliefs about their ability to transfer information between different types of representations. The interest is concentrated on differences among students at primary and secondary education. Results indicated that students at secondary education have less positive beliefs for the use of representations at the learning of mathematics than at primary education. As a consequence they have less positive self-efficacy beliefs about their abilities to use them. Unexpected was their lower performance at solving tasks on fractions for which the information is represented in different forms.

Keywords: representations, beliefs, self-efficacy

Mathematics is a specialized language with its own contexts, metaphors, symbol systems and purposes (Pimm, 1995). From an epistemological point of view there is a basic difference between mathematics and other domains of scientific knowledge as the only way to access mathematical objects and deal with them is by using signs and semiotic representations (Duval, 2006). Cognitive development is related with metacognitive and affective development. One's behavior and choices, when confronted with a task, are determined by her/his beliefs and personal theories, rather than her/his knowledge of the specifics of the task. Thus, students' academic performance somehow depends on what they have come to believe about their capability, rather than on what they can actually accomplish.

The relationship between cognition and affect has the last decades attracted increased interest on the part of mathematics educators, particularly in the search for causal relationship between affect and achievement in mathematics (Young, 1997). This is due to the fact that the mathematical activity is marked out by a strong interaction between cognitive and emotional aspect. The affective domain is a complex structural system consisting of four main dimensions or components: emotions, attitudes, values and beliefs (Goldin, 2001). At the present study we focus on students' beliefs and mainly their self-efficacy beliefs in using different types of representations in mathematics learning and understanding. We concentrated our attention on the notion of fractions.

Fractions are among the most essential (Harrison & Greer, 1993), but complex mathematical concepts that children meet in school mathematics (Charalambous & Pitta-Pantazi, 2007). An important factor that may contribute to students' difficulties in learning fractions is the transition from primary in secondary school with all the changes that this encompasses in mathematical teaching and learning.

THEORETICAL BACKGROUND

Self-efficacy beliefs

Beliefs is a multifaceted construct, which can be described as one's subjective "understandings, premises, or propositions about the world" (Philipp, 2007, p. 259). According to Pehkonen and Pietila (2003) there are several difficulties in defining concepts related to beliefs. Some researchers consider beliefs to be part of knowledge (e.g. Pajares, 1992), some think beliefs are part of attitudes (e.g. Grigutsch, 1998), and some consider they are part of conceptions (e.g. Thompson, 1992).

The construct of self-efficacy beliefs constitutes a key component in Bandura's social cognitive theory; it signifies a person's perceived ability or capability to successfully perform a given task or behavior. Bandura (1997) defines self-efficacy as one's perceived ability to plan and execute tasks to achieve specific goals. He characterized self-efficacy as being both a product of students' interactions with the world and an influence on the nature and quality of those interactions. Self-efficacy beliefs have received increasing attention in educational research, primarily in studies for academic motivation and self-regulation (Pintrich & Schunk, 1995). It was found that self-efficacy is a major determinant of the choices that individuals make, the effort they expend, the perseverance they exert in the face of difficulties, and the thought patterns and emotional reactions they experience (Bandura, 1986). Furthermore, self-efficacy beliefs play an essential role in achievement motivation, interact with self-regulated learning processes, and mediate academic achievement (Pintrich, 1999).

Multiple representations in mathematics teaching and learning

The representational systems are fundamental for conceptual learning and determine, to a significant extent, what is learnt (Cheng, 2000). Learning involves information that is represented in different forms such as text, diagrams, practical demonstrations, abstract mathematical models, simulations etc (Schuyten & Dekeyser, 2007). Recognizing the same concept in multiple systems of representations, the ability to manipulate the concept within these representations as well as the ability to convert flexibly the concept from one system of representation to another are necessary for the acquisition of the concept (Lesh, Post, & Behr, 1987) and allow students to see rich relationships (Even, 1998). Recently the different types of external representations in teaching and learning mathematics seem to become widely acknowledge by the mathematics education community (NCTM, 2000). Given that a representation cannot describe fully a mathematical construct and that each representation has different advantages, using multiple representations for the same mathematical situation is at the core of mathematical understanding (Duval, 2006). The necessity of using a variety of representations or models in supporting and assessing students' constructions of fractions is stressed by a number of studies (Lamon, 2001). The geometric shapes used for introducing the continuous model of fractions are distinguished into two types: the circular model which is based on the use of circles and the linear model which is based on a rectangle divided into a number of equal part (Boulet, 1998).

An issue that has received major attention from the education community over the last years refers to the students' difficulties when moving from elementary to secondary school and to the discontinuities in the curriculum requirements, the use of teaching approaches, aids and methods. According to Schumacher (1998) the transition to secondary school is accompanied by intellectual, moral, social, emotional and physical changes. Pajares and Graham (1999) investigated the extent to which mathematics self-beliefs change during the first year of middle school. By the end of the academic year, students described mathematics as less valuable, and they reported decreased effort and persistence in mathematics. The findings of the Deliyianni, Elia, Panaoura and Gagatsis's (2007) study suggest that there is a noteworthy difference between elementary and secondary education in Cyprus concerning the representations used in mathematics textbooks on fractions. There are also differences in the functions the various representations in the school textbooks fulfil.

The present study investigated Grade 5 to Grade 8 students' beliefs about the use of different representations for the learning of the fractions and their self-efficacy beliefs about the use of those types of representations. That means that it explores the differences of students' beliefs at primary and secondary education concerning the use of different types of representations.

METHOD

The study was conducted among 1701 students of 10 to 14 year of age who were randomly selected from urban and rural schools in Cyprus. Specifically, students belonging to 83 classrooms of primary (Grade 5 and 6) and secondary (Grade 7 and 8) schools (414 in Grade 5, 415 in Grade 6, 406 in Grade 7, 466 in Grade 8) were tested.

A questionnaire was developed for measuring students' beliefs about the use of different types of representations for understanding the concept of fractions. The questionnaire comprised of 27 Likert type items of five points (1=strongly disagree, 5=strongly agree).

The reliability of the whole questionnaire was very high (Cronbach's alpha was 0.88). The items of the questionnaire are presented at Table 1.

At the same time a test was developed for measuring students' ability on multiple representation flexibility as far as fraction addition is concerned. The test included 22 fraction addition tasks that examine multiple-representation flexibility and problem-solving ability. There were treatment, recognition, conversion, diagrammatic problem-solving and verbal problem-solving tasks (further details for the tasks can be found at the paper of Deliyianni et al. (2007). Cronbach's alpha for the test was 0.87.

The test and the questionnaire were administered to the students by their teachers at the end of the school year in usual classroom conditions. Right and wrong or no answers were scored as 1 and 0, respectively. Solutions in treatment, recognition and translation tasks were assessed as correct if the appropriate answer, diagram, equation or shading were given respectively, while a solution in the problems was assessed as correct if the right answer was given.

RESULTS

The analysis of students' responses to the items of the questionnaire resulted in six factors (KMO=0.933, $p<0.001$) with eigenvalues greater than 1 (Table 1). The first factor corresponded to students' self-efficacy beliefs about conversion from one type of representation to another. The second factor was associated with their general self-efficacy beliefs in mathematics. The third factor represented their beliefs about the use of the number line, while the forth factor represented their beliefs about the use of models, materials or representations. The fifth factor corresponded to students' beliefs about the use of diagrams in problem solving and the sixth factor to their self-efficacy beliefs about the use of verbal representations.

Item	F1	F2	F3	F4	F5	F6
I can easily find the diagram that corresponds to an equation of fractions.	.53					
I can easily solve tasks than ask toconverse the part of a diagram into an equation.	.62					
I can easily find the diagram that corresponds to an equation of decimals.	.67					
I can easily find the equation of fraction addition that corresponds to a part of a surface of a rectangle.	.63					
I can easily find the equation of fraction addition which is presented with arrows in number line.	.58					
I am very good in solving tasks with decimals.		.70				
I am very good in problem solving fractions.		.78				
I can easily solve tasks with fractions.		.79				
I can easily solve equations of fraction addition.		.70				
I can easily solve equation of decimal addition.		.56				
Number line helps me in problem solving with fractions.			.68			
Number line helps me in solving equations with fractions.			.68			

My teacher usually uses number line in order to explain us the operations of fractions.			.64			
Number line helps me in solving equations with decimals.			.64			
A good student in mathematics can present the solution of a problem by many different ways.				.55		
For the problem solving the use of equation is necessary.				.65		
In mathematics the use of materials (fraction circles, dienes cubes etc) is useful mainly for students at primary education				.59		
The diagrams (number line, rectangle etc) are useful for executing operations.				.42		
If I have to explain how I have solved a problem with decimals, I prefer to use an equation.				.57		
If I have to explain how I have solved a problem with fractions, I prefer to use a diagram.					.65	
When I solve a problem with fractions, I use the number line for executing the operations.					.44	
When I solve a problem with fractions by using a diagram, I then try to solve it by using an equation, as well.					.49	
When I solve a problem with decimals I use a diagram.					.67	
I can easily explain how I have solved a problem with decimals by using a diagram.					.47	
I prefer solve problems with decimals which present the data verbally.						.79
I can easily explain verbally how I have solved a problem with decimals.						.69
Eigenvalues	7.87	2.48	1.92	1.58	1.25	1.17
Percentage of variance explained	24.6	9.76	6.77	5.01	4.20	3.34
Cumulative percentage of explained variance	24.6	34.3	41.1	46.1	50.3	53.6

Table 1: Factor loading of the six factors against the items associated with participants' beliefs

Analysis of variance (ANOVA) indicated that there were statistically significant differences in respect to grade for the factors F1, F2, F5 and F6. Specifically in the case of F1 there were differences at the means ($F_{3,1547}=9.09$, $p<0.001$) between students' self-efficacy beliefs to converse flexibly the concept of fraction addition from one representation to any other who were attending the Grade 8 with the students of the Grades 5, 6 and 7 ($\overline{X}_5=3.63$, $\overline{X}_6=3.58$, $\overline{X}_7=3.53$, $\overline{X}_8=3.37$). In the case of the F2 the statistically significant differences ($F_{3,1574}=31.615$, $p<0.001$) were between the Grade 5 with Grades 7 and 8, the Grade 6 with the Grade 7 and 8 ($\overline{X}_5=4.08$, $\overline{X}_6=3.93$, $\overline{X}_7=3.70$, $\overline{X}_8=3.56$). Students at the Grade 8 seemed to have less positive beliefs for the significance of using different types of representations (F5). There were statistically significant differences ($F_{3,1597}=6.209$, $p<0.001$) between Grade 5 and Grade 8, Grade 6 and Grade 8 ($\overline{X}_5=3.29$, $\overline{X}_6=3.24$, $\overline{X}_7=3.17$, $\overline{X}_8=3.09$). In the case of their preference for using verbal explanations the differences ($F_{3,1671}=21.036$, $p<0.001$) were between Grade 5 with Grade 7 and 8 and Grade 6 with Grade 7 and 8. Therefore, most of the differences revealed were between the students at primary education and the students at secondary education.

Very impressive and unexpected were the descriptive results of the students' mathematical performance at the test. As it is obvious in Figure 1 students at the Grade 7 have lower performance than the students at the Grade 6.

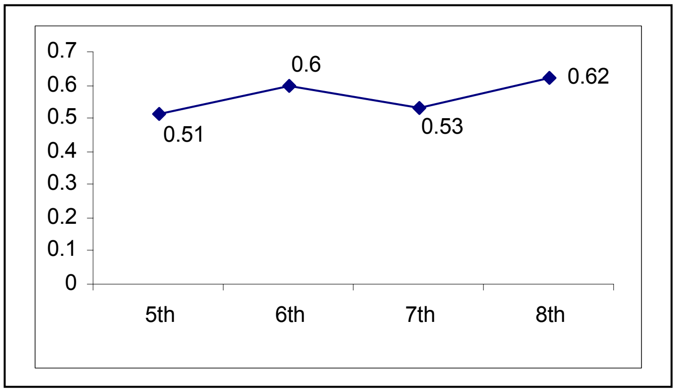

Figure 1: Students' of different grades performance on the mathematical test.

Students were cluster, by using cluster analysis, according to their performance at the test into three groups. Group 1 consisted of the 426 students with the lowest performance, Group 2 consisted of 788 students with medium performance and Group 3 consisted of 487 students with the highest performance. Analysis of variance (ANOVA) with independent variable the three groups and dependent variables the six factors, which were comprised from the abovementioned factor analysis, indicated statistically significant differences in respect to F1 ($F_{2,1547}$=51.819), F2 ($F_{2,1474}$=74.903), F4 ($F_{2,1609}$=12.057) and F6 ($F_{2,1671}$=8.844). In all cases the first group had the most negative beliefs and self-efficacy beliefs and the third group had the most positive beliefs. That means that students with high mathematical performance had at the same time positive beliefs for the use of representations and high self-efficacy beliefs.

Finally students were clustered into two groups according to their general self-efficacy beliefs in mathematics (F2), by using cluster analysis. The group with higher self-efficacy beliefs consisted of 1047 students ($\overline{X}$=4.31) and the second group consisted of 528 students ($\overline{X}$=2.82).T-test analysis between the two groups in respect to the other five factors indicated that there were in all cases statistically significant ($p<0.01$) differences (Table 2). Students with higher general self-efficacy beliefs in mathematics had at the same time more positive beliefs for the use of different forms of representations and more positive self-efficacy beliefs for the use of those representations and their ability to transfer their knowledge.

	t	df	$\overline{X}_1$	$\overline{X}_2$
F1	22.82	1463	3.81	2.97
F3	6.508	1527	3.30	2.99
F4	13.897	1507	4.09	3.57
F5	9.151	1499	3.32	2.96
F6	14.616	1565	3.48	2.73

Table2: Students' with high and low self-efficacy beliefs differences in respect to their beliefs about the use of representations

DISCUSSION

The main emphasis of the present study was on investigating students' self-efficacy beliefs for mathematics in relation to their beliefs about the use of representations for understanding the concept of fraction. The analysis of the data confirms earlier findings that young students have high self-efficacy beliefs (Bandura, 1986) and that they tend to overestimate their abilities. However those beliefs decreased at the secondary education. It seems that students' sense of efficacy diminishes somehow when they compare their abilities with classmates and even more in relation to their mathematical performance as it is revealed by their final grades at mathematics. The influence of those active experiences is too strong and with immediate results. Accepting that the most important step is getting individuals to become aware of their own processes, strengths and limitations in order to have an accurate self-representation, it seems that the specific result is important for the learning of the concept of fractions. Nevertheless it is not positive generally, because there are too many other concepts at the teaching of mathematics at secondary education for which students have to use flexibly different types of representations. For example the concept of function admits a variety of representations, each of which offers information about particular aspects of the concept without being able to describe it completely (Elia et al., 2008).

Interesting and unexpected was the differences between students' performance in the use of different forms of representations at primary and secondary education and mainly the lower performance at secondary education. A possible explanation for the lack of improvement regarding their mathematical performance observed are the differences regarding the representations and their functions in mathematics textbooks used in primary and secondary education in Cyprus (Deliyianni et al., 2007). Furthermore, the secondary school students may had not created referential connections between corresponding elements and related structures in a way that promotes understanding of this concept during their primary schooling. Their difficulties increased in secondary education since no emphasis is placed on learning with multiple representations.

Results confirmed that students with low performance in mathematics have at the same time negative beliefs for the use of different forms of representations because they cannot use them fluently and flexibly as a tool to overcome obstacles while solving tasks and handling the whole situation. It seems that there is a need for further investigation into the subject with the inclusion of a more extended qualitative and quantitative analysis. Most mathematics textbooks today make use of a variety of representations more extensively than every before in order to promote understanding (Elia, Gagatsis & Demetriou, 2007). Much more research is needed for the students' beliefs about the role of those representations regarding different mathematical concepts in relation to their self-efficacy beliefs for using them as a tool for the better understanding of the concepts.

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Acknowledgements

This paper draws from the medium research project MED19 that investigated the role of multiple representations in mathematics learning during the transition within and between primary and secondary school, which is supported by the University of Cyprus.

THE EFFECT OF ACHIEVEMENT, GENDER AND CLASSROOM CONTEXT ON UPPER SECONDARY STUDENTS' MATHEMATICAL BELIEFS

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The influence of achievement, gender and classroom context on students' mathematical beliefs were analysed from survey data from 1436 Finnish upper secondary school students. The results indicate that students of the same class tend to have similar effort, enjoyment of mathematics and evaluation of teacher. Students' mathematical confidence is influenced by gender while their perception of their competence mainly relates to their achievement in mathematics.

Keywords: beliefs, gender, secondary school, multilevel analysis

INTRODUCTION

Mathematical beliefs are on the one hand considered as individual constructs that are generated by individual experiences. On the other hand, beliefs are considered to be constructed socially, in a shared social context of a classroom. Which is more important? Are all beliefs constructed in the same way or are some beliefs socially constructed while some others are purely individual?

In Finnish research on affect in mathematics education the focus has clearly been on the level of human psychology, and only a few studies have explored also the social level (Hannula, 2007). One reason for this is most likely that differences between schools and geographic regions are low and the social variables have generally less pronounced effect on achievement in mathematics in Finland than in most other countries (OECD-PISA, 2004). Finland is also culturally rather homogeneous. Hence, it is not surprising that comparative studies between different groups of students within Finland have not been popular, gender being an exception to the rule. One study on regional effects indicated that students in capital province choose advanced syllabus more often than students in another province (Nevanlinna, 1998). This indicates that geographical differences in mathematics related beliefs may exist.

A general international trend has been that gender differences in mathematics achievement are disappearing. Gender differences in overall achievement of 15-year olds have disappeared also in Finland, but robust gender differences still exist in their affect towards mathematics (Hannula, Juuti & Ahtee, 2007). When attitude towards mathematics has been constructed as a single variable, studies generally have found boys to hold a more positive attitude towards mathematics (e.g. Saranen 1992). However, when different dimensions of attitude have been separated, interesting variations have been found. For example, all studies have not found gender differences in 'liking of mathematics' (Kangasniemi, 1989). Gender difference has been clearer in how difficult mathematics is seen (Kangasniemi, 1989) and quite robust in students' self-confidence in mathematics (Hannula & Malmivuori, 1997; Kangasniemi, 1989; Hannula, Maijala, Pehkonen & Nurmi, 2005). Lower self-confidence among female students has been found even on level of individual tasks, in case of both correct and incorrect answers (Hannula, Maijala, Pehkonen & Soro, 2002). Class-level factors are seen to influence students' self-confidence, and these seem to be more relevant to girls' than to boys' self-confidence (Hannula & Malmivuori 1997).

Although Finland scored to the top in PISA achievement scores, Finland was also characterised by less favourable results on the affective measures. Finnish students' lack interest and enjoyment in mathematics, they have below average self-efficacy, and low level of control strategies. As a more positive result, levels of anxiety were also low. In Finland affect was an important predictor of achievement. Mathematical self-concept was the strongest predictor of mathematics performance, and this correlation was strongest among countries in the study. The study also revealed that gender differences favouring males in affect were larger in Finland than in OECD on average. (OECD-PISA, 2004)

In a study of elementary and secondary teachers' beliefs Pekka Kupari identified two types of mathematics teachers, traditional and innovative teachers. The traditional teacher emphasises basic teaching techniques and extensive drill, while the innovative teacher emphasises student thinking and deeper learning. (Kupari, 1996)

Moreover, Riitta Soro (2002) found out in her study that most mathematics teachers held different beliefs about students based on student's gender. Girls were seen to employ inferior cognitive skills and succeed because of their diligence, while boys were seen to be talented in mathematics but lacking in effort. However, there were also teachers who did not hold such gendered beliefs.

As there are quite different teachers, one would expect this to have an effect on beliefs of their students. If this is the case, then we are likely to find significant amount of variation of students' beliefs to be attributable to the class they study in. Moreover, this variation might be different for male and female students.

In this report we shall explore more deeply which aspects of mathematical beliefs are most affected by shared classroom context or gender, and which seem to be individual constructs, for which gender and class are poor predictors of the belief.

THEORETICAL FRAMEWORK

In the literature, beliefs have been described as a messy construct (Pajares, 1992). There are many variations for characterisations of belief concept (Furinghetti & Pehkonen, 2002). In this article we consider mathematical beliefs as "an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior" (Schoenfeld 1992, 358). Op 't Eynde, De

Corte and Verschaffel (2002) provide a framework of students' mathematics-related beliefs. Constitutive dimensions are object (mathematics education), self, and context (class), which further lead to several sub-categories:

1) Mathematics education (mathematics as subject, mathematical learning and problem solving, mathematics teaching in general),

2) Self (self-efficacy, control, task-value, goal-orientation), and

3) The social context (social and socio-mathematical norms in the class,). With regard to the social context, Op 't Eynde & DeCorte (2004) found out later that the role and functioning of one's teacher are an important subcategory of it.

In an earlier study (Rösken, Hannula, Pehkonen, Kaasila and Laine, 2007), we have explored the structure of mathematical beliefs among upper secondary school students. Our studies confirmed partially the aspects of mathematical beliefs that they suggested.

Mathematical beliefs and mathematics learning have a two-way interaction. On one hand, the way of teaching mathematics will gradually influence students' mathematics related beliefs. There exist research results that a teacher's mathematics beliefs are in connection with his pupils' beliefs (e.g. Crater & Norwood 1997; Philippou & Christou 1997). On the other hand, pupil's mathematical beliefs act as a filter influencing all their thoughts and actions concerning mathematics.

These different findings can be summarised on a model where there the three levels of classroom context, gender and individual are differentiated in the process of belief development (Figure 1).

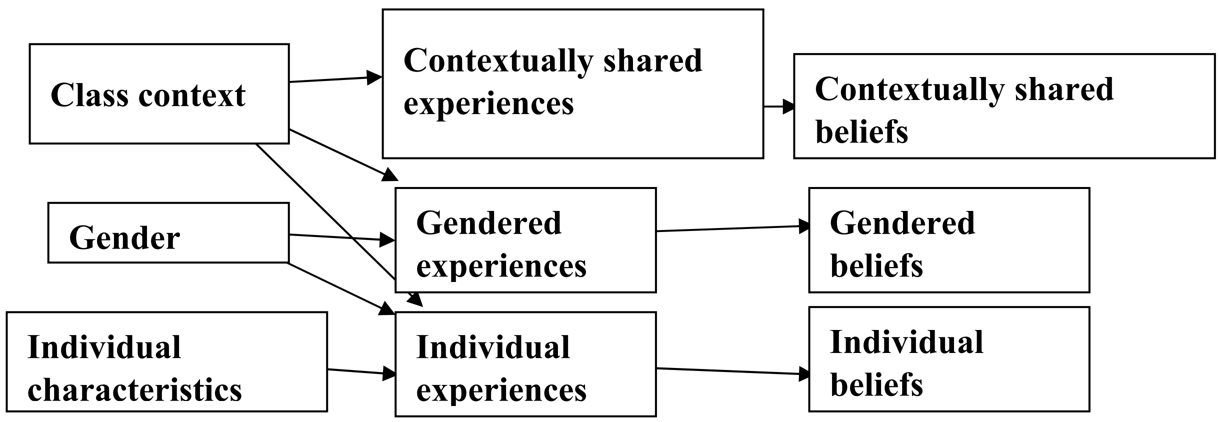

Figure 1. A model for generation of mathematical beliefs.

To begin from the most general level, there are experiences that people of the same class share. These are, for example, the personality of the teacher, the physical classroom and the implemented curriculum. These influence all students in a class and are the origin of shared experiences. Although the students are in the same classroom, their experiences may be quite different. One origin of these differences is the individual life histories that each student brings into the classroom. These life histories influence the way the students position themselves in the classroom, the way they engage with mathematics, teacher and peers and the way they interpret their experiences in the classroom. Yet, these individual experiences are partly shaped by the shared events in the classroom. This is illustrated with an arrow from classroom context to individual experiences.

There are several subsets of students in the class who have something they share and which distinguishes them from the other students (e.g. ethnicity, social class, achievement level, hobbies, social subcultures). One of such subsets is generated by students' gender. Gender is seen to play a significant part in the experiences in the classroom and in the beliefs that students develop (e.g. Hannula et. al, 2008). Also most teachers' have different beliefs about boys and girls as mathematics learners (Soro, 2002). Therefore it is reasonable to make the claim that social environment in mathematics classrooms is not the same for male and female students. Moreover, as teachers and classes are different, these gendered experiences may vary from one class context to another.

METHODS

Instrument and Participants

The view of mathematics indicator has been developed in 2003 as part of the research project "Elementary teachers' mathematics" financed by the Academy of Finland (project #8201695). It has been applied to and tested on a sample of student teachers and was slightly modified for the present sample. That is, items addressing specifically aspects of teaching mathematics like View of oneself as mathematics teacher (D1-D6) and Experiences as teacher of mathematics (E1-E7) were removed. More information about the development of the instrument can be found e.g. in (Hannula Kaasila, Laine & Pehkonen, 2006).

The participants in our study came from fifty randomly chosen Finnish-speaking upper secondary schools from overall Finland, including classes for both, advanced and general mathematics. The respondents were in their second year course for mathematics in grade 11. Altogether 1436 students from 65 classes (26 general and 39 advanced) filled in the questionnaire and gave it back. The response rate was higher among advanced mathematics courses.

Through an exploratory factor analysis we obtained a seven-factor solution that counts for 59 % of variance and provides factors with excellent internal consistency reliability (Table 1). We related three factors to personal beliefs since a clear self-relation aspect regarding competence (F1), effort (F2) and confidence (F7) can be found. Two factors we related primarily to social context variables, namely teacher quality (F3) and family encouragement (F4), one to more emotional expressions concerning enjoyment of mathematics (F5) and one to mathematics as a subject; that is, difficulty of mathematics (F6). A description of factor analysis as well as all components and their loadings can be found in another report. (Rösken et. al, 2007)

Name of the component	Sample item	Number of items	Cronbach's alpha
Competence	Math is hard for me	5	0.91
Effort	I am hard-working by nature	6	0.83
Teacher Quality	I would have needed a better teacher	8	0.81
Family Encouragement	My family has encouraged me to study mathematics	3	0.80
Enjoyment of Mathematics	Doing exercises has been pleasant	7	0.91
Difficulty of Mathematics	Mathematics is difficult	3	0.82
Confidence	I can get good grades in math	5	0.87

Table 1. The 7 principal components of students' view of mathematics.

A GLM univariate analysis was performed on SPSS. The seven belief factors were the dependent variables, gender was a fixed factor, and class a random factor. Mathematics grade was a covariant. Students of advanced and general mathematics courses were analysed separately, and partial η^2 is used as a measure of effect size. It should be noted that although partial η^2 is a reliable estimate within a sample, it does not provide reliable estimate for the whole population. Because all variables did not confirm with the assumptions of normality, we made also a nonparametric Kruskal Wallis test to test the statistical significance of the grouping effect.

RESULTS

The GLM univariate analysis indicated several statistically significant effects (Table2 and Table 3). However, the assumption of equal variance did not hold true in all cases and nonparametric tests were necessary to confirm results (see below).

	General mathematics											
	Grade			Gender			Group			Gender x Group		
	F	Sig.	η^2	F	Sig.	η^2	F	Sig.	η^2	F	Sig.	η^2
Competence*	326,16	,000	,35	,12	,729	,00	1,58	,111	,61	,97	,507	,04
Effort	172,22	,000	,27	3,10	,087	,09	2,03	,041	,67	1,15	,278	,06
Teacher Quality	41,86	,000	,08	10,37	,003	,22	2,95	,004	,75	,92	,577	,05
Family Encouragement	,75	,388	,00	2,20	,147	,06	1,05	,456	,51	1,08	,359	,06
Enjoyment of Mathematics	196,65	,000	,30	2,94	,096	,08	1,65	,107	,62	1,00	,470	,05
Difficulty of Math*	194,80	,000	,30	4,73	,036	,12	1,90	,057	,65	,94	,550	,05
Confidence*	86,40	,000	,16	23,29	,000	,41	1,06	,444	,51	1,02	,433	,05

Table 2. GLM univariate analysis for general mathematics students (gender*group, grade as covariate). η^2 is partial η^2. *) variance in groups was not equal (Levene's Test of Equality of Error Variance)

Most of the mathematical beliefs were related to the mathematics grade the student had. A simple correlation was calculated to determine the direction of the correlation (correlation table is not reprinted here). All correlations were positive, except or correlation between grade and perceived difficulty of mathematics.

Regarding gender differences, the GLM Univariate analysis indicated that for both advanced and general syllabus female students were less confident and they perceived teacher quality lower and mathematics more difficult than male students. The effect was strongest in self-confidence.

The analysis indicated a strong group effect for teacher quality. In groups of general mathematics there was also a strong group effect on effort and in groups of advanced mathematics a strong group effect on enjoyment. Moreover, there was a gender and group interaction effect for enjoyment among advanced mathematics courses, indicating stronger group effect for female students.

	Advanced mathematics											
	Grade			Gender			Group			Gender x Group		
	F	Sig.	η^2	F	Sig.	η^2	F	Sig.	η^2	F	Sig.	η^2
Competence*	332,61	,000	,30	1,09	,301	,02	1,63	,077	,63	1,08	,355	,05
Effort*	254,72	,000	,25	,13	,717	,00	1,02	,479	,51	1,13	,278	,05
Teacher Quality*	53,34	,000	,07	5,83	,019	,10	7,26	,000	,88	1,14	,274	,05
Family Encouragement	1,20	,274	,00	,34	,561	,01	1,50	,116	,61	,73	,877	,03
Enjoyment of Mathematics	175,78	,000	,18	,30	,591	,01	2,41	,005	,71	1,49	,036	,06
Difficulty of Mathematics	254,08	,000	,24	34,27	,000	,40	1,67	,066	,63	1,24	,160	,05
Confidence	115,86	,000	,13	75,07	,000	,60	1,29	,228	,57	1,28	,132	,05

Table 3. GLM univariate analysis for advanced mathematics students (gender*group, grade as covariate). η^2 is partial η^2. *) variance in groups was not equal (Levene's Test of Equality of Error Variance)

Because all variables did not confirm with the assumptions of normality, we made separate analysis to confirm some of the disputable results above (Table 4). Unfortunately this analysis did not allow a simple means to control for effect of achievement. The results confirmed the group effects partially. For students of general mathematics the statistically significant group effects were different for male and female students. For male students, groups had an effect on competence and effort, whereas for female students the group effect was found on teacher quality and confidence. This confirms the group effect on effort for male students and teacher quality for female students. The observed group effects on competence and confidence may actually be effects of grade.

For advanced mathematics a statistically significant group effect was found for teacher quality, effort, and enjoyment. This confirms the results of GLM Univariate analysis. Moreover, for female students only, a group effect on confidence was found.

Kruskal Wallis Test Statistics for group differences									
Course, Gender		Compe-tence	Effort	TQ	FE	Enjoy	Difficulty	Confidence
General, male	χ^2	36,39	46,10	27,053	21,96	25,16	26,56	20,38
	df	25	25	25	25	25	25	25
	Asymp. Sig.	,066	,006	,353	,638	,453	,378	,727
General, female	χ^2	30,64	24,70	66,369	47,61	31,12	23,41	43,72
	df	25	25	25	25	25	25	25
	Asymp. Sig.	,201	,479	,000	,004	,185	,554	,012
Advanced, male	χ^2	35,25	58,61	96,81	38,20	51,06	56,99	39,51
	df	36	36	36	36	36	36	36
	Asymp. Sig.	,504	,010	,000	,370	,049	,014	,316
Advanced, female	χ^2	40,71	52,04	140,12	33,8	99,700	47,43	54,14
	df	35	35	35	35	35	35	35
	Asymp. Sig.	,233	,032	,000	,523	,000	,078	,020

Table 4. Kruskal Wallis Nonparametric Test for the group effect on mathematical beliefs among male and female students in general and advanced mathematics courses. TQ = Teacher quality, FE = Family encouragement

CONCLUSIONS

The results of these analysis confirmed that there is a certain level of agreement in certain mathematical beliefs among students of same class. Most pronounced this was for perceived teacher quality. In our earlier studies on teacher education students (e.g. Hannula et. al, 2006) we were not sure whether the variation in respondents beliefs about their teacher's quality was an effect of their own mathematical achievement or if it reflected actual differences in the teaching they had received. This study confirms that students' belief of their teacher's quality is shared among students of the same class and therefore it is likely to be generated by shared experiences in the classroom context. Yet, also student's gender and achievement had an effect on this evaluation of the teacher. This provides evidence for the suggested interaction between levels in the model (Figure 1).

Shared classroom context seemed to have an effect also in students' effort (general mathematics) and enjoyment (advanced mathematics). This is indicating that through choices in instruction, it is possible to create a 'culture' in the classroom that is motivating or enjoyable. However, we can not rule out the possibility that these differences between classes be effect of geography or some other variable that differentiates these groups.

An interesting finding was that there was a gender and group interaction effect for enjoyment among advanced mathematics courses, indicating stronger group effect for female students. This might relate to the anecdotes that students still occasionally tell about chauvinistic mathematics teachers they have had. The small effect size (6%) indicates that this is not a major problem on the level of educational system. However, for those female students who have to suffer through these classes it may be a big problem. Alternatively, this might indicate that there are such teachers in Finnish upper secondary schools that are able to create lessons that female students find especially enjoyable.

It is worth to note that gender had a stronger influence on confidence in mathematics than mathematics grade. The same is true also for and perceiving mathematics difficult in advanced course. In this sense these beliefs are truly gendered beliefs.

The findings provide support for the presented model and give indication to the origin of the measured beliefs (Figure 2). The effects of context and gender were surprisingly strong and the results support the hypothesis of social origin of beliefs.

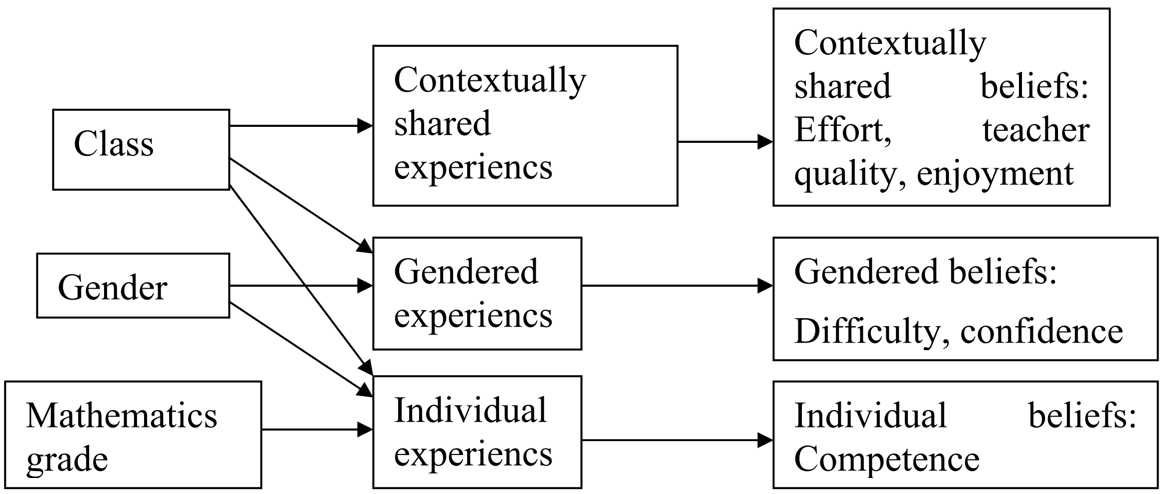

Figure 2. Empirically confirmed contextual, gendered and individual beliefs.

Enjoyment of mathematics, self-confidence in mathematics and self-efficacy beliefs are often considered as closely related aspects of attitude towards mathematics. This study highlights the different origin of these three aspects of attitude towards mathematics. Hence, it seems worthwhile to separate these different aspects also in future studies.

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EMOTIONAL KNOWLEDGE OF MATHEMATICS TEACHERS – RETROSPECTIVE PERSPECTIVES OF TWO CASE STUDIES

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Abstract

In this paper we provide a partial description of certain facets and experiences that are central to the development of emotional knowledge from the retrospective perspectives of two highly experienced mathematics teachers in middle and high school. One of the study participants refers to the emotional knowledge she developed over the years regarding her interactions with her students, while the second participant also refers to the emotional knowledge she developed regarding her interaction with the school principal. Both indicate the differences in their emotional reactions between the first practice years and the years after. The differences are seen primarily in the type and in the intensity of their emotions. While negative feelings mostly accompanied the first years, later years were accompanied by more positive emotions.

1. Introduction

Teaching and emotions are inseparable. Emotions are dynamic parts of ourselves, and whether they are positive or negative, all organizations, including schools, are full of them (Hargreaves, 1998). In his literature review, Zembylas (2007) asserts that although "teacher knowledge" has become a major area of exploration in educational research, limited attention is given to the emotional aspects of teaching. While Shulman's (1987) work on pedagogical content knowledge (PCK) was further investigated and discussed by many researchers, teachers' understandings of emotional aspects of teaching and learning continued to be ignored. Zembylas argues that "any effort to expand current conceptions of PCK should include the connection between PCK and *emotional knowledge* (EK) in general – that is, a teacher's knowledge about/from his or her emotional experiences with respect to one's self, others (e.g. students, colleagues), and the wider social and political context in which teaching and learning takes place" (p. 356). Furthermore, Zembylas continues, in order to teach well, "teachers must be able to connect their emotional understanding with what they know about subject matter, pedagogy, school discourses, personal histories, and curriculum" (p. 364). In this paper we provide a partial description from a study we conducted that focused on themes identified by teachers as central to their development of EK. We present two case-studies of mathematics teachers, each of whom has more than 30 years of teaching experience.

2. Theoretical background

In the process of determining mathematics teachers' qualifications, teacher educators focus on various types of knowledge identified as essential for good teaching: content knowledge, didactical knowledge, knowledge about students, and knowledge of class management (Shulman, 1987; Shulman, 2000). Often these types of knowledge are discussed, separately on the assumption that teachers are capable of integrating them into a coherent whole. However, issues concerning emotional aspects of teaching and their interrelations with the above knowledge types, are rarely discussed in mathematics teachers' training programs.

Planes and types of EK. Zembylas (2007) finds a reciprocal relationship between PCK and EK, and argues that the latter "occurs on different *planes* as there are different *types* of EK that are aspects of PCK" (p. 358). These planes are: individual, relational, and socio-political. The individual plane refers to how teachers experience and express their EK on the personal plane; the relational plane refers to how teachers use EK in their relationships with students; and the socio-political plane refers to EK of the institutional and cultural context of schooling and its influence on teachers' curricular decisions and actions. There is no hierarchical order between the three planes. Their boundaries are blurred, and mutual influence and interaction exist between them.

Positive vs. negative emotions. Smeltzer (2004) studied the emotions of beginning teachers, and discerned positive and negative emotions according to their characteristics and forcefulness, as they appeared in the teachers' reactions. The categories of positive emotions include: joy-happiness, fulfillment-reward-satisfaction, competence-confidence-motivation, and surprise-fun. The categories of negative emotions include: frustration-anger, incompetence-anxiety-fear-doubt, exhaustion-stress, and disappointment-discouragement-sadness. Smeltzer also found that the most dominant and intense category of emotion is frustration-anger. It comes as a result of the turmoil beginning teachers, experience as defeat, distress, or displeasure. The incompetence-anxiety-fear-doubt category represents low self-efficacy, expressed by feelings of inadequacy, uneasiness, apprehension, worry, hesitancy, or uncertainty. The exhaustion-stress category characterizes weariness, fatigue, and energy loss. The disappointment-discouragement-sadness category refers to the most desperate and desolate of emotions such as unfulfilled expectations, sorrow, low spirits, disheartenment, and dashed hopes.

The categories of positive emotion were found to be of less frequency and intensity. The joy-happiness category represents the delight, pleasure, and contentment experienced in the early years of teaching. The fulfillment-reward-satisfaction category extends the joy-happiness category, representing a deeper and more intense degree of gratification. The competence-confidence-motivation category signifies teacher self-efficacy identified by assurance, certainty, and proficiency. The least dominant and intense of all the emotional classifications is the surprise-fun category that refers to unanticipated and spontaneous experiences in teaching. In the present study the research participants recounted various emotions that can be generally grouped into positive and negative headings. Moreover, these emotions can also be further categorized according to Smeltzer's types which were previously mentioned.

3. The study

Our study focuses on experienced mathematics teachers, each of whom who has more than 30 years of teaching experience. The aims of our study are to characterize: (*i*) facets and experiences that are central to the development of EK from retrospective perspectives; (*ii*) interrelations between EK and PCK; and (*iii*) the evolvement of teachers' EK during their years of practice from retrospective perspectives. In this paper we provide a partial description of the results from the first part of our study. We also present certain facets and experiences of the emotional component of teaching that are central to the development of EK, as shown in these two case-studies.

3.1 The study participants

Twelve mathematics teachers with more than 30 years of teaching experience each were interviewed. In this paper we will briefly present the narratives of only two of them: Betty (56) and Rose (55), both who teach mathematics in middle-high school. We chose to make use of their stories because more than the other participants, Betty and Rose were able to identify the "causes and effects" that impacted their emotions and the development of their EK. In section 4 we present excerpts from their actual narratives.

3.2 Method

Data collection. We asked the twelve teachers to tell us their stories, with deliberate attention given to emotional aspects of teaching and EK. The interviews were open. We asked the teachers several general questions (for example – why they chose to become teachers), and following their narratives we asked for further clarification. We were careful not to direct them, or to interfere in their associative train of thought. The interviews were tape-recorded. Each interview lasted between 3 to 4 hours and took place in an informal setting, such as the teacher's home or Cafeteria.

Data analysis. Scanning the transcripts of the recorded interviews, we first picked out all the excerpts which included expressions of emotion. Then we differentiated between various types of emotion according to the addressee of the emotional reaction, namely: emotional reactions towards students, the school principal or other colleagues.

Being aware of the small size of our sample, we cannot say that the data collected represents the general emotional profile of the teachers in our country.

However, it does shed light on some important aspects of the teaching experience that should be considered.

4. Results and discussion

In this section we make use of the narratives of Betty and Rose to characterize some of the important facets and experiences that emerged in relation to EK development. In the scope of this paper we focus merely on EK with respect to students and school principal.

Betty's story

Betty is 56 years old and has more than 29 years of teaching experience. Betty was born and raised in Lebanon. She remembers her classmates "standing tensely and quietly in their places until the teacher entered the class and gave us permission to sit down. All the students behaved politely and respected the teachers, and there were no disciplinary problems…When I came to Israel I knew it was a different country with a different culture but I could not anticipate the extreme differences."

Betty immigrated to Israel when she was 16 years old. When she was 18, she began to study computer science. After graduation she worked as a computer programmer for two years in a large commercial company, and then was offered a position as a mathematics teacher in a middle-high school. She accepted the offer. Betty chose to begin her story as a mathematics teacher with a description of her first lesson in the school:

> "Although it happened many years ago I remember it as if it were yesterday. This was my first day at school and I had to teach mathematics in one of the 11^{th} grade classes. I opened the door and I was shocked. All the students were half-sitting, half-lying on the tables and no one even bothered to turn his/her head toward me when I entered the classroom. I felt discouraged. I asked the students to sit properly so that we could start the lesson and they said: "This is how we behave!" I felt hopeless and speechless but after a few seconds I said: "If you do not follow my request, I will leave the classroom." One of the boys went to the door lay down on the floor and said: "Over my dead body!" The rest of the students laughed. I was very close to tears and felt very frustrated and hopeless. But I knew that if I showed any sign of weakness I would not be able to teach this class again. So with my remaining bit of strength I insisted that they follow my instructions which eventually they did. I must admit that from time to time I ask myself what I would have done had they had kept misbehaving…
>
> Unfortunately, I had to face similar situations several times during my first two years of teaching. I felt like the students were testing me, looking to see how consistent my behavior was…However the second time is never like the first. The first time you confront a certain situation which was not anticipated, the emotional effect is very powerful since it is accompanied by a sense of helplessness. The first time it happens to you, you do not know how to respond,

you feel a lack of proper communication skills, and your self-esteem plunges. However, when you face a similar situation again, knowing that you have already survived such an experience, your emotional reaction (ER) is less intense. You feel like you already know how to handle the situation successfully."

Betty claims that although the first years were difficult she chose not to quit her job: "I had many moments when I asked myself why keep on suffering? However, emotionally, I could not afford to give up. It was actually like admitting that I was not capable of handling a class. I could not bear this thought…It was my pride [smiling] that prevented me from quitting."

Betty's description of her first lesson is full of negative emotional expressions: shock, disrespect, hopelessness, and frustration. These emotions resulted in a sense of "being pushed to the corner," which affected her ER and her decision to use the threat of leaving the classroom against the students. After the students laughed, her emotions intensified to such an extent that Betty was close to tears. The fact that Betty chose to open her story with this lively and unpleasant memory demonstrates how powerful these emotional impressions were. Betty, however, quickly regained her composure and repressed her negative emotions. She chose to use an alternative ER, and then insisted that the students follow her instructions. Although this alternative reaction was successful, the pestering thought of "what would have happened if…" occupied her thoughts for years. It appears as if some sort of "emotional sequence" in Betty's mind remained unsolved.

According to Betty, ERs decrease in their intensity due to the building of EK. The second time she had to face such an episode in the teaching environment, she already knew what to do and how to react. Emotions can either paralyze one's actions or serve as a starting point for learning how to transform them into an actual response. This is the meaning of building EK. In Betty's case, EK that was translated into communication skills with students and knowledge about classroom management. In the ensuing years Betty asserts that she continued to suffer from negative emotional experiences and reactions within the classroom. Building her EK actually sustained her through the inner emotional struggle of whether to give up and thus lose her pride or whether to learn to confront her emotions and regulate and navigate her way through them. Gradually Betty built her self-image as a teacher:

"During the first few years of my teaching I remember that my students kept asking me personal questions. I believe this was their way to get to know me and to adjust their behavior to my expectations. At the beginning I was flattered and I cooperated with them. But then I realized that they interpreted this cooperative behavior of mine to mean I was their friend. When I had to be authoritative they were confused. So I realized that I had to operate differently - to be nice to them not as a friend but as a teacher. In fact, my image as a mathematics teacher was built during that period… I believe that after the first two years at the school my image as a mathematics teacher was solidified and the students conveyed that information about me to new incoming students."

Learning to reflect on her EK also enabled Betty to establish her image as well as her status as an appreciated teacher. Although she was tempted to cooperate with the students and to provide them with personal information, she chose to remain nice to them, but not too friendly. We might say that these were Betty's first steps in developing emotional understanding (Denzin, 1984). Betty concluded her story:

> "The main difference between my functioning as a beginning and as an experienced teacher is that as a beginning teacher the types of knowledge I had were disconnected, isolated. I had no idea how to integrate my content knowledge, pedagogical knowledge, and EK. Moreover, I wasn't even aware of the fact that such integration was essential to my success as a teacher. I believe that my reflections on the complexity of class management and student-teacher relations was most dominant in developing my EK and in developing my ability to synthesize these types of knowledge. Only after I was able to balance between these types of knowledge did the intensity of my ERs significantly decrease, no longer being the dominant aspect of my teaching."

Betty's reflection on her evolution as a teacher focuses on the importance of merging academic content, pedagogical, and emotional knowledge. In the beginning her deficiencies in EK created a situation according to which her emotions governed and directed her actions, and they were highly intense. With time, her ability to regulate her emotions, reflect on them to generate EK, minimized their intensity and dominancy, and enabled her to recognize EK as equally important as other types of knowledge. It was, however, only after she realized that all types of knowledge were interconnected that she felt she became a good teacher.

Rose's story

Rose is 55 and she has 32 years of teaching experience. Rose’s parents were both teachers. Her father was a mathematics teacher. Rose claims that "since I was a child I knew I would never be a teacher. I saw my parents working very hard and I didn't want to be like them." When she was 18 she started studying statistics at the university. She recalls: "I hated every moment there. The teachers were bad. We were more than 100 students in a class, and the teachers didn't know us personally. I was shy, and in such a large class I was embarrassed to ask questions or provide answers." By the end of the year, after failing most exams, she started to wonder whether she had chosen the right profession. Before the beginning of the school year her father suggested that she work as a substitute teacher in his school until the beginning of the university's academic year. She accepted the suggestion "just to save some money." However, "the moment I entered the class I knew – this is what I wanted to do! It was something about the chemistry with the students." Rose left the university and started to study in a small college, where she graduated as a mathematics and physics teacher: "I loved the college. There were no more than 10 prospective teachers in a class, and our teachers knew each of us personally. They encouraged me to ask questions and listened to what I had to say." After her graduation she started to teach mathematics in a middle-high school:

> "I was young and naïve, and at the beginning I didn't realize that I was sent to teach classes no other teacher wanted. There were many disciplinary problems, but it didn't bother me. The other teachers didn't understand how I managed to survive these students…When I reflected on my experience at the university and the college, I realized that the alienated attitude at the university as opposed to the close and warm relations between the teachers and students in the college had a tremendous influence on my ability to persist in my studies. So I guessed that if I treated each student warmly and personally, not as a problematic person but as an individual, I would be able to see beyond my immediate emotional difficulties that might stem from disciplinary problems. And it worked… I knew that many students hated mathematics and found it very difficult. It was very important for me to reduce their fears. I knew this was one of the keys to my success as a teacher… Nothing however prepared me for the struggle with the school management. I never realized why the principal of the school was hostile. He didn't speak nicely to me and didn't support me as a new teacher. I tried very hard not to let this affect my work with the students. For me, closing the door of the classroom was like entering an airplane and landing in a different country… As I said, I was naïve and I had nothing to do with intrigues. By the end of the year the principal told me that he didn't want me to teach high-school classes anymore, only middle-school classes. He didn't explain why. He said that because I didn't teach the high level classes he didn't consider me important for the school. I felt insulted and humiliated, and although I loved the students I couldn't bear this humiliation and decided to leave this school."

Rose left the school with "hard feelings. My self-esteem was harmed, and I was confused. I didn't realize what had been disrupted." She found a job in another school, but the supervisor of the former school pleaded to return. She acceded to his request on the condition that she continue to teach her students. Rose feels that "I returned to that school as a winner. I gained back my self-esteem. However, the principal couldn't accept the fact that he was forced to have me back against his will. Emotionally, it was very hard to arrive to school every day. I had no idea how to confront him." Three years later her father told her that there was a vacant position in his school and she "went back to where it all started." This new school was highly selective in those days, and she started to work with "totally different students."

From Rose's story it appears that she had a high emotional self-awareness when she started to teach. Reflecting on her emotional experiences as an undergraduate student, she realized that personal and attentive relations with students are essential for developing their readiness to learn. The fact that by the time she started to teach she had already gained some relevant EK helped her handle successfully problematic disciplinary situations, and not to consider them threatening. In fact, we might say that even if there were any conflicts with the students, Rose put them aside since she was emotionally more occupied by an unexpected front – the bad attitude of the school principal. As a new teacher in school she expected to receive supportive attention from the school management in general and from the school principal in particular. The principal's attitude hurt her feelings and gave rise to feelings of humiliation and insult in her. Her lack of EK regarding relations with management prevented her from confronting her emotions and coping successfully with the situation she encountered. Rose was not able to resolve the situation, and therefore, with her damaged self-esteem, she chose to leave the school. Trying to recover her self-esteem Rose agreed to return to the school, but during the following three years she did not manage to further develop her EK with respect to teacher-management relations, and she decided to leave the school again, this time forever.

As regards to her relationships with students, Rose believes that she had "a breakthrough when my daughter entered middle-school":

> "It happened fourteen years ago, and I realized that my approach to the students was too academic. I didn't really know their emotional world. I understood that when they were angry or in bad mood it wasn't because they wanted to struggle with me, but merely because they were teenagers with emotional distresses. I became more curious about their emotional lives. I wasn't angry when they didn't do their homework. I talked to them personally and tried to be more attentive to their emotions… I tried to develop awareness about what might insult them, to recognize those with whom I could be cynical with, those who needed my encouragement, and those who needed my embrace. I stopped punishing them, because I didn't want to insult them… This emotional approach turned out to be beneficial for them as well as for me. I started to enjoy teaching more… to emphasize values and emotions, and to treat them as equal partners… As I said before, many students are afraid of mathematics, and I became more sensitive to this emotion, and I kept looking for various didactical approaches to help them overcome their anxiety."

Rose's further development of her EK as a teacher occurred when she started to develop her EK with respect to her own daughter. From her, Rose became aware of the reasons that underlie her students' anger and dispositions and started to be more involved in their emotional lives. Her new EK directed her towards developing personal emotional relationships with the students on the basis of each student's personality. Although she was already aware of their fear of mathematics, it was only after she established her EK that she was able to successfully integrate her EK and her didactical knowledge as well as her knowledge about the curriculum.

Five years ago the principal of the school retired, and a new principal started to administrate Rose's school: "This principal is bad for school. Since his first day at school he gathered around him 'yes-men' and formed cliques…I refused to join the 'right' clique and, like other teachers in my condition, I have to deal with his harassment. However, unlike my first school, I don't let it ruin me emotionally. I believe I have learned how to control my emotions, to neutralize them when necessary. I don't take it personally. He has his own personal problems, and I can't be responsible for that."

Rose's last excerpt shows that throughout the years she developed her EK regarding teacher-management relationships. When she had to face hostile behavior for the second time, she was already prepared and her ER towards the situation was not as intense as it had been the first time.

5. Conclusions

Teaching is an emotional practice and the use of emotions can be helpful or harmful (Hargreaves, 2000). Thus there is a need to learn about teachers' EK in order to be able to redirect it in desirable directions.

EK is about developing emotional understanding. The last term is constituted from two words which come from totally different areas. *Emotional* refers to activities ruled by instincts and intuition, while *understanding* refers to activities ruled by logic and cognition. The combination of these two terms implies the need to control and lead the emotions by cognitive means, such as understanding. Moreover, while didactical and content knowledge can be acquired in teacher training programs, EK is dynamically built as a result of human interaction. Moreover, EK is subjective and varies from one person to another. Both Betty and Rose describe EK as a knowledge base that is gradually built and which comes as a result of human interaction. When Betty and Rose made their initial steps as teachers, they were well equipped with didactical and curricular knowledge. Their preliminary EK however was influenced by their previous experiences as learners: in Betty's case – her experience as a pupil in school and in Rose's case – her experience as an undergraduate. Both Betty and Rose refer to EK concerning their interaction with students while Rose refers in addition to EK concerning her interaction with the school's principal. Considering Zembylas' (2007) distinction between the three planes of EK, although Betty and Rose refer to the individual, relational and socio-political planes of emotion, in our paper we relate merely to personal relationships. EK that relates to inter-personal relationships develops as a result of what teachers encounter during their professional lives. Namely, when facing crises in teacher-student or teacher-management relationships, coping with the situation produces an ER which in turn produces a practical reaction that can affect the situation itself. Considering Betty's and Rose's narratives, it appears that ERs differ in their intensity and focal points. The intensity is heavily dependent on the rate of familiarity with the focal point, the teacher's personality, social-cultural background, and more.

That the interviews represent retrospective perspectives of events the teachers experienced many years ago, strengthens the feeling that after all these years they served as milestones in building their EK. It is harder to reflect on ER than on cognitive processes since the first action might involve the exposure of weaknesses and difficulties. It is therefore worthwhile to consider Betty's suggestion to create a kind of support group which can help teachers safely make it through the hard start is unusual, since people often tend to avoid the exposition of their feelings in public.

Both interviewees managed to develop a certain level of ability to reflect on their emotions during their teaching practice. This ability enabled them to develop their emotional understanding regarding their relations with students, the school principal, and other colleagues.

In most professions people face new situations, experience frustration and helplessness, joy and satisfaction, and difficult individuals, among other challenges. The inability to reflect on circumstances and ER, to grow and develop into the profession, can lead one to experience negative feelings such as frustration. These feelings, although essential to the process of growth and development, have a tremendous influence on other aspects of one's personal life (Yaffe-Yanai, 2000). It is therefore important that teachers be able to reflect on their experiences, design and develop their EK, and learn to integrate the different types of knowledge they possess. It would be interesting to listen to the stories of teachers who chose to quit teaching in various phases of their professional lives, and compare their EK to those who persisted.

Our focus is on middle- and high-school mathematics teachers. It is reasonable to assume that elementary school teachers have different stories. It would be also interesting to examine the differences between lower-elementary and upper-elementary school teachers to learn how the students' age influences teachers developing EK.

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CHANGING BELIEFS AS CHANGING PERSPECTIVE

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There is a phenomenon that has been observed in my work with inservice teachers. This phenomenon can be seen as embodying profound and drastic changes in the beliefs of the teachers participating in various projects. In this article I first describe this phenomenon and then more closely examine it using a framework of perspective. This framework allows for the articulation of the changes of beliefs as a foregrounding (or a reprioritization) of already existing beliefs. In doing so, I put forth a theory that allows for beliefs to be seen as both stable and dynamic – but always contextual.

INTRODUCTION

I work with inservice teachers. My reason for doing this is to affect change in these teachers' classroom practices, and ultimately, to affect change in the mathematical experiences of their students. In general, I try to accomplish this change through a focus on teachers' beliefs – beliefs about mathematics and beliefs about what it means to learn and teach mathematics. My assumption is that there is a link between teachers' beliefs and their practice (Liljedahl, 2008) and that meaningful[1] changes in practice cannot occur without corresponding changes in beliefs.

Recently, my main method of operating in this regard is to work with groups of teachers to co-construct some artefact of teaching – a definition, a task, an assessment rubric, a lesson, etc. This has proven to be a very effective method of reifying[2] the fleeting, and sometimes delicate, changes to beliefs that teachers experience within these settings (Liljedahl, in press, 2007). Within this context I am both a facilitator and a researcher. However, I am not a facilitator and a researcher in only the obvious sense. Although it is true that I facilitate the various activities that the teachers engage in – from discussions to the crafting of artefacts – it is also true that I facilitate the environment within which this all takes place. The sort of inservice work that I am involved in is more than simply the delivery of workshops, it is the provision and maintenance of a community of practice in which ideas are provisional, contextual, and tentative and are freely exchanged, discussed, and co-constructed. At the same time, while it is true that as a researcher I am interested in the down-stream effects of the work that I am engaged in (changes in teachers' practice in the classroom,

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[1] Meaningful change is seen as a shift in teaching towards a more reform oriented practice. This change needs to be pervasive and robust.

[2] In this paper *reify* and *reification* is used in the tradition of Wenger (1998) rather than in the tradition of Sfard (1994). As such, reification means to make concrete – to turn some ephemeral aspect of teaching into thing*ness*.

improvement in students' experiences and performance, etc.), it is equally true that I am interested in researching the inservice setting itself. There is much that happens within these settings. It is this later context which is the subject of this paper.

Working as both the facilitator and the researcher interested in the contextual and situational dynamics of the setting itself I find myself too embroiled in the situation to adopt the removed stance of observer. At the same time, my specific role as facilitator prevents me from adopting a stance of participant observer. As such, I have chosen to adopt a stance of *noticing* (Mason, 2006). This stance allows me to work within the inservice setting to achieve my inservice goals while at the same time being attuned to the experiences of the persons involved. I notice, first and foremost, myself. I attend to my choices of activities to engage in and the questions I choose to pose. I attend to my reactions to certain situations as well as my reflections on those reactions, both in the moment and after the session. More importantly, however, I attend to the actions and reactions of the teacher participants both as individuals and as members of a community. I observe intra-personal conflicts, interpersonal interactions, the dynamics of the group, as well as the interactions between individuals and the group. And in so doing, from time to time I notice phenomena that warrant further observation and/or investigation. Often these are phenomena that occur in more than one setting and speak to invariance in individual or group behaviour in certain contexts. Once identified these phenomena can be investigated using methodologies of practitioner inquiry that combine the role of educator with researcher – in this case teacher educator with researcher (Cochrane-Smyth & Lytle, 2004)[3]. Using a methodology of noticing I have observed rapid and profound changes in beliefs among individual teachers within a context of reification (Liljedahl, in press, 2007) and, more recently, among groups of teachers within this same context. It is this later phenomenon that I report on in this paper.

THEORETICAL BACKGROUND

Green (1971) classifies beliefs according to three dichotomies. He distinguishes between beliefs that are primary and derived. "Primary beliefs are so basic to a person's way of operating that she cannot give a reason for holding those beliefs: they are essentially self-evident to that person" (Mewborn, 2000). Derived beliefs, on the other hand, are identifiably related to other beliefs. Green (1971) also partitions beliefs according to the psychological conviction with which an individual adheres to them. Core beliefs are passionately held and are central to a person's personality, while less strongly held beliefs are referred to as peripheral. Finally, Green distinguishes between those beliefs held on the basis of evidence and those held non-evidentially. Evidence-based beliefs can change upon presentation of new evidence.

[3] It should be noted that the main distinction between a methodology of noticing and a methodology of practitioner inquiry is that noticing doesn't presuppose a research question. It is a methodology of attending to the unfolding of the situation while being attuned to the occurrence of phenomena of interest.

Non-evidentiary beliefs are much harder to change being grounded neither in evidence nor logic. Instead they reside at a deeper and tacit level.

A person's belief system can, subsequently, be seen as a collection of beliefs competing for dominance in different contexts. Metaphorically, it is like a scene that is photographed from different perspective, with each perspective allowing something else to be foregrounded. Changes to learners' belief systems can then be seen as changes in perspectives[4]. Green argues that changing learners' belief systems is the main purpose of teaching. I argue that changing teachers' beliefs is the main purpose of inservice education.

METHODOLOGY

The data for the results presented here comes from three different, but similar, contexts in which I worked with groups of teachers in different schools and school districts. The first context (c1) involved a group of grade 5-8 mathematics teachers (n=10) working to design a task that could be used as district wide assessment of grade 8 numeracy skills in a school district in western Canada. This inservice project was comprised of 6 sessions (3 hours long, 3 weeks apart) during which we were to co-construct a working definition of numeracy (later adopted as the district definition) and design and pilot test a number of tasks that would reflect the qualities of our definition. The second context (c2) involved a group of grade 8 mathematics teachers (n=6) from a different district engaged in a very similar project. This time we were attempting to design a task that could measure the numeracy skills of their own students only. This project was comprised of 3 full day meetings 6 weeks apart. The third context (c3) involved all the mathematics teachers (n=18) in a middle school (grades 6-8). In this context we were working to design an assessment rubric that could capture some of the mathematical processes necessary for effective mathematical thinking. This involved a series of 12 one hour meeting held every two or three weeks.

As already mentioned, my method of operating within these inservice environments is through *noticing*. What this means from a more methodological perspective is that there is a great reliance on field notes taken both during the inservice sessions and more prolifically immediately after the inservice sessions. These field notes serve as a record of the things that I have noticed during individual sessions. Of course, they are limited in that they are only a record of that which has been attended to. However, these notes (or *noticings*) then form the basis of what is attended to in future sessions thereby creating an iterative process of refinement of attention. As this process continues phenomena that are deemed to be interesting receive more and more attention. This may simply mean a heightened awareness or anticipation of certain occurrences. Other times this means an adjustment in the facilitation practices in

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[4] This is not to say that changes in beliefs cannot also be seen as changes in beliefs, but for the purposes of this paper I stay with the metaphor of changing perspective.

order to more aggressively pursue the phenomenon. And sometimes it may mean stepping outside my role as a facilitator to investigate the phenomenon more directly as a researcher through methods such as interviews or questionnaires.

As such, the data for this study comes from a number of different sources. First and foremost, are the field notes from each of the aforementioned contexts. These notes increased in detail with each occurrence of the phenomenon. From c2 and c3 there are also transcriptions from interviews with different participants conducted at opportune times during or after certain sessions. These interviews were aimed at uncovering the participants own thoughts about the changes I was observing. The questions were of a semi-structured nature meant to preserve the conversational atmosphere that I had established with all of the participants while at the same time helping to illuminate the phenomenon itself.

THE PHENOMENON

The *exo/endo-spection* phenomenon, as I have come to call it, is comprised of a series of either three or four distinct phases, always in the same sequence, each having its own associated name. The names are an amalgamation – the prefix *exo-* and *endo-* comes from Greek meaning outer, outside, external and inner, inside, internal respectively; while *-spection* comes from the Latin *specere* which means 'to look at'.

Phase 1: exo-spection (x)

The teachers work on an activity which, at the time, occupies their focus. This could be a problem solving exercise or the designing of a lesson, task, or assessment rubric. Whether or not the activity is relevant to their own teaching practice is immaterial as the teachers' focus is on the completion of the task, rather than on the potential for the task to inform their own practice. That is, the teachers are looking at the activity as lying outside of themselves.

Phase 2: eXo-spection (X)

The teachers realize that the problem they have solved, or the lesson or task they have built, is not commensurate with their own classroom context. They see this as a large scale problem bemoaning the poor state of affairs of all students and the educational system in general. They look at the source of the problem as lying far outside of themselves – societal expectations, the curriculum, the evils of external examinations, deterioration of standards, etc. – and speak of systemic reform as the only solution. As such, they are not only pushing the problem further outside of themselves, but also broadening its scope.

Phase 3: eNdo-spection (N)

Suddenly there is a change in the teachers' disposition – the problem, regardless of where it lies, must be solved within their own practice in the scope of the classroom. Now the conversations are about what they can do within their teaching in order to enable their students to be successful in solving a specific problem, completing a specified task, or performing well on a given assessment. The teachers' are no longer pushing the problem, and any subsequent solutions, away from themselves, but are rather bringing it back to their locus of control.

Phase 4: endo-spection (n)

For some teachers there is a final shift of attention to the plight of individual students. The conversations shift from the classroom to a particular student or subset of students, and with it comes a narrowing of focus on their influence as teachers. This final shift is also marked by a subtle shift in discourse from *teaching* to *learning*.

It should be noted that I have deliberately avoided using the term *introspection* which means to examine one's own thoughts and feelings. This is not what I am trying to capture here. *Endo-spection* is not about looking inside oneself, but about looking at something as lying inside of oneself or one's locus of control. Conversely, *exo-spection* is about looking at something as lying outside of oneself or one's locus of control.

In c1, x occurred in the first two sessions, X during the third session, N during the fourth session, and for two participants, n occurred in the last two sessions. In c2, x and X occurred in the first session, N in the second, and for one participant there was evidence of n in the third session. Finally, in c3, x occurred in the first 3 sessions, X in the fourth and fifth session, N in the sixth session, and for some of the participants, n occurred at various times during the last four sessions.

In general, the adoption of an exo-spection stance was uniformly a group position. That is, without prompting, every member of the group adopts an exo-spection stance and the group as a whole adopts an exo-spection stance. The discourse of the group did not deviate from this stance and there was a general sense that there was no need to do so – until there was a sudden transition to the eXo-spection stance. This transition, as well as the transition to eNdo-spection, was initiated by one or two members of the group, but then uniformly taken up by the group as a whole. It is almost as though the initiators were merely articulating what was already in the minds of the other members of the group, or the initiators merely precipitated an inevitable position. Conversely, the shift to an endo-spection stance, although articulated within the group context, was not taken up in the same way.

ANALYSIS

Because, for this paper, I am most concerned with changes in beliefs I will constrain my analysis to those points of greatest change – that is, the transitions between phases ($x \rightarrow X$, $X \rightarrow N$, and $N \rightarrow n$). Further, I will look at these changes through a lens of changing perspectives.

exo-spection to eXo-spection (x → X)

As already mentioned in the description of the exo-spection (x) phase, the teachers are initially contentedly working at completing the task at hand. In c1 and c2 this involved designing a numeracy task that conforms to a taken-as-shared definition of numeracy. In this case the teachers made extensive references to the published curriculum learning outcomes, the rationale that forms the underpinnings of the curriculum, as well as some ministry documents pertaining to the positioning of numeracy vis-a-vis the curriculum. In c3 the tasks that occupied the teachers in the first few sessions were increasingly challenging[5] problem solving activities. Here the teachers were caught up in the excitement of doing mathematics that does not explicitly rely on mastery of specific learning outcomes. This can be seen in Barry's comments during one of the early sessions.

> I love these problems. I mean, it's been a long time since I worked on problems myself, and I really like it. That card trick problem had me scratching my head all weekend. (Barry, c3, session II, field notes)

In either case, the teachers were focused on their own completion of these tasks, without much consideration for how they applied to their own practice.

The transition to X occurred in all three contexts when there was a sudden awakening to the fact that what the teachers were working on was not commensurate to their own classrooms contexts. This is nicely captured in the sudden change of tone in Barry's comments.

> These problems are all fine and good. I mean, I enjoy doing them, but I don't have time for this with my kids. I have WAY too much stuff to get through to play around with these kinds of problems. Besides, my kids don't have enough patience for this kind of work. (Barry, c3, session V, interview transcripts)

It is also seen in the comments of Heidi and Charlotte working in c1.

> I think we're getting it. The task is really starting to look like a numeracy task rather than just a word problem. It's not easy fitting all this stuff about communication, ambiguity, and multiple solutions into a task. But we're getting there. (Heidi, c1, session II, field notes)

> I think these tasks are great, we've done a good job, but parents [of my students] are never going to go for this. The first time I send something like this home the phone will be ringing off the hook. We constantly have to work on drills to get the kids ready for the FSA's [Foundational Skills Assessment – an external high stakes exam, the results of which fold back onto the teacher]. And if we're not we're hearing about it from the parents and not because of the FSA's. They don't care about that, but these parents, a lot

[5] This does not mean an increase in the mathematical complexity of the tasks. What is increasing is the demands on particular problem solving skills required (ability to organize work, communicate thinking, group work, deal with ambiguity, etc.).

> of them are from Asia, and to them drills are important. (Charlotte, c1, session III, field notes)

The beliefs that these teachers are expressing (drills are important, learning outcome is curriculum, what parents want is important, kids are not capable) are not beliefs that have suddenly manifested themselves in latter sessions of the project. These are deep-seated beliefs (primary, core, evidential, tacit, or otherwise) that have been in the background during the teachers' initial encounters with their respective tasks. Working alone, or in a group, on something away from the multifaceted demands and expectations of their job less dominant beliefs (mathematics can be fun, numeracy is important, etc.) were able to come to the fore and inform their work in the initial sessions. But as the reality of their job rushed in on them the more dominant beliefs once again moved to the forefront, eventually paralysing their ability to see their initial work as being relevant to their own practice. However, there is still a wish that relevance could be found, but it is overwhelmed by the deep-seated belief that the problem is systemic AND can only be solved systemically. This can be seen in Adam's remarks.

> Look, I agree that this is all very important. But there is just no way that we can make this work. There just isn't enough time, the kids aren't strong enough, we don't have administrative support, and, at the end of the day, the Ministry of Education just doesn't care. If they did, this is the kind of stuff we would see on the provincial exams. Until we can get them to change everything from the top down it just isn't going to work. I wish it were different, but it isn't. (Adam, c2, session I, interview transcripts)

eXo-spection to eNdo-spection (X → N)

Initially, this transition is what drew my attention to the xXNn phenomenon. After commiserating about the negativity and hopelessness experienced in prior session of c1 there was a sudden rebirth of professional growth. This can be seen in Charlotte's comments in the fourth session of c1.

> We have to keep pushing on in the direction we are going. If we don't design a task that shows what the kids can't do we're not ever going to be able to make any changes. We won't have anywhere to start. (Charlotte, c1, session IV, field notes)

Adam expressed a similar sentiment in the second session of c2.

> In my opinion, these tasks aren't telling me enough. I'd like a task that really showed that these kids don't have a clue how to work together, for example. (Adam, c2, session II, field notes)

He adds details to these comments in a post-session interview.

> I started to think about what we were doing here, with this whole project, and what it is we are trying to accomplish. I then started to think about how little I took away from my own math learning and what it is that is really important. We have an opportunity here to develop some really useful skills, stuff that these kids can use in grade 9, in grade 10, in

> university, in life. They need to learn how to work together, how to deal with problems, how to tough it out, and stuff like that. But in order to do that we need to first show them that we are serious about this stuff. We can't just talk about it, we have to do it, and we have to mark them on it, and we have to start somewhere. (Adam, c2, session II, interview transcripts)

Tracey, also from c2, has a slightly different perspective.

> They loved it. They asked me yesterday when we are going to do another numeracy task. I couldn't believe it. But you know what, they don't have a clue how to work together. So, now I'm working on that in my classroom. (Tracey, c2, session II, field notes)

As did Mary, who brought in samples of students' work.

> As you can see there isn't much here – especially the boys. Like, you have to have a secret decoder ring to figure out what they are doing here. BUT, you know what, they did it. They worked on it and they got answers. Now we have to go forward with it. (Mary, c3, session VI, field notes)

The belief that assessments can be used formatively to inform both the teacher and the students is, again, not new. It has now moved into the forefront, however, buoyed by the realization of what it is that it is important, what the students can (or cannot) do, and what it is that the students enjoy doing. Whereas the transition from x to X can be seen as a regression to the norm (a return to a lower energy level, if you will) that is achieved almost subconsciously, the transition to N is almost wilful in nature. This re-prioritizing of beliefs is taxing and will require much effort and energy to sustain. It requires effort and motivation, and that motivation is found both in the successes of the students and the recapitulation of what is important. Or it can be found in the realization that what has come before isn't working, as is articulated by Phil.

> I'm not sure if this is going to work. But I know for sure that what I've been doing before isn't working and I can continue to blame the system for all its faults or I can decide to do something about it. All I know is that I'm tired of both teaching my students AND learning for my students. Something has to change. (Phil, c3, session VI, interview transcripts)

eNdo-spection to endo-spection (N → n)

As already mentioned, only some of the teachers moved to the final phase of the xXNn phenomenon. Those who did, however, did so for seemingly the same reason – they were focusing on the learning of particular students or subsets of students. This was seen in their discourse about particular cases. Whereas some teachers spoke about cases as being exemplifications of the norm or the outliers within their classroom, these teachers spoke about the individual cases as standing for themselves. This can be seen in both Tracey's and Mary's comments.

> So, I still have this one girl who is just toxic to anyone I put her with. No matter what I do she just will not work cooperatively. I've talked to the councillor and we think it has to do with self-esteem issues. So, I'm starting to think that this is where I should be putting my focus when it comes to her. (Tracey, c2, session 3, field notes)

> In general, the students are doing much better. My work using graphic organizers has really helped. But, I still have a set of boys who just can't figure out which graphic organizer to use, or even that they have to use one. I'm not sure what to do about it, probably just keep working on it. But for now I'm still telling them which ones to use so that they can get through the task. (Mary, c3, session 11, interview transcript)

The belief that students are individuals and, thus, require differentiated instruction is likely not a new belief. However, with the use of formative assessment as an information gathering tool the teachers were giving this belief more and more prevalence.

CONCLUSION

Beliefs are stable patterns of thought, conscious or otherwise (Green, 1971). It is, therefore, unlikely that the teachers in this study changed their beliefs as drastically as the data may indicate. An alternative explanation is that the profound changes in beliefs are not a change at all, but rather a reprioritization of already existing beliefs - an affording of prevalence to less dominant beliefs. Such an explanation allows for both the robustness of beliefs and the possibility of profound change. This idea of reprioritization, or perspective, also allows for a more useful application of Green's organization of beliefs along three dimensions. A person's beliefs are hidden from us. Indeed, they may even be hidden from the person themselves. As such, knowing that beliefs may be central or peripheral, core or derived, evidential or tacit does us no good. Instead, recognizing that in different contexts different beliefs will be foregrounded, wilfully or otherwise, will allow us to think more holistically about belief systems as dynamic and contextual.

The xXNn phenomenon is such a context. Using a methodology of noticing and a framework of perspective I have described and analysed this phenomenon and concluded that the profound changes that are occurring within this context might just be due to a reprioritization of already existing beliefs. Further research into the phenomenon is necessary. There is great potential in analysing it using frameworks of psychology, group dynamics, as well as Gestalt. But it is early days, and this research is still in its exploratory phase. Now that the phenomenon has been identified, articulated, and even anticipated[6], however, more detailed data can be gathered and more thorough analyses can be performed.

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[6] In fact, since gathering the data for the work presented here I have already identified the phenomenon, or subsets of it, within a master's course, a single session of a lesson study cycle, and a 90 minute workshop.

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BELIEFS: A THEORETICALLY UNNECESSARY CONSTRUCT?

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In this paper I analyze different existing suggestions of definitions of the term beliefs, focusing on relations between beliefs and knowledge. Through this analysis I note several problems with different types of definitions. In particular, when defining beliefs through a distinction between belief and knowledge systems, this creates an idealized view of knowledge, seen as something more pure (less affective, less episodic, and more logical). In addition, attention is generally not given to from what point of perspective a definition is made; if the distinction between beliefs and knowledge is seen as being either individual/psychological or social. These two perspectives are also sometimes mixed, which results in a messy construct. Based on the performed analysis, a conceptualization of beliefs is suggested.

Key words: belief, definition, individual, knowledge, social

INTRODUCTION

There exists plenty of research in mathematics education focusing on aspects of beliefs, in recent years evident by books covering this specific topic (e.g., Leder, Pehkonen, & Törner, 2002b). However, Thompson (1992) points out that although the topic has been popular in educational research for many years, little attention has been given to theoretical aspects of the concept of beliefs. Specifically for mathematics education, Op't Eynde, De Corte, and Verschaffel (2002) note the same type of lack of theoretical studies about beliefs.

In many studies, the term 'belief' is not explicitly defined, but it is assumed that the reader knows what is meant (Thompson, 1992). For some purposes this might suffice, and in general different types of definitions, from informal to extended types, could be suitable depending on the situation (McLeod & McLeod, 2002). In addition, a theoretical perspective can focus on different aspects, for example by being more or less philosophically or psychologically oriented. When Schommer (1994) discusses different types of beliefs as key concerns in the conceptualization of epistemological beliefs, she argues that interesting results, perhaps of a more applied type, can be achieved also without explicit focus on the more philosophical aspects, but that the inclusion of such aspects would improve the conceptualization of beliefs. A philosophical perspective can include what McLeod and McLeod (2002) describe as part of a more elaborate definition, such as relations to nearby concepts. For beliefs, this elaboration could include relations between beliefs, knowledge, and different affective constructs.

When studying beliefs, instead of analyzing and arguing around different types of definitions of beliefs, it seems most common to describe different suggestions found in the literature and then choose one of these or create your own for the study in question (if a definition is at all given). Even if it is perhaps impossible to create a general definition that is suitable for all types of research (as noted by Abelson, 1979; McLeod & McLeod, 2002), there is a need to discuss and analyze different suggestions of definitions. In the present paper the focus is on such analyses.

Purpose

As the title of the present paper implies, I am taking a critical perspective regarding the concept of beliefs and suggestions of how this construct can be defined. This critical stance has evolved from informal, personal reflections when having read different types of studies of beliefs, and similarly as Pajares (1992), having noted a certain messiness regarding definitions and properties of beliefs. I have not only noted such messiness when looking at the breadth of different studies, where plenty of different suggestions for definitions or properties exist, but also when trying to analyze the internal coherence of singular articles regarding definitions and properties of beliefs.

The main purpose of the present paper is to dig deeper into these reflections, in order to see what types of problems seem to exist when trying to define beliefs and also if and how these problems can be resolved. In particular, I will suggest a type of reconceptualization of beliefs, emerging from noted problems around (1) the point of perspective taken when defining and describing properties of beliefs, and (2) suggested relationships between beliefs and knowledge.

It is important to note that I am not suggesting that the ideas presented here should be seen as final in some sense, but that they primarily constitute a starting point in my attempts to reconcile with some experienced problematic issues, for continued discussions and reflections and for continued work on a larger research project (see Österholm, in press). Also, I am not suggesting that I am presenting an entirely new perspective, regarding the mentioned reconceptualization, but as can be seen by references given throughout the present paper, others have presented similar suggestions, although sometimes done from other perspectives or focusing on somewhat different aspects of beliefs.

Research about beliefs

Historically, the interest in educational research in the study of beliefs seems to come from realizing that a focus on “purely cognitive” factors (in particular, content knowledge) is not sufficient when trying to describe and explain students’ problem solving activities (Pehkonen & Törner, 1996; Schoenfeld, 1983) or teachers’ classroom behavior (Speer, 2005). The relationship between (content) knowledge and beliefs is thus a central aspect. This relationship is also the most commonly referred to when discussing the definition of beliefs, and different views about this relationship can also be seen as a major reason for experiencing beliefs as a messy construct (Pajares, 1992).

Since there can be different types of knowledge, such as procedural or conceptual, while beliefs are usually formulated as statements, the comparison between knowledge and beliefs can focus on factual, declarative knowledge.

BELIEFS – AS SEEN FROM DIFFERENT POINTS OF PERSPECTIVES

Abelson (1979) describes a cultural dimension of beliefs; that if all members of some type of group have a specific belief, then they might not label it as a belief but as knowledge. This cultural dimension corresponds to what other authors describe as a social property of knowledge (e.g., Op't Eynde et al., 2002; Thompson, 1992); that for something to be seen as knowledge it has to satisfy some type of truth condition – a condition that is negotiated and agreed upon within a community (of practice). Thus, depending on what social community you belong to, you can have different views on what is seen as knowledge and what is seen as belief. From this perspective, when focusing on social aspects, the difference between belief and knowledge can be defined by saying that knowledge fulfills the mentioned social criteria but that beliefs do not, or perhaps *cannot*, since there can exist statements that cannot be evaluated using existing criteria within a certain community.

This relative property of beliefs highlights the importance of taking into account from what perspective a labeling of something as a belief or as knowledge is being done. In addition, there is also the possibility of changing perspective when deciding on the definition of beliefs, from defining beliefs from a social perspective to defining beliefs from an individual perspective. For example, when Leatham defines beliefs he describes the relationship between belief and knowledge by seeing that

> there are some things that we "just believe" and other things that we "more than believe – we know." Those things we "more than believe" we refer to as knowledge and those things we "just believe" we refer to as beliefs. (Leatham, 2006, p. 92)

This type of definition describes the relationship between beliefs and knowledge as a psychological property. A somewhat different defining property of beliefs, but also from the individual perspective, is given by Abelson (1979); that the believer is aware that others may believe differently. This property includes a social dimension but the distinction between beliefs and knowledge is still being done from the individual perspective, and is psychological in nature. From this perspective, when focusing on the individual, the difference between belief and knowledge can be defined by seeing beliefs as something related to uncertainty, either in relation to other parts of an individual's beliefs/knowledge or in relation to what others claim to believe/know.

Sometimes an author suggests some defining properties of beliefs that are from an individual perspective and some other properties that are from a social perspective. For example, I have mentioned Abelson (1979) when describing both these perspectives, and Pehkonen and Pietilä (2003) also include both these perspectives when differentiating between beliefs and knowledge. The simultaneous use of these different perspectives when defining a concept could be a cause for creating a messy construct. However, it is often difficult to decide if all suggested properties should be seen as part of a homogenous definition or as something that can be inferred from a (sometimes implicit) definition or from empirical results.

From this analysis we can see that a central distinction in the discussion of beliefs and knowledge is from what perspective a definition or description is given, whether these concepts are construed as individual or social. This distinction deals with whether the decision regarding differences between belief and knowledge is located in the individual (i.e., that it is psychological in nature) or if it is located in the social community. Independently of which of these perspectives is used when defining beliefs, there is also another aspect of different perspectives; that different persons can have different views on what is regarded as knowledge and what should be labeled as belief, that is, there is a relative property of beliefs. This property is caused by taking the relationship between beliefs and knowledge as a starting point when defining beliefs and is also based on a general view of knowledge (which has previously not been stated explicitly in the present paper), that knowledge is "not a self-subsistent entity existing in some ideal realm" (Ernest, 1991, p. 48), but that knowledge is seen either as an individual construction (what Ernest label as subjective knowledge) or as a social construction (what Ernest label as objective knowledge).

TYPES OF DEFINITIONS OF BELIEFS

Sometimes it can be difficult to analyze some of the suggested definitions and properties of beliefs since the authors do not always motivate or describe these defining properties in detail. For example, it is sometimes mentioned, without further explanation, that beliefs can be conscious or subconscious (e.g., Leatham, 2006; Pehkonen & Törner, 1996), but since the concept of consciousness in itself is very complex (e.g., see Velmans, 1991) it is difficult to interpret such a suggested property of beliefs. In particular, the interpretation becomes more difficult if some definition of beliefs has not been given, or if no connection is made between a suggested property of beliefs and a given definition.

One way to define beliefs is to focus on the claim that a person believes that (or has the belief that) a certain statement is true. The question of what you mean by such a claim deals with the definition of beliefs. For example, a belief can be seen as a type of knowledge that is "subjective, experience-based, often implicit" (Pehkonen & Pietilä, 2003, p. 2), or as a personal judgment formulated from experiences (Raymond, 1997, p. 552). However, many such definitions seem to be of an informal type (as labeled by McLeod & McLeod, 2002), since they most often do not explicitly describe what is meant by all words used in the definition and how these words/properties create a construct different from nearby concepts.

Another way to define beliefs, or at least to describe some properties of beliefs, is to focus on relationships between different beliefs, and thereby describe characteristic properties of so-called belief systems. Certain suggested differences between belief systems and knowledge systems can then be taken as a characterization of beliefs. In the literature it seems common to refer to Abelson (1979) and Green (1971, as cited in for example Furinghetti & Pehkonen, 2002; Leatham, 2006; Op't Eynde et al., 2002; Pehkonen & Pietilä, 2003; Raymond, 1997) who both have proposed such differences between the two kind of systems. Since references to belief systems seem quite common in the mathematics education literature, I will in the next section analyze the notion of belief system regarding the view of knowledge that is implicitly, and sometimes explicitly, created through the separation of belief and knowledge systems.

While a definition that focuses on a singular belief/statement can be done from both an individual and a social perspective, implicit in the type of definition that focuses on belief systems seems to be a view that such systems are psychological constructs.

Properties of belief systems – creating an idealized view of knowledge

There is no consensus in the research community on the positioning of beliefs on a cognitive-affective scale (Furinghetti & Pehkonen, 2002), but it is sometimes claimed that a difference between belief and knowledge systems is that the former has, or at least has a relatively stronger, affective component (Abelson, 1979; Speer, 2005). However, it is unclear why, for example, a certain belief about mathematics teaching should have a greater affective component than the knowledge of the relationship between the diameter and the circumference of a circle. The situation (or the several situations) when the knowledge about the circle has been dealt with could very well have been strongly loaded with affect, for example from the joy of discovering this relationship or the dislike of having another fact to memorize. Such existing affective components of knowledge are also pointed out by Pajares (1992).

Also, it is seldom explained in detail how or why beliefs should be regarded as 'more affective' than knowledge, and when McLeod (1992) describes a framework for the study of affect, it is pointed out that beliefs are not emotional in themselves but that the role of beliefs is one (central) factor when attitudes and emotional reactions to mathematics are formed.

Some suggest that belief systems are more episodic in nature than knowledge systems; that beliefs have a closer connection to specific situations or experiences (Abelson, 1979; Speer, 2005). This property seems to lie close to the clustering property suggested by Green (1971, as cited in Leatham, 2006), which permits the belief system to consist of clusters of beliefs that can be more or less isolated from each other. Leatham (2006) describes this property as a means to explain the contextualization of beliefs and that a person can hold different beliefs that can seem to contradict each other, if these beliefs belong to different clusters. However, learning and thereby knowledge is also always situated and context dependent, “resulting in clusters of situated knowledge” (Op't Eynde et al., 2002, p. 25).

Another suggested difference between belief and knowledge systems is that belief systems are built up using quasi-logical principles while knowledge systems are built up using logical principles (Green, 1971, as cited in Furinghetti & Pehkonen, 2002). For example, it is claimed that relationships between beliefs cannot be logical “since beliefs are arranged according to how the believer sees their connections” and also that “knowledge systems […] cannot contain contradictions” (Furinghetti & Pehkonen, 2002, p. 44). If a person’s knowledge system is not built up around how this person sees the connections between different components of the system, it seems unclear exactly who or what is creating the structure within the system. In this case knowledge is perhaps not referred to as an individual, psychological construct but seen as a social construct. However, also when seeing knowledge from such a perspective it becomes difficult to reconcile with the statement that knowledge systems cannot contain contradictions, since the history of mathematics includes examples that such contradictions have existed, for example regarding the connection between convergence of series and the limit of the general term (see Leder, Pehkonen, & Törner, 2002a, p. 9). You could explain this by viewing knowledge as something absolute and thus maintaining that knowledge systems cannot be contradictory, by seeing contradictions as stemming from beliefs and not from knowledge.

In summary, regarding the relationships between beliefs and knowledge based on existing suggested properties of belief systems, knowledge is described as less affective, less episodic, and more logical and consistent. These properties create an idealized picture of knowledge, as something pure and not ‘contaminated’ with affect or context.

A PROPOSED CONCEPTUALIZATION

Based on the analysis about different types of definitions of beliefs that can be made from different points of perspectives, I here discuss a conceptualization of beliefs that take into account the criticism that has been put forward. I am not suggesting that this conceptualization is necessarily suitable for all types of studies or situations, but that it is one way to relate to some of the problems that seem to exist when defining and describing beliefs.

Beliefs are seen as being related to uncertainty in some way. From some observer’s perspective a statement can be labeled as a belief for different reasons, but all related to some degree of uncertainty, as described in the following three examples.

The first example is that if a statement cannot be included in, or directly related to, some (traditional) existing (scientific) content domains, such as mathematics or pedagogy, it can be labeled as a belief. For example, Ernest (1989) and Schoenfeld (1998), who do not explicitly discuss the definition of beliefs, describe beliefs and knowledge as two separate categories. Included in these categories are *knowledge* about teaching and learning, and *beliefs* about the nature of teaching and learning, where the former can be included in the domain of pedagogy while the latter perhaps cannot (but perhaps can be included in the domain of philosophy).

Another example is that uncertainty can also exist within a certain domain. Thompson (1992) describes the simultaneous existence of several competing, and possibly contradicting, theories within education, which he sees as a contributing cause for the difficulties of separating between beliefs and knowledge in the study of teaching and learning.

The last example of a reason for labeling something as a belief is if a statement contradicts something that is part of some scientific domain. For example, this is done by Szydlik (2000) who discusses content beliefs, which for example include to see the existence of gaps in the real line.

All these examples are from a social perspective since they relate to domains (i.e., communities of practice), but the property of uncertainty was also mentioned earlier when discussing beliefs defined from an individual perspective. Thus, uncertainty can be seen as a more general aspect of beliefs, regardless of from what perspective the concept is defined.

Unlike uncertainty, an aspect that can differ depending on from what perspective beliefs are defined is whether a belief, when compared to knowledge, is seen as a different type of psychological object. From a social perspective it becomes difficult to motivate that beliefs and knowledge refer to such different types of objects since the difference by definition is a social construction. Therefore, when studying the behavior of individual persons (such as teachers' activities in classrooms or students' problem solving activities) the social perspective does not seem suitable when defining beliefs. This has also been highlighted by other authors, for example by arguing that

> individuals (for the most part) operate based on knowledge as an individual construct. That is, their actions are guided by what they believe to be true rather than what may actually be true. (Liljedahl, 2008, p. 2)

Others have also suggested that one should focus on the study of conceptions as a whole, which includes what some label as beliefs and knowledge (e.g., Thompson, 1992). However, there could be a reason to study beliefs as defined from an individual perspective, such that beliefs and knowledge from this perspective can be seen as psychologically different types of objects, since experienced differences in the degree of uncertainty could affect behavior differently. Empirical studies seem necessary for deciding if there is a reason to make such a distinction or if it is more reasonable to see the whole of a person's conceptions.

These presented perspectives on beliefs mainly focus on singular statements and not on properties of a system of beliefs compared to a system of knowledge. This type of conceptualization is chosen because of the problems noted about the systemic view when defining and describing properties of beliefs, in particular the tendency to create an ideal and problematic view of knowledge. Also, the presented perspectives put an emphasis on the person making a claim about relationships between beliefs and knowledge, which some authors also have noted, but have not taken as a more fundamental aspect.

CONCLUSIONS

In the present paper, two main issues have been highlighted through the analysis of existing suggested definitions and properties of beliefs:

(1) The important issue of explicitly focusing on the point of perspective taken when defining and describing properties of beliefs, in particular the difference between taking a social or individual perspective regarding where the difference between belief and knowledge is located.

(2) The problematic issue of trying to define, in an objective manner and focusing on the individual, the difference between beliefs and knowledge through the separation of belief and knowledge systems.

Due to these issues one can question the necessity of the concept of beliefs, since the difference between beliefs and knowledge is not construed as so absolute, but that the meaning of the concept can be relative with respect to the person labeling something as a belief. In this way, beliefs are not seen so much as being used for making an important theoretical distinction between belief and knowledge, but more seen as a linguistic tool to signal what type of object/statement is in focus, as seen from the person making a claim about beliefs. Thus, the notions of belief and knowledge may say more about an observer than they do about some important theoretical distinction between two types of entities, or about some important distinction, “within” the person being observed. In this sense, the concept might have lost some of its theoretical importance.

The most central point in my analysis and criticism is directed towards certain contradictory aspects in the existing literature, in particular that a common psychological perspective presented through the distinction between belief and knowledge systems implies a more idealized view of knowledge than what is existent in the social perspective of knowledge. Most often, when aspects of both these perspectives are mentioned, there is no in-depth analysis of possible relationships or contradictions between these aspects. Even when Op’t Eynde et al. (2002) perform a more in-depth analysis of the social perspective, they also claim the existence of a psychological difference between beliefs and knowledge, by mentioning the quasi-logical property of beliefs. I see this use of a mixture of different perspectives as a central cause for the creation of beliefs as a messy construct. Thus, a main topic when defining beliefs is to decide, based on what is being studied, which perspective is the most suitable one when defining beliefs, the social or the individual, and then to be consistent within this one perspective.

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MATHEMATICAL MODELING, SELF-REPRESENTATION AND SELF-REGULATION

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The aim of the present study was to investigate the improvement of students' self-representation about their self-regulatory performance in mathematics by using mathematical modeling. Three materials were developed and administered at 255 11th years old students, for mathematical performance, self-representation and the use of self-regulatory strategies for problem solving. A webpage with the proposed model (the model of Verschaffel, Greer & De Corte, 2000) was constructed and used individually by students. Results indicated that the program created a powerful learning environment in which students were inspired in their own experiences. Although the program improved their cognitive and self-regulatory performance, it reproduced the differences among students in respect to their cognitive and metacognitive performance.

Keywords: self-regulation, self-representation, mathematical modeling

In the last decades, children's early understanding of their own as well as others mental states has been intensively investigated, reflecting growing interest for the concept of metacognition (Bartsch & Estes, 1996). In psychological literature, the term metacognition refers to two distinct areas of research: knowledge about cognition and self-regulation (Boekaerts, 1997). Self-regulation refers to the processes that coordinate cognition. It reflects the ability to use metacognitive knowledge strategically to achieve cognitive goals, especially in cases where someone has to overcome cognitive obstacles.

As regards the relationship between academic self-concept and academic achievement, extant literature supports both direct and indirect relationships between them; however, the range of correlations reported is a function of several factors (Guay, Marsh & Boivin, 2003). Age is a factor that affects this relationship (Dermitzaki, Leonardi, & Goudas, 2008). In young students, academic self-concept is usually very positive and not highly correlated with external indicators, such as skills and achievement (Guay et al., 2003). Veenman and Spaans (2005) assumed that metacognitive skills initially develop on separate islands of tasks and domains. Beyond the age of 12, these skills will gradually merge into a more general repertoire that is applicable and transferable across tasks and domains. The present work is concentrated on the improvement of metacognitive performance on the domain of mathematics and more specifically on the improvement of self-regulatory behavior.

Learning mathematics, as an active and constructive process, implies that the learner assumes control and agency over his/her own learning and problem solving activities (De Corte, Verschaffel & Op´t Eynde, 2000). Knowing when and how to use cognitive strategies is an important determinant to successful word problem solving (Teong, 2002). Metacognitive behavior can be applied in every stage of the problem solving activity (Lerch,2004). For example before starting solving a particular problem, students can ask themselves questions like what prior knowledge can help them develop a solution plan for the particular task; during the application of the solution plan the students monitor their cognitive activities and compare progress against expected goals. Finally, after reaching a solution, the students may need to look back, to check for the reasonableness of outcomes and integrate newly acquired knowledge to existing.

Problem solving procedure and the use of mathematical modeling

Studies on solving mathematical word problems refer to various conditions that cause transfer to occur, for example, providing solved examples (e.g. Bassok & Holyoak, 1989), having a scheme (Nesher & Hershkovitz, 1994), and providing feedback (Hoch & Loewenstein, 1992). The first step in solving a problem is to encode the given elements (Davidson & Sternberg, 1998). Encoding involves identifying the most informative features of a problem, storing them in working memory and retrieving from long-term memory the information that is relevant to these features. Incomplete or inaccurate metacognitive knowledge about problems often leads to inaccurate encoding and could generate learning obstacles.

A specific strategy frequently taught in math classes in order to enhance problem solving ability, is to use analogy in order to create a mental model of similar problems. In this regard, the students are expected to extract the relevant facts from the statement of the problem, compare it to their knowledge base, relevant to the problem domain, and recognize similarities between the new problem and problems they have previously encountered, and abstract the proper entities and principles. Empirical findings show that students fail to see the underlying principles unless they are explicitly pointed out (Panaoura & Philippou, 2005).

The modeling of open-ended problems have been of interest to mathematics educators for decades. Mathematical modeling of problem solving is a complicated procedure which is divided into different stages (Mason, 2001). When a mathematical modeling task is offered in a school the goal generally is not that students learn to tackle only that particular task. Rather, students are expected to recognize classes of situations that can be modeled by means of a certain mathematical concept, relation or formula, and to develop some degree of routine and fluency in mapping problem data to the underlying mathematical model and in working though this model to obtain a solution (Van Dooren, Verschaffel, Greer & De Bock, 2006).

A characteristic is that the modeling process is not a straightforwardly sequential activity consisting of several clearly distinguishable phases. Modellers do not move sequentially through the different phases of the modeling process, but rather run through several modeling cycles wherein they gradually refine, revise or even reject the original model. The present paper discusses the impact of the use of the mathematical model proposed by Verschaffel et al. (2000) on the development of students' self-representation about their self-regulatory behavior in mathematics. The main stages of the model are: 1) Understanding the phenomenon under investigation, leading to a model of the relevant elements, relations and conditions that re embedded in the situation (situation model), 2) Constructing a mathematical model of the relevant elements, relations and conditions available in the situation model, 3) Working through the mathematical model using disciplinary methods in order to derive some mathematical results, 4) Interpreting the outcome of the computational work to arrive at a solution to the real – word problem situation that gave rise to the mathematical model, 5) Evaluating the model by checking if the interpreted mathematical outcome is appropriate and reasonable for the original problem situation, and 6) Communicating the solution of the original real – word problem.

At the first phase of the problem solving procedure by the use of the mathematical model students have to consider and decide what elements are essential and what elements are less important to include in the situation model. In the next phase, the situation model needs to be mathematised i.e. translated into mathematical form by expressing mathematical equations involving the key quantities and relations. Students need to rely on another part of their knowledge base, namely mathematical concepts, formulas, techniques and heuristics. After the mathematical model is constructed and results are obtained by manipulating the model, numerical result needs to be interpreted in relation to the situation model. At this point, the results also need to be evaluated against the situation model to check for reasonableness. As a final step, the interpreted and validated result needs to be communicated in a way that is consistent with the goal or the circumstances in which the problem arose.

Nowadays problem solving skills have become a prominent instructional objective, but teachers often experience difficulties in teaching students how to approach problems and how to make use of proper mathematical tools. Many teachers of mathematics teach students to solve mathematical problems by having them copy standard solution methods. It comes as no surprise, therefore, that many students find it difficult to solve new problems, especially problems within a context (Harskamp & Suhre, 2006). Attempts to improve problem solving should focus on episodes students neglect when solving problems. The aim of the present study was to develop students' (5^{th} grade) problem solving ability and to enhance their ability to self-regulate their cognitive performance in order to overcome cognitive obstacles when they encounter difficulties while trying to solve mathematical problems. One of the main emphases was to oblige students reflect on their cognitive processes while trying to solve the problems and encounter difficulties in order to self-regulate their behavior. We hypothesized that the development of self-representation in order to be more accurate regarding the students' strengths and limitations would improve their self-regulatory behavior in mathematics. Especially for the problem solving procedure we hypothesized that the better distinction of problems and the clustering of those problems according to their similarities and differences would have as a consequence the better transfer of knowledge and strategies from the one domain to the others and from general situation to the specific ones.

METHODOLOGY

Participants: Data were collected from 255 children (107 experimental group and 148 control group), in Grade 5 (11 years old) from five different urban elementary schools. The participation at the program were voluntary because we had used the extra time students stayed at school for the program of the Ministry of Education, called "day-long school".

Procedure: The main emphasis was on the development of the program for the use of the proposed mathematical model, the training of students on the model and the evaluation of its results. At the first phase of the study three materials were constructed for pre and post test. The first one was about students' self-representation, the second for mathematical performance and the third one for their behavior while trying to solve mathematical problems. The first one comprised of 40 Likert type items of five points (1 = never, 2 = seldom, 3= sometimes, 4= often, 5= always), reflecting students' self-representation about mathematical learning. The responses to the questionnaire provide insight into students' self-representation which refers to how they perceive themselves in regard to a given mathematical problem. The reliability of the whole questionnaire was very high (Cronbach's alpha was .87).

The second questionnaire comprised of 20 mathematical tasks on counting, geometry, statistics and problem solving. In order to be sure about the suitability of the tasks we asked primary education teachers to assess their relation to the teaching content. All items in the mathematical performance questionnaire were scored on a pass-fail basis (0 and 1). The reliability of the mathematical tasks was high (Cronbach's alpha was 0.85).

The third questionnaire comprised of ten couples of sentences and students had to choose which one expressed better their cognitive behavior while they were encountering a difficulty in problem solving. All the questionnaires were first used at a pilot study in order to examine their construct validity. All the tasks were presented in paper and pencil form and were individually administered.

Then an intervention program was developed in order to propose the use of the mathematical model (Figure 1) for problem solving, proposed by Verschaffel et al.

(2000). The emphasis was on the understanding that different stages of problem solving would have as a consequence the use of different cognitive procedures and that the cognitive obstacles could be encountered by realizing the cognitive interruptions at one or more of those stages and mainly by self-regulating the cognitive performance. For example a self-regulatory strategy is the ability to recognize the "inner" mathematical similarities and differences of mathematical problems in order to transfer cognitive and metacognitive strategies among different domains. For the purpose of the project we had constructed a web page which was visited individually by each student of the experimental group (107 students) during 20 "meetings". One of the main emphases was to oblige students rethink their cognitive processes while trying to solve the problems and encounter difficulties in order to monitor their performance.

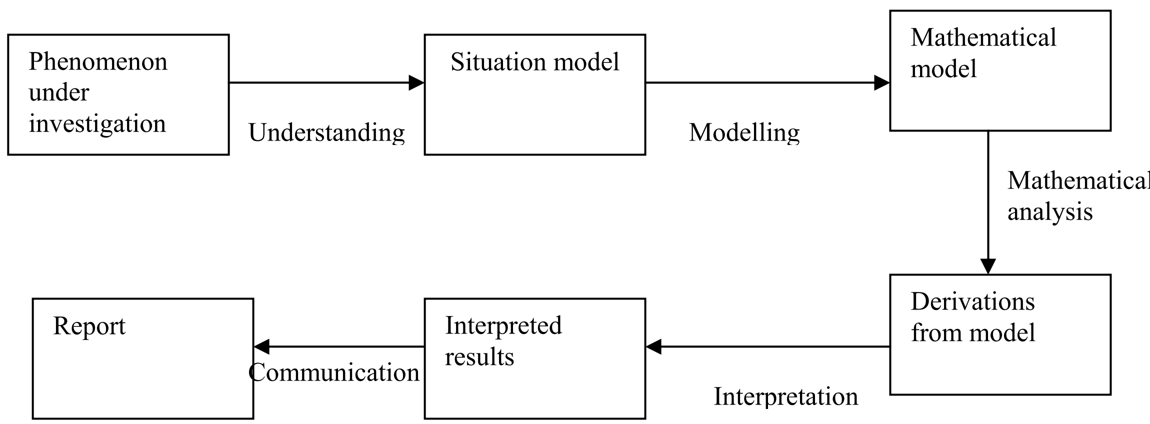

Figure 1: The mathematical model proposed by Verschaffel et al. (2000)

We had organized twenty "individual meetings" of the students with the webpage in order to work with the model (almost 20 minutes each meeting). Using the model used the first four "meetings" for the familiarization with the environment of the computer and for understanding the whole idea of the webpage for the problem solving procedure. The ten following "meetings" concentrated on different stages of the proposed mathematical model. For example at the stage of "understanding the problem" students had to solve problems with not enough data, or with more than the necessary data, they had to answer specific questions about the data of the problem, they had to explain in their own words the problem, to summarize it etc. At the stage of "modeling" they had to work on the classification of mathematical problems by explaining the criteria they used in order to classify the problems. There were problems with the same situational characteristics or the same context in order to oblige students to be concentrated on the structural mathematical characteristics. At the last six "meetings" students should solve mathematical problems by using all the stages of the mathematical model. In each stage the "cartoon" that was the hero of the webpage asked questions such as "How did you get that? This isn't a better solution? (for a proposed solution). Do you have any better solution?", in order to force students to self-regulate their cognitive performance. We wanted to have a reflection at all the stages of their work. The students' responses were recorded automatically at a database with details such as when they had worked on the specific task and for how long.

RESULTS

The data about self-representation (1st questionnaire) were first subjected to exploratory factor analysis in order to examine whether the presupposed factors that guided the construction of the items of the first questionnaire were presented in the participants' responses. This analysis resulted in 6 factors with eigenvalues greater than 1, explaining 65.56% of the total variance. After the content analysis, according to the results of the exploratory factor analysis items were classified in the following factors:

F1: general self-image about mathematics, F2: self-representation about problem solving abilities, F3: self-representation about the strategies used in order to self-regulate the cognitive performance, F4: self-representation about students' spatial abilities in mathematics, F5: self-representation about the degree of concentration on problem solving procedure, F6: the preference for different types of representations

We concentrated on the three factors which were related with self representation in respect to problem solving and self-regulation (F1, F2 and F3). The comparison of the means of the three factors between the pre and post tests for the experimental and the control group were statistically significant in all cases ($p<0.001$). Nevertheless the improvement was highest for the experimental group in the case of the second and the third factors (Table 1). It is obvious the increase of the control group as well as a consequence of the age development and the impact of teaching and learning (those were factors that could not be controlled). However the improvement was in all cases higher in the case of the experimental group.

	pre - test		post - test	
	experimental	control	experimental	control
F1	3.92	4.00	4.00	4.07
F2	3.22	3.25	3.69	3.57
F3	2.76	2.78	3.35	3.20

Table 1: The means of the experimental and the control group for the three factors at the pre and post test.

At the same time for the experimental group the improvement was highest in the case of the general mathematical performance ($\overline{X}_{1exp}$=0.27, $\overline{X}_{2exp}$=0.63, $\overline{X}_{1control}$=0.27, $\overline{X}_{2control}$=0.52) and the problem solving performance ($\overline{X}_{1exp}$=0.20, $\overline{X}_{2exp}$=0.47, $\overline{X}_{1control}$=0.20, $\overline{X}_{2control}$=0.39). Specifically the highest differences were found in the domain of geometry ($\overline{X}_{1exp}$=0.28, $\overline{X}_{2exp}$=0.47, $\overline{X}_{1control}$=0.29, $\overline{X}_{2control}$=0.44) and statistics ($\overline{X}_{1exp}$=0.38, $\overline{X}_{2exp}$=0.69, $\overline{X}_{1control}$=0.38, $\overline{X}_{2control}$=0.64). This result reveals the positive impact of the use of the specific mathematical model on the mathematical performance.

The most important in the case of self-representation is the accuracy of this feature in relation to the real mathematical performance. We have clustered, depended on cluster analysis, the participants in respect to their general self-image about their mathematical performance into three groups. The first group was consisted of 42 students with low self-image ($\overline{X}$=2.55), the second one of 82 students with medium self-image ($\overline{X}$=3.26) and the third one of 99 students with high self image ($\overline{X}$=3.94). There were statistically significant differences between the first and the third group at the initial phase (pre – test) in respect to their real mathematical performance (F=4.716, df=2, p=0.01, $\overline{X}_1$=0.466, $\overline{X}_2$=0.543, $\overline{X}_3$=0.605). After the program the difference of the groups regarding their general self-image in relation to their mathematical performance (post test) was significant only in the case of the experimental group (F=4.447, df=2, p=0.01, $\overline{X}_1$=0.557, $\overline{X}_2$=0.6059, $\overline{X}_3$=0.699). Those results indicated that most students had accurate self-image in respect to their real mathematical performance and they did not seem to overestimate their abilities.

At the same time students' means at the classification of similar mathematical problems according to the mathematical structure of the problems were highest at the post test. The development was statistically higher in the case of the experimental group ($\overline{X}_1$=0.29, $\overline{X}_2$=0.49, t=12.79. p<0.001) than the control group ($\overline{X}_1$=0.29, $\overline{X}_2$=0.41, t=11.69, p<0.001). The difference between the two groups was statistically significant (t=3.32, df=228, p<0.01).

A part of the couples of sentences at the third questionnaire were about the self-regulatory strategies they use in order to encounter difficulties and cognitive obstacles at the problem solving procedure. For the self-regulatory strategies the difference of the means between the two measurements was statistically significant (t=2.93, df=98, p<0.01, $\overline{X}_1$=0.65, $\overline{X}_2$=0.69) only in the case of the experimental group. That means that students tended to develop more self-regulatory strategies or tended to believe that they have to develop those strategies. Even the second learning situation is an important step for the change of cognitive and metacognitive behavior, as well.

Students of the experimental group were clustered according to their self-representation about problem solving ability and their general mathematical ability into three groups (low self-representation: 24 students, medium: 36 students, and high self-representation:

34 students). Analysis of variance (ANOVA) indicated that there was a statistically significant difference concerning their self-representation about the use of self-regulatory strategies in mathematics ($F_{2,93}$ =6.094, p=0.003). As it was expected the mean of the group with the high self-representation was higher (0.80) than the other two groups (medium: 0.63 and low: 0.58). The most interesting result was that the students' with medium and low mathematical performance was increased after the program (low: $\overline{X}_1$=0.83, $\overline{X}_2$=0.87, medium: $\overline{X}_1$=0.90, $\overline{X}_2$=0.94, high: $\overline{X}_1$=0.94, $\overline{X}_2$=0.94). In the case of the improvement on the self-representation about the use of self-regulatory strategies for the three groups the changes were similar (low self-representation: $\overline{X}_{1s}$= 0.50, $\overline{X}_2$= 0.53, medium self-representation: $\overline{X}_1$= 0.64, $\overline{X}_2$= 0.67, high self-representation: $\overline{X}_1$= 0.80, $\overline{X}_2$= 0.84). This stability or low increase may indicate that students realized their difficulties and limitations and did not tend to overestimate their abilities in using strategies.

DISCUSSION

Results confirmed that providing students with the opportunity to self-monitor their learning behavior in the case of encountering obstacles in problem solving through the use of modeling is one possible way to enhance students' self-representation about the self-regulatory strategies they use in mathematics and consequently their mathematical performance. It seems that the program with the use of the model created a powerful learning environment in which students were inspired in their own experiences. Nevertheless it is obvious that students with high self-representation about their mathematical abilities in the initial phase were at the same time students with the most self-regulatory strategies after the impact of the intervention program, as well. That means that although the program improved the metacognitive performance and the mathematical performance of the experimental group, further research is needed in order to find ways to change the initial differences among students.

For the development of a more accurate self-representation about mathematical performance and self-regulation in problem solving teachers must create a powerful learning environment, in which children are allowed and inspired to, their own learning experiences. According to the self-regulated learning approach students are self-regulating when they are aware of their capabilities of the strategies and resources required for effectively performing a task (Paris & Paris, 2001). Learners, who decide to ask a more competent person for assistance when faced with a task, indicate that they realize their difficulties and try to find out ways to overcome them. The accurate self-representation about the strengths and limitations is a presupposition for the development of self-regulation. Instruction should mainly lead students to self-questioning as a systematic strategy in helping them control their own learning and organize by themselves the different occasions they may encounter. In the area of mathematics, a number of important questions about metacognition remain unanswered.

Much more research is needed to study the different aspects of metacognition in a more systematic and detailed way. We suggest specifically that further research could focus on interactive computer programs that may be designed to provide feedback and hints to assist students in becoming more aware of their cognitive and metacognitive processes. It would be optimistic and naïve to claim that such types of intervention programs would develop the self-regulatory strategies of all students. Possibly different models and programs are suitable for different groups of students.

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Acknowledgements

This paper draws from the Research Project ALPAS, which was funded by the Cyprus Research Foundation with the contribution of Erik De Corte and Wim Van Dooren (University of Leuven).

ENDORSING MOTIVATION: IDENTIFICATION OF INSTRUCTIONAL PRACTICES

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This paper presents some results of a larger study that investigates the relationship between instructional practices in the mathematics classroom and students' motivation and their achievement in mathematics. Data were collected from 321 sixth grade students through a questionnaire comprised of three Likert-type scales measuring motivational constructs, a test measuring students' understanding of the fraction concept and an observation protocol for teachers' instructional practices in the classroom. Findings revealed the importance of multi-level modelling in the analysis of instructional practices suggested by achievement goal theory and mathematics education research that promote both students' motivation and achievement in mathematics.

INTRODUCTION

Research on achievement motivation provides substantial evidences of instructional practices that foster students' motivation (Anderman et al., 2002; Turner et al., 2002). These instructional practices are alike the ones developed by mathematics educators to achieve both learning and motivational outcomes (Stipek et al., 1998). Motivation is treated in mathematics education as a desirable outcome and a means to enhance understanding (Stipek et al., 1998). In broad, the socio-constructivist perspective on learning (Op't Eydne et al., 2006) underlines the interplay between cognitive, motivational and affective factors but also it highlights the influence of the specific classroom context in the whole process.

In this respect, the present study investigates variations in instructional practices and their impact on students' achievement motivation and outcome. Understanding the interplay between the characteristics of a particular instructional setting, and students' achievement-related goals and outcomes is an important direction for both motivational and mathematics education research (Anderman et al., 2002; Stipek et al., 1998). In the next section we consider the basic concepts and define the research questions.

THEORETICAL BACKGROUND AND AIMS

Motivation

Motivation cannot directly be observed but it can be noticeable only by its interaction with affect, cognition and behaviour. Hannula (2006) defines motivation as the preference to do certain things and to avoid doing some others. In regards to students' motivation four basic theories of social-cognitive constructs have so far been identified: achievement goal orientation, efficacy beliefs, personal interest in the task, and task value beliefs (Pintrich, 2003). In this study we conceptualise motivation according to achievement goal theory because it has been developed within a social-cognitive framework and it has studied in depth many variables which are considered as antecedents of students' motivation constructs. Some of these variables are students' competence based variables, such as need of achievement or fear of failure, self-based variables, such as self efficacy beliefs, and demographic variables, e.g. gender (Elliot, 1999). In addition, one of the strengths of goal orientation theory in understanding students' motivation is that it explicitly considers the role of teachers and instructional contexts in shaping students' goal orientations. Thus a major tenet of goal theory is that students' adoption of personal goals is influenced even in part, by the goal structures promoted by the classroom and boarder school environments (Anderman et al., 2002).

Achievement goal theory is concerned with the purposes-goals students perceive for engaging in an achievement-related behaviour and the meaning they ascribe to that behaviour. A mastery goal orientation refers to one's will to gain understanding, or skill, whereby learning is valued as an end in itself. In contrast, a performance goal orientation refers to wanting to be seen as being able, whereby ability is demonstrated by outperforming others or by achieving success with little effort (Elliot & Church, 1997). Recently, there has been a theoretical and empirical differentiation between performance-approach goals, where students focus on how to outperform others, and performance-avoidance goals, where students aim to avoid looking inferior or incompetent in relation to others (Cury et al., 2006).

These goals have been related consistently to different patterns of achievement-related affect, cognition and behaviour. Being mastery focused has been related to adaptive perceptions including feelings of efficacy, achievement, and interest (Anderman et al., 2002; Elliot & Church, 1997; Cury et al., 2006). Although the research on performance goals is less consistent, this orientation has been associated with maladaptive achievements beliefs and behaviours like low achievement, fear of failure and superficial cognitive commitment, i.e. the use of 'surface' learning strategies such as copying, repeating and memorizing (e.g. Cury et al. 2006). Efficacy beliefs encountered as an antecedent variable in the achievement goal theory, refers to the beliefs in one's capabilities to organize and execute the courses of action required to manage prospective situations (Bandura, 1997).

Instructional practices

Environmental factors are presumed to play an important role in the goal adoption process and eventually in students' achievement (Anderman et al., 2002). Elliot & Church (1997) underline that if the achievement setting is strong enough it alone can establish situation-specific concerns that lead to goal preferences for the individual, either in the absence of a priori propensities or by overwhelming such propensities.

Earlier studies on achievement goals specify various classroom instructional practices as contributing to the development of different types of goals and consequently, eliciting different patterns of motivation and achievement outcomes (e.g. Ames, 1992). Goal orientation theorists lying on a large literature on classroom motivational environments focus on six categories that contribute to the classroom motivational environment. The categories, represented by the acronym TARGET refer to task, authority, recognition, grouping, evaluation and time. *Task* refers to specific activities, such as problem solving or routine algorithm, open or closed questions in which students are engaged in; *Authority* refers to students' level of autonomy in the classroom; *Recognition* refers to whether the teacher values the progress or the final outcome of students' performance and how the teacher treats students' mistakes (as a a part of the learning process or as cause for punishment); *Grouping* refers to whether students work with different or similar ability peers; *Evaluation* refers to how the teacher treats assessment, giving publicly grades and test scores, or focusing on feedback as a means for improvement and mastery; *Time* refers to whether the schedule of the activities is rigid or flexible.

This framework has been adapted and developed by goal theory researchers working within classroom context (Anderman et al., 2002; Turner et al., 2002). Using classroom observations and qualitative analysis, they found that instructional practises in classrooms in where students adopted mastery goals differed from instructional practises in classroom characterized by students' low mastery goals or high performance goals. Specifically, according to the task variable, in mastery oriented classrooms teachers used an active instructional approach, ensuring that all students participated in classroom talk and adapted instruction to the developmental levels and personal interests of their students, while in low mastery oriented classrooms, learning was processed by students listening to information and following directions (Anderman et al., 2002; Turner et al., 2002). Regarding authority, in high mastery oriented classrooms teachers engaged the class in generating the rules, while in low mastery oriented classrooms the teachers presented their rules to the students (Anderman et al., 2002). In high mastery classrooms teachers emphasized the intrinsic value of learning, while recognition practices were characterized by warm praise, which was also task oriented, clear, consistent and credible (recognition). High levels of genuine enthusiasm, positive affect and enjoyment by these teachers with respect to engaging in academic tasks was also observed. In low mastery oriented classrooms teachers used punishment and threats with students who did not do what they were told (Anderman et al., 2002). In high mastery orientation classrooms students had considerable freedom within the classroom-e.g. talking to classmates (autonomy) and peer collaboration (grouping) (Anderman et al., 2002). Reversely, in high mastery classrooms teachers emphasized students' performance, relative performance and differential prestige (evaluation) while in low mastery classrooms teachers emphasized test scores and grades or students' differential performance on tasks (evaluation). Moreover teachers in high mastery classrooms valued the time during the lesson referring to time allocation for different activities (time) while students in the low mastery oriented classrooms were allowed to work on their paces (Anderman et al., 2002).

In mathematics education domain, Stipek et al. (1998) in a relevant study referring to instructional practices and their effect on learning and motivation found that affective climate was a powerful predictor of students' motivation and mastery orientation. Students in classrooms in which teachers emphasized effort, pressed students for understanding, treating students' misconception and in which autonomy was encouraged reported more positive emotions while doing math work and enjoying mathematics more than other students while they also scored higher in a fraction test. Teachers' provision of substantive feedback to students rather than scores on assignments was also associated with mastery orientation.

Despite the apparent utility of the list concerning the classroom practices both by achievement goal researchers and mathematics educators, very few studies have examined these practices in relation to students' perceptions of achievement goals and outcomes in the ecology of regular classroom. To the best of our knowledge none of these studies had employed multilevel statistical tools for the identification of teachers' practices that influence students' specific goals and vis-à-vis students' achievement. In this respect the purpose of this study was:

- To test the validity of the measures for the six factors: fear of failure, self-efficacy, interest, mastery goals, performance-approach goals and performance-avoidance goals, in a specific social context.
- To construct and test the validity of an observational protocol that includes convergent variables referring to instructional practices in the classroom from the mathematics education domain and the achievement motivation one.
- To identify instructional practices suggested by achievement motivation theory and mathematics education theory that affect students' motivation (mastery and performance goals) in the mathematics classroom applying multilevel analysis.

METHOD

Participants were 321 sixth grade students, 136 males and 185 females from 15 intact classes and their 15 teachers. All students-participants completed a questionnaire concerning their motivation in mathematics and a test for achievement in the mid of the second semester of the school year.

The motivation questionnaire comprised of six sub-scales measuring: a) mastery goals, b) performance goals, c) performance avoidance goals, d) self-efficacy, e) fear of failure, and f) interest. Specifically, the questionnaire comprised of 35 Likert-type 5-point items (1- strongly disagree, and 5 strongly agree). The six-item subscale measuring mastery goals, the five-item subscale measuring performance goals, the four-item subscale measuring performance-avoidance goals, as well as the five item subscale measuring efficacy beliefs were adopted from the Patterns of Adaptive Learning Scales (PALS) (Midgley et al., 2000); respective specimen items in each of these four subscales were, "one of my goals in mathematics is to learn as much as I can"

(Mastery goal), "one of my goals is to show other students that I'm good at mathematics" (Performance goal), "It's important to me that I don't look stupid in mathematics class" (Performance-avoidance goal), and "I'm certain I can master the skills taught in mathematics this year" (efficacy beliefs). Students' fear of failure was assessed using nine items adopted from the Herman's fear of failure scale (Elliot & Church, 1997); a specimen item was "I often avoid a task because I am afraid that I will make mistakes". Finally, we used Elliot and Church (1997) seven-item scale to measure students' interest in achievement tasks; a specimen item was, "I found mathematics interesting". These 35 items were randomly spread through out the questionnaire, to avoid the formation of possible reaction patterns.

For students' achievement we developed a test measuring students' understanding of fractions. The tasks comprising the test were adopted from published research and specifically concerned students' understanding of fraction as part of a whole, as measurement, equivalent fractions, fraction comparison and addition of fractions with common and non common denominators (Lamon, 1999).

For the analysis of teachers' instructional practices we developed an observational protocol for the observation of teachers' mathematics instruction in the 15 classes during two 40-minutes periods. The observational protocol was based on the convergence between instructional practices described by Achievement Goal Theory and the Mathematics education reform literature. Specifically, we developed a list of codes around six structures, based on previous literature (Ames, 1992; Anderman et al., 2002; Stipek et al., 1998), which were found to influence students' motivation and achievement. These structures were: task, instructional aids, practices towards the task, affective sensitivity, messages to students, and recognition.

The structure *task* included algorithms, problem solving, teaching self-regulation strategies, open-ended questions, closed questions, constructing the new concept on an acquired one, generalizing and conjecturing. We checked whether teachers made use of instructional aids during their lesson. *Practices towards the task* included the teacher giving direct instructions to students, asking for justification, asking multiple ways for the solution of problems, pressing for understanding by asking questions, dealing with students' misconceptions, or seeking only for the correct response, helping students and rewording the question posed. Behaviour referred to *affective sensitivity* included teachers' possible anger, using sarcasm, being sensible to students, having high expectations for the students, teachers' interest towards mathematics or fear for mathematics. *Messages to students* included learning as students' active engagement, reference to the interest and value of the mathematics tasks, students' mistakes being part of the learning process or being forbidden, and learning being receiving information and following directions. Finally, *recognition* referred to the reward for students' achievement, effort, behavior and the use of external rewards by the teachers.

During the two classroom observations lasted for 40 minutes for each teacher, we identified the occurrence of each code in each structure.

RESULTS

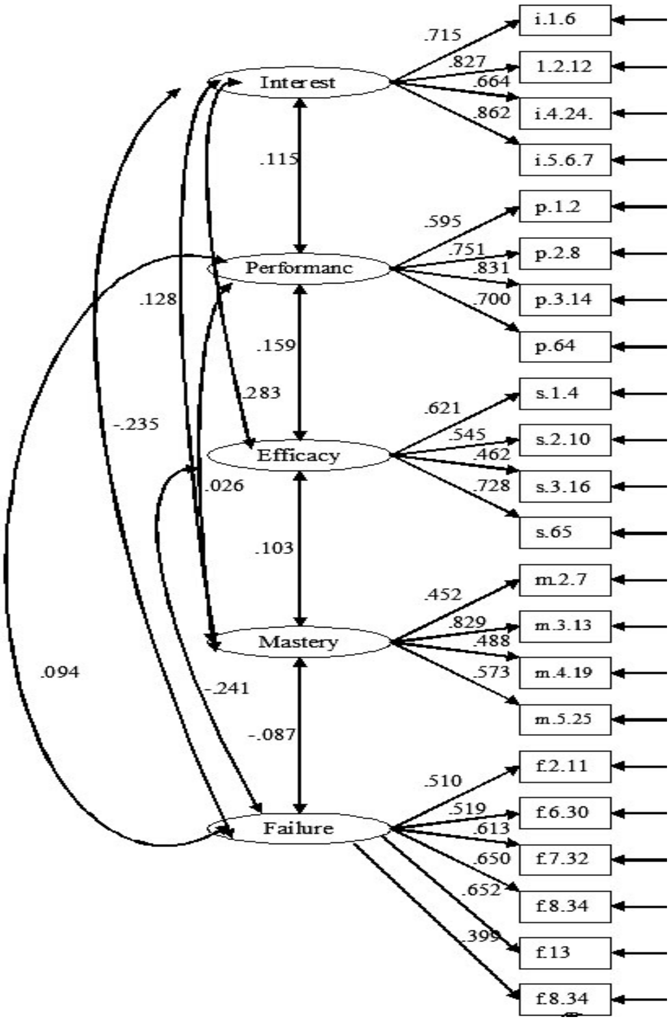

Fig 1: The factor model of students' motivation with factor parameter estimates.

With respect to the first aim of the study, confirmatory factor analysis was conducted using EQS (Hu & Bentler, 1999) in order to examine whether the factor structure yields the six motivational constructs expected by the theory.

By maximum likelihood estimation method, three types of fit indices were used to assess the overall fit of the model: the chi-square index, the comparative fit index (CFI), and the root mean square error of approximation (RMSEA). The chi square index provides an asymptotically valid significance test of model fit. The CFI estimates the relative fit of the target model in comparison to a baseline model where all of the variable in the model are uncorrelated (Hu & Bentler, 1999). The values of the CFI range from 0 to 1, with values greater than .95 indicating an acceptable model fit.

Finally, the RMSEA is an index that takes the model complexity into account; an RMSEA of .05 or less is considered to be as acceptable fit (Hu & Bentler, 1999). A process followed for the identification of the six factors including the reduction of raw scores to a limited number of representative scores, an approach suggested by proponents of Structural Equation Modelling (Hu & Bentler, 1999). Particularly, some items were deleted because their loadings on factors were very low (e.g. for the factor interest the item i.3.18. and for the factor fear of failure the item f.5.28) and some other items were grouped together because they had high correlation with each other (e.g. for the factor fear of failure the items f.1.5 and f.3.17). From the analysis the factor performance-avoidance goals failed to be confirmed. Then in line with the motivation theory, a five-factor model was tested (fig. 1). Items from each scale are hypothesized to load only on their respective latent variables. The fit of this model was (x^2 =691.104, df= 208, p<0.000; CFI=0.770 and RMSEA=0.086). After the addition of correlations among the five factors the measuring model has been improved (x^2 =343.487, df= 198, p<0.000; CFI=0.931 and RMSEA=0.049).

Concerning the second aim of the study, analysis of the observations involved estimating the mean score of each code for the two 40 minutes observations using the SPSS and creating a matrix display of all the frequencies of the coded data from each classroom. Each cell of data corresponded to a coding structure. From a first glance, the observational protocol succeeded in detecting differences in teachers' practises during the mathematics lessons. Notably, teachers 4, 9, 13, 15 used more algorithmic tasks than the others, while teachers 2, 4, 7 used more problem solving activities than their other colleagues. Open-ended questions were used more by teachers 3, 5 while teachers 8 and 14 used more the closed type of questions. Very few teachers made use of the visual aids (4, 7, and 8). From the category practices towards the task justification of students' answers were asked from almost all teachers expect from teachers 2, 3, 10, 13. Press for understanding characterized teachers' 6 and 13 practices, while asking for multiple problem solutions was not popular to this sample of teachers. Teacher 5 was characterized by her willingness to help students. Regarding teachers' affective sensitivity, teacher 1 expressed anger while teacher 7 showed great sensitivity to students. Concerning the structure messages all teachers apart from teachers 1 and 15 treated students' erroneous responses as part of the learning process, while the other codes regarding this category were met rarely during these lessons. Regarding recognition, teachers 1 and 7 rewarded students for their performance.

According to the third aim of the study, the identification of instructional practices suggested by achievement motivation theory and mathematics education that affect students' mastery and performance goals, we applied Multilevel analysis using the program MLwin (Opdenakker & Van Damme, 2006). Multilevel analysis is a methodology for the analysis of data with complex patterns of variability, with a focus on nested sources of variability: e.g. students in classes, classes in schools etc. The main statistical model of multilevel analysis is the hierarchical linear model, an extension of the multiple linear regression model to a model that includes nested random effects. Multilevel statistical models are always needed if a multi-stage sampling design has been employed (a sample of pupils and a sample of teachers) because the clustering of the data should be taken into consideration avoiding the drawing of wrong conclusions (Opdenakker & Van Damme, 2006). The simplest case of this model is the random effects analysis model (empty model). When a two level hierarchy is used (e.g. students at the first level and teachers at the second level), this model exhibits only random variation between groups and random variation within groups. Estimating the variance at the distinguished level (e.g. students and teachers) it is possible to see which level is important for the estimation of the variance. For example if the estimation variance at student level (level one) is much higher that the estimation of the variance at the teacher level, then this means that differences between students with respect to the characteristics under study are largely related to individual students and not to the teachers. The empty model can be expanded by the inclusion of explanatory variables. With the explanatory variables, we try to explain part of the variability of the dependent variable. It is possible to explain variability at level one as well as in a next-step at level two (Opdenakker & Van Damme, 2006).

The first test in the analysis regarding variables that influence the development of mastery goals was to determine the variance at the student level and teachers' level without explanatory variables (empty model 0). The variance at each level reached statistical significance ($p<0.05$) and this implied that MLwiN could be used to identify the variables which were associated with achievement in each subject. Regarding mastery goals, student effect was much higher than teachers effect (91% and 9% respectively). Following the procedure we added in model 1 student demographic variables. Model 1 explained 2% of the total variance and the likelihood statistics (X^2) showed a significant change between the empty model and the model 1 ($p<0.05$). From the three variables (education mother-father and gender) only gender had significant effect on students' mastery goals. The variance was explained solely to student level (2%). In model 2 all affective variables according to achievement goals theory were added to the model. Specifically the antecedent variables fear of failure and efficacy beliefs were added to the model and also performance goals. Model 2 explained 26% of the total variance and X^2 showed a significant change between model 1 and model 2 ($p<0.001$). The antecedent variables had a significant effect to the model, with fear of failure to have negative effect, while performance goals did not have any effect. From the 26% of the total variance 23% was at the student level and 3% at the teacher level. In Model 3 we added teachers' educational background but it turned out not to have any statistical significant effect on students' mastery goals. Then we added to the model teachers' practices concerning the structure Task and again they did not have any statistical significant effect to the model. We continue adding the other categories of teachers' practices. The only one that had negative statistical effect on students' mastery goals was the absence of visual aids. Model 3 explained 2% of the total variance and X^2 showed statistical significant change between model 2 and model 3. The variance was explained exclusively to teacher level.

We followed the same process to identify variables that had significant effect on students' performance goals. We ended that from student level, fear of failure and self efficacy had significant effect on students' performance goals while from teacher level the practice, "teacher rewords the question asked" had significant effect to students' performance goals.

Next, we followed Stipek et al. (1998) process grouping instructional practices in each of the six categories regarding the observational protocol together with the ratio of open-ended questions to closed questions. The ratio related to the questions had significant negative effect on students' performance goals.

Figure 2 presents the results of the multilevel analysis in identifying variables that explain the variance of students' mastery and performance goals. Sign + indicates regression at the significance level $p<0.5$, while ++ indicates regression at the significance level $p<0.001$. Dotted arrows indicate negative regression.

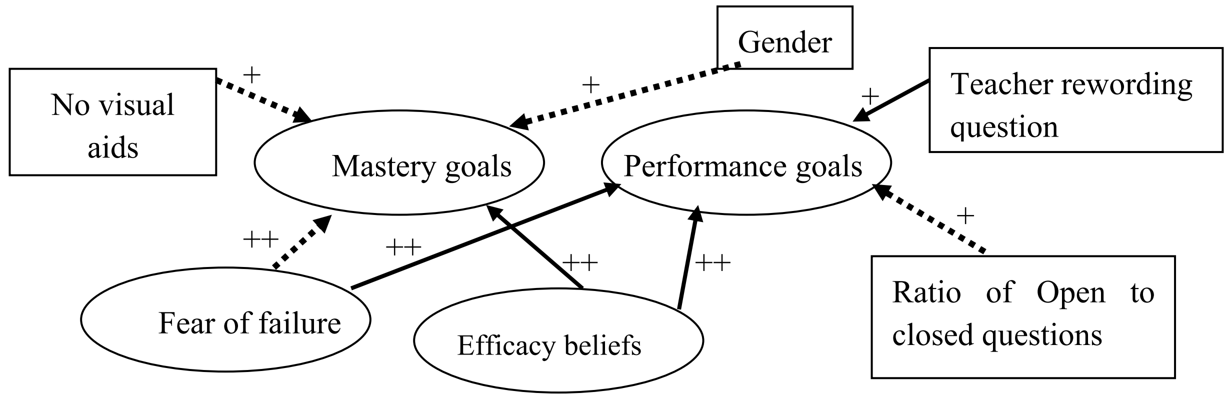

Fig 2: Results of the Multilevel analysis on mastery and performance goals.

CONCLUSION

Regarding the first aim of the study, data revealed that factors referred to the five of the six motivational constructs were confirmed in the Cypriot environment. The factor regarding performance-avoidance goals failed to be confirmed in contrast to the results of other studies (Cury et al., 2006). This may be due to students' age-usually this factor is confirmed in elderly students or to the different cultural context.

Regarding the second aim of the study, the data revealed important differences in the instructional practices used in the mathematics classrooms in line with other studies (Anderman et al., 2002; Pantziara & Philippou, 2007; Stipek et al., 1998). However the need for in-depth analysis of these practices born due to the study's evidence that while in some classrooms teachers applied the practices suggested by motivation and mathematics education to foster students' motivation, students' motivation was high while their mathematics performance was poor.

As far as the third aim is concerned, taking into consideration the clustering of the data in the multi-stage sampling (sample of pupils and sample of teachers) we applied the multilevel analysis to identify variables that have significant effect on students' achievement goals. The results revealed that more effect on students' motivation had students' variables (gender, fear of failure, efficacy beliefs) while only few of the numerous instructional practices suggested by other studies (Anderman et al., 2002; Stipek et al., 1998) found to have effect on students' motivation. This may be due to the new analytical tools used considering the variance between the different level of the depended variables or to the small number of teachers involved in the study. Whatever the case is, further research is needed using multilevel analysis in domains regarding achievement goals and mathematics education for the identification of instructional practices that endorse motivation and achievement in mathematics.

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HAVING TO TEACH GEOMETRY: ON VISUALISATION AND TEACHERS' DEFENCES

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Abstract

The nature and presence of geometry in the school curriculum in England has undergone several changes over the past 40 years and today's curriculum includes some Euclidean-style geometric reasoning that was absent in the late 1980s and 1990s. Consequently, teachers of mathematics currently in secondary schools in England have a wide range of geometrical backgrounds, including many who did not have any school education in Euclidean geometry. This paper reports on an in-service geometry-for-teaching course and focuses on two themes – visualisation and psychoanalytic defences – that are briefly theorised and discussed in the context of teachers having to teach geometry.

Setting the scene

"Ah-ha, I see it now" a member of the class says softly, smiling, nodding slightly; another member of the class, face a resentful stare, cries out "I never see things, I'm rubbish at geometry". What was, respectively, seen and not seen, was a two-dimensional Euclidean geometry theorem; those who saw, or did not see, were secondary mathematics teachers studying our 'geometry for teaching' MA module[1]. The first teacher's positive disposition contrasts with the second teacher's disturbed and defensive state. In this paper, the source of data comes from the class of twelve school teachers who took this geometry module in the summer of 2008.

What's the issue? Visualisation is central to geometrical reasoning, requires a different way of thinking and processing information to much school mathematics. Geometry is difficult to teach because of (a) the nature of visualisation for geometrical reasoning and the affective states that are conducive to visualisation, and (b) teachers' lack of preparedness for this way of thinking, their consequent lack of confidence in modelling this way of thinking in front of their pupils in their classrooms and their resulting psychoanalytic defences that make it even more difficult for them to visualise geometrical theorems and relationships.

BACKGROUND

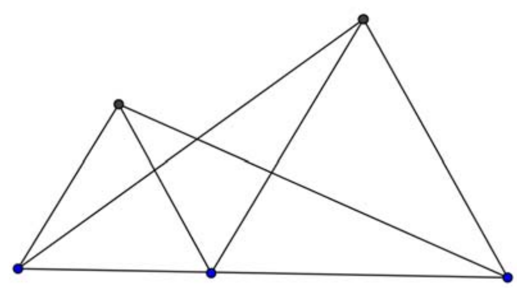
The mathematics curriculum in England has changed several times over the past 40 years and geometry has appeared in different guises. Post second world war, secondary education in England was generally divided into the academic grammar schools - where geometry was taught as the formal system of Euclid – and, for the majority of the age cohort, secondary-modern schools where learning for practical application predominated. By the late 1960s, influenced by the Modern Mathematics Movement (Howson, 1982), there were moves to "free geometry from the shackles of

Euclid" so when the academic schools and the secondary schools merged (mostly by the early 1970s), geometry in schools in England was generally introduced through transformations (Howson, *op. cit.*) a pragmatic compromise between formality and applicability. Furthermore, this transformational geometry was often experienced descriptively (e.g., 'what sort of rotation is that?') rather than structurally (e.g. 'what group represents the isometries of a square?'). More recently, concerns have been raised by mathematicians (Royal-Society, 2001) and mathematics educationalists (Hoyles & Küchemann, 2002) about young peoples' competence in the key mathematical practice of providing proofs of conjectures in order to establish theorems. As elementary Euclidean geometry is an area of mathematics where proofs can be quite succinct, and visual/tangible/manipulable representations of theorems are frequently available, geometry has quite recently been returned to the curriculum (QCA, 2008). So in England we have teachers, usually in their mid-twenties to late-thirties, who learnt a descriptive geometry at school, who are now in the position of having to teach Euclidean geometry. These teachers are often in positions of seniority/responsibility, and, as one head of a school mathematics department remarked, it is "scary to find a big gap in my knowledge". For many such teachers, mathematics has been experienced as being focused on numerical and algebraic concepts in which routinisation of procedures and understanding of constructs has been emphasised rather than a holistic, 'whole picture' view, characteristic of geometric thinking.

VISUALISATION IN GEOMETRIC REASONING

Turning now to geometrical thinking, in particular '(having a) visualisation', by which I mean having a visual appraisal of a geometric configuration that leads directly to seeing (the truth of) a theorem or a key feature of a geometric relationship. The sense of the word visualisation I am using is in the spirit of René Thom's "a theorem is above all the object of a vision" rather than in the sense of Van Hiele's 'level 1' of geometric learning where 'visualization' refers to visual recognition of shapes but lack of awareness of their properties (Van Hiele, 1986). Van Hiele uses the term 'insight' for the experience that grasps a geometric result from an holistic appraisal. This term has a similar meaning to 'visualisation', the difference being that Van Hiele's term requires a discursive potential which visualisation does not; as in Thom's conception, the theorem is seen directly, (Rodd, 2000).

For example, in the diagram where on the horizontal, two equilateral triangles are constructed and two more line segments drawn in, it may be possible for you to see two congruent triangles; the fact of their identicalness can be (but might not be!) visualised, i.e. seen as true.



Recent work in psychology suggests that visualisation, in the sense outlined above, employs affect in different ways from number/algebra. Linnenbrink and Pintrich, from an admittedly limited number of empirical studies, offer a general, though not universal principle: a positive mood tends to support a broad perspective that enables having an holistic appraisal of a situation, (and visualisation is a type of holistic appraisal of a geometric situation), while a negative mood tends to focus in on details (Linnenbrink & Pintrich, 2004). Evidence for this principle includes a study that found that young children learnt about shape more successfully when in a positive mood (*ibid.* p77), unlike middle school students' computational learning where "positive affect was related to lower levels of achievement" (*ibid.* p75). So Linnenbrink and Pintrich's cognitive-affective lens suggests that although the occurrence of instances of visualisation can't be predicted, the potential for having a holistic visual appraisal of a geometrical situation is linked to affective states like being in a positive mood which facilitates having a broad perspective.

This resonates with our work with the teachers on the course: in several sessions, we observed that a great deal of 'play-time' was needed for the participants to see geometrical configurations in more holistic, visual ways. I use 'play-time' to signal that the participants were free to explore the materials or the starting points in their own ways, that the atmosphere was relaxed and convivial and that, because over half the course was taught over whole days (Saturdays), we could give longer to a given task than had been anticipated in our planning (and cut other tasks). An example of a task that took time to be appraised visually was the following: we asked "in a unit cube, what is the distance from a vertex to the plane that is defined by the three adjacent vertices? (in class, a diagram was used). Though several participants were able to calculate an answer quickly with a familiar formula, some participants did not have this knowledge. Responding to this situation, we encouraged all participants to develop alternative approaches to employing a formula using different representations, like model-making. They worked in groups in a classroom where the atmosphere supported both discussion and contemplation and even those who 'knew the formula' remarked that this experience had helped them better understand the situation. In this vertex-to-the-plane task, the in-the-know participants were quick to calculate and they used the available time to develop a more visual approach to geometrical reasoning (rather than, for example, to hone their algebraic skills or to solve similar, more difficult problems). They were enjoying themselves, but, in the case reported, I cannot be sure whether or not their positive moods facilitated their own capacities for visualisation or whether the lack of time pressure was more important – clearly 'good mood' and 'lack of time constraints' are not independent.

This leads on to considering how teachers model doing mathematics, particularly problems like the one mentioned, in their classrooms. There are surely many reasons why teachers would quickly calculate, rather than explore different visual representations (other than the ever-present reason of curriculum pressures). In the 'white-board jungle' of the secondary classroom, teachers need to project a teacherly identity to their pupils and defend themselves from adolescent challenge. As a mathematics specialist, this teacherly identity includes being a personifier of mathematical knowledge and so, for those whose geometrical confidence is weak, they might well rely on their fluent and to-the-point algebraic/numeric skills, rather than opening themselves to challenge by taking rather a long time to appraise a geometrical configuration.

These considerations lead to trying to understand more about the teacher's mind, in particular, defences that are invoked when having to teach geometry. This will hopefully start to explain why visualisation is difficult for teachers to do spontaneously in the classroom and difficult for teachers to educate their students to visualise.

IDEAS FROM PSYCHOANALYSIS TO FATHOM TEACHERS RELATIONSHIPS WITH GEOMETRY

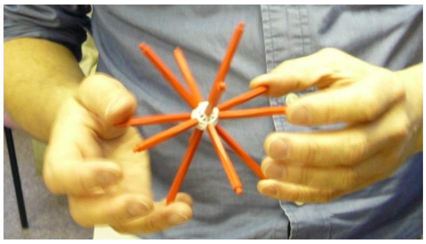
The psychoanalytic point of view has centrally the idea that one survives and defends oneself by reacting to phenomena by ingesting or by spitting out. These reactions start in infancy as experiences of the physical processes of taking in and expelling; then they develop into the psychological defence mechanisms of projective and introjective identification where, for example, the mother's mind is projected onto or is taken within. (There is also in psychoanalysis the family-historical dimension of becoming a mind that is tagged as the Oedipus Complex; this dimension is not pursued here, not that its importance is denied, but because integrating a teacher's family and background is beyond the scope of this paper.) In a rather ideal sense, using the psychoanalytic terms, a teacher projects something 'good' for pupils to ingest, then the teacher introjects the pupils' subsequent projections (that may be 'good' or 'bad') and the cycle continues.

Projective and introjective identifications are linked with a person's 'states of mind' which are relevant to both affect and cognition. I am drawing on the Kleinian and post-Kleinian tradition, (Waddell, 1998), that names 'states of mind' in terms of life-stages: infancy, childhood, latency, adolescence, adulthood, old age, yet the "complexity [of states of mind] … is that they are not naturally linked to the chronology of developmental stages" (*ibid.* p11); the states of mind are emotionally experienced at all ages (*ibid.* p8). Each of these states of mind is subject to fluctuations between that of self-interest and that of empathetic concern. Some self-interest (is perceived as) having a need to project (spit out) the bad and some self-interest (is perceived as) having a need to introject (drink in) the good. If self-interest, emotionally experienced as an infantile urge for survival, is not felt under threat, then the mind can project to others' interests and concerns and subsequently introject the feedback from those others, and develop empathy. Clearly, in teaching this concern for others is central, but in reality, sometimes teachers have to defend their own self as a priority. In our case here, the teachers who had to teach geometry to adolescents had to defend themselves doubly: against the typical (i.e., age-appropriate) challenge from their students and their own lack of having been fed (sufficient) geometrical nutrients during their education.

The state of mind of an infant – that we all continue to feel throughout life – experiences objects (both people and events are called 'object') in extreme terms and 'splits', chaotically, between (what is experienced as) good and (what is experienced as) bad. This is referred to as 'paranoid-schizoid': anxiety ('paranoid') to do with self-preservation is felt as either all good or all bad ('schizoid'). In contrast, the depressive (meaning 'considered' rather than 'sad') position occurs when the mind sees others as separate (unlike a newborn who does not distinguish itself from its mother), the mind is more balanced, more ambivalent and does not experience extreme feeling. The paranoid-schizoid position stimulates the person to defend themselves and the depressive position gives opportunity for developing relationships. What these defences or relationships are like depends on whether projective or introjective mechanisms are employed by the psyche and the nature of those mechanisms in the particular circumstance.

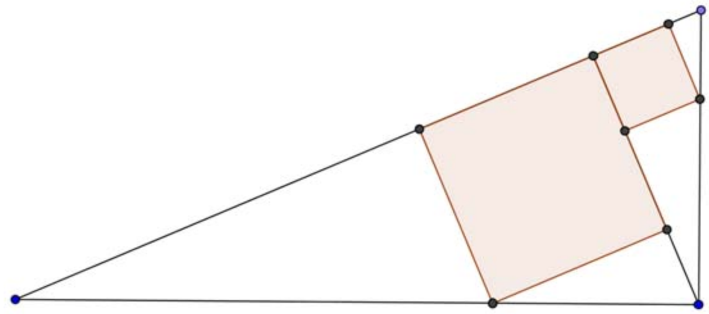
During the course, we noticed various ways that teachers defences manifest themselves. The examples presented in the next section illustrate (a) being blocked, (b) fantasising skill, and (c) identifying with the 'Other', here, pupils who are positioned as the not knowing.

Defences related to geometry: examples from the course



(a) Observation of the difficulty some of the teachers had with geometric reasoning started from the first session when the participants were given the Zome (Zome, 2008) materials to play with and get to know each other through companionable problem-posing/solving. We observed that some teachers with strong mathematics identities and good qualifications started by counting as opposed to any process that could be said to be visualising. The photo shows one such student's first model; he was unable to tell how many sticks he'd used by relational means (for example, by seeing the shape made by a certain grouping of a few (some subitizable number) sticks and then recognising a pattern of groupings). He was not happy with his lack of geometrical seeing. Eventually he 'gave in' and reasoned numerically by pulling out the sticks one by one and counting. During this session, there was a very friendly atmosphere with people moving freely from talking with others (about shared tasks or on general chat) to pursuing an idea of their own, but it did not seem enough to facilitate visualisation. Yet, it was our first meeting of a masters course that self-consciously privileged mathematical knowledge and that may have set up expectations for performance that, consciously or subconsciously, produced a 'blocking' anxiety.

(b) About a third of the way through the course, we organised a computer-room Cabri session. The teachers had been asked whether they had had experience with Cabri and no one said they had not. Yet, when asked to produce a dynamic version of the figure illustrated[2] (in which a perpendicular is dropped from the vertex opposite the hypotenuse in a right angled triangle and two squares constructed, as shown), the level of expertise was much lower than anticipated. For example, several participants found it difficult to devise a method to construct the squares so they retained the required properties when dragged in Cabri. Also they resisted testing their constructions with the dragging capability. In this situation defences are employed to protect the professional self: the teachers were doubtless aware that mathematics teachers 'ought' to be skilled in pedagogical technologies, like Dynamic Geometry Software. However, they (several of the participants) had not ingested the details of how DGS generally, or Cabri in particular, is used in practice. Instead, their knowledge *that* there is such a technology was fantasised as knowledge *of* it as a tool for geometric thinking.



(c) There is a tremendous exclusion potential in geometric thinking as it is hard for one person to get another to 'see' the way a configuration of a diagram or model yields an insight and this sets up various types of defences. For example, one of the participants got frustrated on several occasions when she was unable to visualise a theorem or geometrical relationship. On these occasions, she defended herself psychologically by saying that it was useful, in a way, to feel de-motivated and excluded (as a consequence of not visualising something), as it furthered empathy with her students: "its how the kids feel". By using her lack of visualising as a positive feature vis. à vis. having good relationships with her students, this teacher seems to protect herself from not visualising.

The above three scenarios are examples of instances of defences against geometry that we captured in some way. There would have been other instances being employed that passed us by that might have been noticed by others or might now be noticed by ourselves in a subsequent course. Examples of behaviours that we might interpret as constituting a defence cannot be expected to occur predictably. Serendipity and the preparedness of the observers to notice such occurrences is what affords opportunities to mark such behaviours as data. This raises methodological considerations for further work in this area (which are outside the scope of this present paper).

FOR FURTHER DISCUSSION

On a tension between psychoanalytic and cognitive viewpoints

For those of us who teach or work at geometry, we know that the occurrence of a novel visualisation cannot be predicted, it is experienced as a creative leap – "ah-ha, I see it now!". Dick Tahta wrote extensively about geometry and how geometry comes into being for people and in his chapter 'Sensible Objects' (Tahta, 2007) he draws on the work of the psychoanalyst Bion (*ibid.,* p210) to conceptualise how a visualisation might happen. Tahta conjectures that the position that is conducive to processing visual stimulus and projecting a visualisation is more akin to the paranoid-schizoid than the depressive one. The explanation is that taking in a 'visual whole' is like the all-or-nothing of the infant's experience and that creativity requires the intensity of the infant's state of mind: this sort of creativity is associated with holistic thinking. Thus, of the two positions, the paranoid-schizoid and the depressive, which fluctuate within us all, it is the former that is more conducive to visualisation. As the paranoid-schizoid position is orientated to survival, being demanding and defensive, so the creativity essential for visualisation, in a further, deeper way, stimulates the teacher's defences.

This psychoanalytic take on conditions for visualisation contrasts with the cognitive psychologists' 'good mood': the 'paranoid-schizoid' position is not associated with a relaxed pleasantness that 'good mood' signals! This should not be treated as a contradiction to fix but as something to consider and come to understand better. After all, can these psychoanalytic 'positions' be compared with moods?

Our situation is complicated because, in the classroom, a teacher of geometry has to operate as the parental figure with respect to teaching the pupils, which is associated with a considered ('depressive') position, as in the 'reflective practitioner' (Schön, 1983). And a teacher of geometry is also to be an autonomous geometrician and able to visualise aspects of geometric configurations, which at least according to Tahta is associated with an intense-holistic ('paranoid-schizoid') position. Furthermore, visualisation is not all there is to geometry. After all geometry - the study of space and shape and measurement - is a part of mathematics and mathematical language, notation and reasoning in general is pertinent to the geometrical domain.

All teachers ready themselves for their teaching, not only by planning lessons but by having been prepared and preparing themselves. Teachers' preparation includes, of course, their own education. Returning to the particular situation of our teachers: what were the "loved and trusted resources" (Waddell, *op. cit.,* p 150) that were ingested as they learnt mathematics? In many of these teachers' mathematical mind development, the loved and trusted resources, the reliable entities that they wanted to teach and share, were numbers and algebraic objects and routines, rather than geometric theorems. School mathematical work on number and algebra is usually routinisable and focussed on detail and thus it is more consonant with considered (depressive) analytic processes than it is with the broad sweep of geometrical visualisation which involves intense-holistic (paranoid-schizoid) appraisal. In order to teach geometry, (i.e., in order to teach geometry in a way that includes working on opportunities for pupils to develop their visual appraisal of geometric situations), 'objects' of geometry need to be available and trusted too. Our intention in the course was to facilitate trust in the objects of geometry by play, camaraderie and pursing a chosen geometric topic to depth (for an assignment). Play and camaraderie could be considered as good-mood enhancing practices and the assignment allowed the possibility for more intense experience.

On professional identity

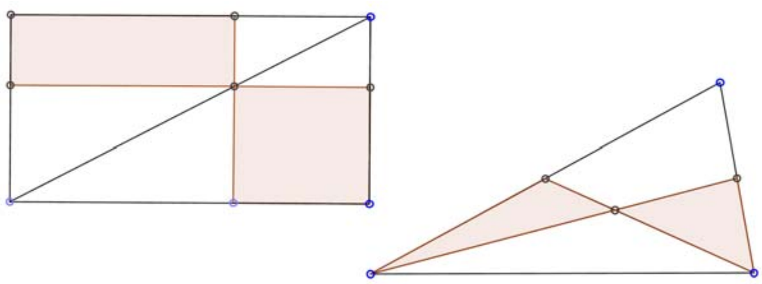
There have been some studies on primary teachers from a psychoanalytic perspective, (Hodgen, 2004). Primary teachers are generalists who are required to teach mathematics and very few who choose to teach younger children chose mathematics specialisms at university or even in high school. Stories of primary school teachers' psychic struggles to establish a mathematics teacher identity are, for all the interest in the detail, not that surprising; it makes sense to help primary teachers overcome their alienation from mathematics so that they can teach with honest enthusiasm and empathise, not just recognise, children's delight in their mathematical enquiries.

However, the teachers relevant to this discussion are secondary specialists who, by and large, have identities that include being a competent mathematician and 'in control' of their subject matter. They teach adolescents; the adolescent state of mind challenges order, unlike the state of mind typical of the younger 'latency' child, (the primary pupil), which craves order (Waddell, *op. cit.,* p11). So the site in which the secondary teachers have to preserve 'the self' is different from that of the primary teachers: the young people secondary teachers work with are likely to see lack of knowledge as a weakness (of course, any particular classroom situation could be different, but at secondary level some sophisticated craft knowledge is needed if a teacher positions themselves as not knowing). So what happens? Geometry is algebraised and visualisation marginalised and teachers tend to adopt the dominant culture and defend themselves against having to perform that which they do not practice.

In conclusion, on in-service provision

How do we resolve such conundrums as: being a mathematics subject leader but defensively unconfident in visual approaches? How do we work with teachers who avoid geometric thinking – either by being numerico-algebraicists or by being defensive-protective – yet want to engage with geometry or need to teach it?

In our course we took it as read that establishing good relationships between participants was an important foundation, not only for mutual support and informal peer teaching, but crucially for the motivating energy that pleasurable camaraderie brings. We found that tasks that seemed very simple, (e.g., compare the areas of the shaded regions in the rectangle and the triangle respectively, as illustrated), elicited conversations that, for example: explored the relationship between the visual and the analytic, asked 'how many different ways could the result be shown and which of these are proofs?', and reflected on 'how do we and our pupils develop our geometrical reasoning?'



This paper has been concerned with teacher education in geometry in England, where teachers established in a successful career find themselves having to teach Euclidean geometry which they had not themselves learned at school. They have been charged to teach geometry to adolescents within a results-oriented culture, where there is 'no time' for waiting for the 'insight' either from teacher or student. These teachers are capable and confident of their capability in solving equations or performing mental arithmetic in front of their classes, but geometry, because of its visual aspect, seems particularly to need a holistic approach, involving non-routinisable ways of thinking that they have not experienced much themselves. Helping these teachers overcome their defensiveness and gain intellectual resources related to visualisation that they can trust to use in the classroom is a challenge in geometry teacher education.

Acknowledgements

Thanks to Dietmar Küchemann with whom I designed and taught the course that is referred to in this article.

I'd also like to acknowledge the importance, in thinking about the subject matter of this article, of working with the group of people involved in the 2006-07 seminar series *Mathematical Relationships: identities and participation* (http://www.lancs.ac.uk/fass/events/mathematical relationships/).

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[1] In this paper, the first person plural is used to describe aspects of the course that Dietmar Küchemann and I taught in the summer term 2008 for our MA in mathematics education at the Institute of Education, University of London; first person singular voice is used to develop the argument: teachers who have to teach geometry without much geometrical education themselves defend themselves against their lack of knowledge and this process of defending militates against their developing abilities to visualise geometrical theorems.

[2] A problem from John Rigby.

Categories of affect – some remarks

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Cognitive concepts were not able to explain some of the effects in mathematics learning especially differences in performance. Therefore researchers started to investigate the influence of affects to the learning process using the concepts beliefs, attitudes, emotions and values. This paper discusses questions connected with the use of these concepts.

Introduction

McLeod (1992) wrote in his survey paper "Research on Affect in Mathematics Education: Reconceptualization" that beliefs, attitudes and emotions are used in mathematics education research to describe a wide range of affective responses to mathematics. The terms and concepts are often transferred from psychology to mathematics education. But McLeod refers also to problems to describe the affective domain and to use the concept transferred from an other field:

> Terms, sometimes have different meanings in psychology than they do in mathematics education and even within a given field, studies that use the same terminology are often not studying the same phenomenon.... Clarification of terminology for the affective domain remains a major task for researchers in both psychology and mathematics education. (McLeod, 1992; 576)

Especially for beliefs and attitudes there have been efforts to clarify the meanings of these concepts. In the paper within the book "Beliefs: A Hidden Variable in Mathematics Education" Furinghetti and Pehkonen describe a process to come to a common shared characterization of the beliefs concept. Even if it was not possible to reach this goal they found as a consequence that it is important to distinguish between deeply rooted and surface beliefs and take the degree of certainty as a characteristic of beliefs:

> Deep-rooted beliefs (such as a research paradigm) are self-evident and, therefore their certainty is 100%. They form the "axioms" in the individual's worldview.
> (Furinghetti and Pehkonen, 2002; 54)

Törner developed a four-component-definition of beliefs and sees beliefs B as a quadruple $B = (O, C_0, \mu_i, \sum_j)$, whereby O is the belief object, C_0 the content set of mental associations, μ_i the membership degree function and $\sum_j$ the evaluation map (Törner, 2002).

Considering the problem of definition in the case of "attitude toward mathematics" we find an analogous situation that Di Martino and Zan describe (Di Martino and Zan, 2001; Zan and Di Martino, 2008):

> …the lack of clarity that characterizes research on attitude and the inadequacy of most measurement. (Di Martino and Zan, 2008; 197)

In their analysis they found in papers three types of definitions to attitude toward mathematics: A “simple” definition where attitude toward mathematics is seen as a positive or negative emotional disposition toward mathematics, a multidimensional definition where three components constitutes attitude – emotional response, beliefs regarding the subject and behaviour related to the subject, and a bi-dimensional definition where attitude toward mathematics is seen as a pattern of beliefs and emotions associated with mathematics.

The lack of clarity what means beliefs or attitude toward mathematics has also consequences for research in the affective field. So writes Sfard:

> Finally, the self-sustained “essences” implied in reifying terms such as *knowledge, beliefs,* and *attitudes* constitute a rather shaky ground for either empirical research or pedagogical practices – a fact of which neither research nor teachers seem fully aware. (Sfard, 2008; 56)

And also Hart referred to this problem and wrote that:

> Research on the affective domain in mathematics education is in need of a strong theoretical basis that will be developed only through sustained, systematic efforts over time.
> (Hart, 1989; 38)

That means we have to rethink the situation of the state of concepts used in research to affect and it seems necessary to consider the problem in a more general way: What is the problem of defining concepts and what is the role of research methods in this process? Can results from other fields help us to understand the categories of affect in a better way?

General aspects of concepts

Niss (1999) refers in his paper “Aspects of the Nature and State of Research in Mathematics Education” to a crucial fact of all research:

> It is important to realise a peculiar but essential aspect of the didactics of mathematics: its *dual nature*. As in the case with any academic field, the didactics of mathematics addresses, not surprisingly, what we may call *descriptive/explanatory* issues, in which the generic questions are ‘what *is* (the case)?’ (aiming at description) and ‘*why* is this so?’ (aiming at explanation). Objective, neutral answers are sought to such questions by means of empirical and theoretical data collection and analysis without any explicit involvement of values (norms). (Niss, 1999; 5)

If the duality to describe and to explain is an essential aspect the research field of didactics of mathematics this duality we must also found in the mean that we use to formulate our research questions and our results. That means we must found it in the structure of language used in didactics of mathematics to formulate and present results. Furthermore we must have a look at the concepts used mathematics education because these concepts must reflect this duality. Concepts are therefore on the one side a mean to describe phenomena and on the other side must concepts also include explaining aspects. If we look at the research in mathematics education we find many papers with deep considerations to mathematical concepts (see for instance the

Special Issue “Semiotic Perspectives in Mathematics Education” in Educational Studies in Mathematics Education, Saenz-Ludlow and Presmeg, 2006) but questions to concepts used in mathematics education and the state of their definitions are not really in the focus of interest. As discussed in the case of concepts used in research to affect in mathematics education some of these concepts are transferred from other fields in the field of mathematics education, but there is a lack of clarity what is the meaning of these concepts in mathematics education. Furthermore we found often the situation that the meaning differ from researcher to researcher. Of course we can found a common kernel in the meaning but some aspects are different.

Let us discuss the problem of the concepts to affect from a more general point of view considering the position of concepts in the scientific research process especially the relationship to the meaning of a concept. My starting point is the definition of a concept given by Sfard:

> A concept is a symbol with its use. (Sfard, 2008; 111)

Sfard uses the term “symbol” to include in the concept definition more signifiers than words and the use means the use of a symbol in a discourse (Sfard, 2008; 236). This extension of the term “meaning of a symbol” to its use in a discourse process allows the attention direct to more perspectives like emotional reactions than it was possible in the classical meaning concept of Frege (Kilpatrick, Hoyles, Skovsmose, & Valero, 2005). Otte refers to the important fact that all our perceptions include elements of interpretation as well as of generalization and therefore all knowledge is in a certain sense indirect knowledge and is a function of symbols and representations (Otte, 2005; 231). That means to understand concepts is a cognitive activity that is connected with intuition:

> Thom, and Bruner as well, intend to draw attention to the fact that we cannot develop our cognitive activities if we do not believe in the reality of our intuitions, and that this intuitions or mental states nevertheless may be treacherous and without objective validity or reference. Subjective meaningfulness and objective validity may not coincide. (Otte, 2005; 231)

This quotation refers also to the problem how an individual requires a concept. In mathematics education research we find to this problem two answers how this problem is seen. Following the ideas of Piaget the intellectual growth results from a direct interaction between the individual and the world whereas the position of social constructivism see:

> …that whatever name is given to what is being learned by an individual – *knowledge*, *concept*, or *higher mental* function – all these terms refer to culturally produced and constantly modified outcomes of collective human efforts. (Sfard, 2008; 77)

We have to accept that knowledge and concepts are outcomes of a cultural process and both cannot be learned outside of a discourse community. This means that a learner needs help from an experienced person (Lave and Wenger describe this learning process as “legitimate peripheral participation” (Lave and Wenger, 1991) but we have to consider the individual part of this process. Lakoff and Nunez refer to the important role of metaphors in this process:

> One of the principal results in the cognitive science is that abstract concepts are typically understood, via metaphor, in terms of more concrete concepts. This phenomenon has been studied scientifically for more than two decades and is in general as well establishes as any result in cognitive science (though particular details of analysis are open for further investigations). One of the major results is that metaphorical mappings are systematic and nor arbitrary. (Lakoff and Nunez, 2000; 40 – 41)

This transfer of metaphors is important to have in mind if we use concepts from other fields like the concept of attitude because these concepts are combined with metaphors to understand the concepts in this field. In mathematics education we must specify the metaphors for their use in our field.

A second crucial point is strongly connected to our use of language. We use words or symbols those are a result of a process of objectification and this words or symbols produces the illusion that they are in the same category as things but they have no empirical evidence:

> After objectification, we often interpret metastatements, that is, statements about discourse, as statements about the extradiscoursive world (…) This *ontological collapse* (a) may produce *illusory dilemma*, (b) can result in *phony dichotomies* leading to tautologies disguised as causal explanations, and (c) is likely to lead us to *consequential omissions*; blinding us to potentially significant phenomena that cannot be described in ontologically "flattered" terms. (Sfard, 2008; 57)

Referring to this fact we must have in mind that concepts used in mathematics education research and formulated by words have no empirical evidence and therefore no reference objects and get their meaning through all metaphors and associations those we have in mind in connection with the symbol for the concept. A consequence of this fact is that a concretisation of a concept through a questionnaire get a crucial position in the process of what does the concept mean. In many cases we can see that the concepts are in a certain sense reduced to the instrument for measuring the concept. For example if we consider the concept of "mathematical literacy" used in the PISA study. The results of PISA items used in the test are now "mathematical literacy" even if the items cover only a small part of associations those could be associated with the description of mathematical literacy (Raber and Schlöglmann, 2008).

Concepts of affect in mathematics education research

If we look at the position of concepts in research processes they are necessary to describe and explain phenomena. Studying affect was a consequence of the fact that cognitive concepts were not able to explain some of the effects in mathematics learning (McLeod, 1992). Especially differences in the outcomes of mathematics learning were in the focus of interest. To explain these differences researchers used affective concepts like attitudes and beliefs. Differences in mathematical performance were also a consequence of differences in attitudes or beliefs.

Considering the general remarks to concepts in the last chapter we have to hold in mind that for the use of concepts in mathematics education research three components are important. Firstly the concept definition (independently from the formal state of this definition (see McLeod and McLeod, 2002) for the case of beliefs), secondly all associations and metaphors combined with the concepts definition and thirdly the research methods those are used to investigate and measure the concept. If we compare this situation with the problem of mathematical concepts in mathematics education research we find the concept definition and the concept images (or concept tree (Sfard, 2008)) but we must not consider the position of research methods.

Let us start with a definition of affective categories using the definition of Goldin:

> (1) *emotions* (rapidly changing states of feeling, mild to very intense, that are usually local or embedded in context),
> (2) *attitudes* (moderately stable predispositions toward ways of feeling in classes of situations, involving a balance of affect and cognition),
> (3) *beliefs* (internal representations to which the holder attributes truth, validity, or applicability, usually stable and highly cognitive, may be highly structured),
> (4) *values, ethics, and morals* (deeply-held preferences, possibly characterized as "personal truth," stable, highly affective as well as cognitive, may also be highly structured). (Goldin, 2002; 61)

Looking at this definition it is important to take in account that the categories of affective representations refer to the fact that those are descriptions of mental systems situated in the brain. These mental systems are activated in all situations in those mathematics is involved and these systems influence thinking and acting of a person in these situations (Furinghetti and Pehkonen, 2000; Hannula, 1998). If we look sharper to these categories we have to distinguish between emotions and the other categories. Whereas emotions refer to a general mental system that is working in connection with local phenomena and gives an answer to a local situation more independently from the specific of the subject, are the categories attitudes, beliefs, and values, ethics, and morals the result of an evolutionary process representing the consequences of grappling with mathematics as well in school as outside of school (for instance in everyday life, job, etc.). Analyzing the definitions of attitudes, beliefs and values, ethics, and morals we find some keywords –intensity, stability, structure and truth. The intensity is often described by words like "hot" or "cool" (McLeod, 1992) a metaphor that is usually used to describe affective states:

> Affection, for example, is understood in terms of physical warmth. (Lakoff and Nunez, 2000; 41)

The term "stability" or "balance" refers also to a metaphor coming from physics and describing a state of equilibrium. In our case this term is used in twofold meaning. On the one side should it grasp the problem that a mental system leads to the same result for a longer period and on the other hand it describes a balance between the affective and cognitive system. Structure refers also to an ordered situation in the mental system with a clear distinction to other systems. Truth is a metaphor borrowed from the logic but used here in an individual sense that all

utterances made by an individual are subjectively seen as true. This has important consequences for the interpretation of results of investigations.
Considering the research methods researchers have developed many methods according to the complexity of the phenomena (for beliefs research see (Leder and Forgasz, 2002) and in investigations are often used more than one research method. On the whole we can see three groups of methods – quantitative, qualitative and observational methods. The basis for quantitative methods are all kinds of questionnaires and the results are handled by statistical methods. Qualitative methods are mostly based on texts (protocols of interviews, essays, protocols of narratives and protocols of observations) and are looking for key words those expressing affective or emotional reactions (see for instance (Tsamir and Tirosh, in press; Evans, 2002). Observations can also look for key words and more signs of the emotional state like all signs of body language. (there are also studies those use physiological facts but the number of these studies is very small).

Affective categories from the neuroscientific perspective

For further insights, we ought to look at the situation from the neuroscientific point of view. According to this view, there exist two different systems, cognition and emotion. Both exist as a result of biological evolution, with the aim of aiding the individual's survival (Wimmer and Ciompi, 1996; Damasio, 1999; LeDoux, 1998; Roth, 2001). Although located in different parts of the brain (Damasio, 1999; LeDoux, 1998; Roth, 2001), there are connections between both systems that allow interactions. A very important consequence of the existence of these two systems is that we have to distinguish between "feeling" and "knowing that we have a feeling" (Damasio, 1999; 26); or "emotional reactions" and "conscious emotional experience" (LeDoux, 1998; 296).
Furthermore, we should note that although all processes on the neuronal level are unconscious, some of these processes lead to conscious results. We are aware only of these conscious parts of the processes. For remembrances, too, two memory systems exist with respect to emotions: an implicit emotional memory and an explicit memory of emotions (LeDoux, 1998). The implicit emotional memory operates unconsciously, is strongly connected to arousal systems and may often lead to bodily reactions. The explicit memory of emotional situations contains all the conscious knowledge of emotional situations, emotional reactions to objects, persons and ideas etc.. The most important consequence of this is that this memory system is part of the cognitive memory and there is no distinction between a remembrance of an emotion and a remembrance of cognitive content (LeDoux, 1998). The fact that memory of emotions is cognitive has important consequences (Schlöglmann, 2002):

1) We have knowledge about our feelings, their origin and their effect. This knowledge is stored in memory systems as cognitive knowledge.

2) Memory of emotions is open to “rational” manipulation. That means we are able to think about our emotional remembrances, and that all verbal statements about emotional facts are controlled by cognition.
3) Knowledge of our affect with respect to objects and situations allows us to handle our affect at least in controlled situations (see Goldin’s example of the roller coaster experience (Goldin, 2002; 62)).
4) Humans are able to “construct” their remembrances in a way that they are able to live with this memory. Part of this process is forgetting unpleasant facts more easily than pleasant ones: our memory has suppression mechanisms to handle unpleasant remembrances (Roth, 2001).

Assimilation and accommodation processes lead to affective-cognitive schemata (Ciompi, 1999). The affective component is stored in two memories: in the implicit memory that works unconsciously but influences our action and thought (Damasio developed the concept of “somatic marker” to explain this (Damasio, 2004; Brown and Reid, 2004)); and in the explicit memory that stores all the knowledge of affect with respect to people, objects and situations. Affective-cognitive schemata always contain both the unconscious and the conscious components. Repeated assimilation and accommodation processes in relation to a special problem leads to consolidation of the unconscious reactions, as well as to more and more conscious knowledge of feelings and emotional reactions. It provides information on the outbreak of emotional reactions and allows the development of strategies for handling such situations. Malmivuori (2001, 2004) describes the functioning of self-concepts by distinguishing further an unconscious and a conscious share in the regulatory process:

> Thereby, affective regulation represents lower level or more automatic self-regulatory processes with weak self-control beliefs or personal agency and lower state of self-awareness, while active regulation of affective responses relates to enhanced self-control beliefs and high personal agency with efficiently integrated self-regulatory processes and promoted self-awareness. (Malmivuori, 2004; 117).

Assimilation and accommodation processes are also a necessary prerequisite to develop hierarchical structures – meta-structures. This meta-knowledge allows us to use our affects in a conscious way. As we know, emotional reactions are also used by humans to produce desired results in social processes (Goldin, 2002; Schlöglmann, 2006)).

If we consider the categories of affect and the fact that affects are related to two memory systems, the explicit or declarative memory (sharper the episodic memory (stores our personal facts) and the semantic memory (stores our knowledge)) and an implicit emotional system that works unconscious. This system needs an appropriate stimulus for activation and the effect of this activation is at least at the beginning not conscious controlled. As a consequence in investigating affects we can find emotional reactions only if the implicit emotional system is activated through a stimulus. If we look at our research methods especially on questionnaires the items are formulated by the researchers with the aim to grasp all important aspects of the definition and formulate those in form of questions. The attention of the person who has to answer these questions is directed to find an appropriate answer or to value on a scale. Analyzing the items used in questionnaires mostly there are no stimulus for the emotional system and therefore the categories measured by questionnaires seems to be “cool” and “stabil”. Only if there are questions about liking or disliking mathematics or some aspects of mathematics (Zan and Di Martino, 2008) we can expect more emotional motivated results.

Signs for activation of the emotion system we can only see in observations, for instance in problem solving situations (Goldin 2000) and with methods those are able to stimulate also the emotion system. In this sense we find signs for highly emotional states in situation where qualitative and observational methods are used. For instance in interviews with open questions (Tsamir and Tirosh in press), in narratives about own experiences (Ingleton and O’Regan 2002, Stroop 1998), or in essays describing own experiences. In these situations we can get very hot emotional reactions.

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HUMOUR AS MEANS TO MAKE MATHEMATICS ENJOYABLE

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The traditional educational system is constructed in such a manner that it excludes humour as a unique live process of getting knowledge and understanding. Informational communications lays in the basis of logical thinking instead of vivid dialogue with informative purpose. Present work represents an intermediate stage of research of influence of CheCha-maths method. In particular, humour as the affective factor in mathematical reflection is being considered. While using this method, arising of positive emotions can influence how teaching material is perceived, facilitate creation of joyful atmosphere in the classroom and help to maintain pupils' creative state of mind.

Key words: Problem solving, emotions, humour, classroom climate, motivation

INTRODUCTION

"…the comic thought which is starting with the contradiction, strengthened by imagination, is capable to deliver pleasure by training, induces the pupil to participation in dialogue and interaction... Game and laughter are higher expressions of living and rejoicing of life" (Munjiz, 1996).

Anyone who has paid attention to great speakers would know that humour is an excellent method for finding sympathy from the audience and making them open to your message. Also every teacher knows that sense of humour is necessary if you wish to win the hearts of students. Research has established that one's affective state has an effect on cognitive processes (see e.g. Hannula, 2006a). How should this inform teaching? Should the teachers focus on creating an entertaining show for their pupils? Or would the teachers change their lessons into therapy sessions?

In this paper, we will present a teaching approach that is built around math problems that are for the pupil at the same time **Che**erful (entertaining, funny, cool) and **Cha**llenging (difficult). We call these this CheCha-mathematics.

THEORETICAL FRAMEWORK

CheCha-math method is based on three educational approaches: acknowledging affect in math learning (Hannula, 2006a), using humour in teaching (Grecu, 2008) and use of open problems in math teaching (Pehkonen, 2004, 2008).

Affect in mathematical thinking and learning

In order to study affect in math education in contexts of actual classrooms there are three main elements to pay attention to: cognition, emotion, and motivation. Achievement without motivation is not sustainable, and neither is motivation without enjoyment. All three domains have a more rapidly changing state-aspect and more stable trait-aspect. (Hannula, 2006a)

One "fundamental principle of human behavior is that emotions energize and organize perception, thinking and action" (Izard, 1991). Research has confirmed a positive relationship between positive affect and achievement. It seems that the affective outcomes are most important during the first school years, as they are less likely to be altered later on. Two key elements of a desired affective disposition are self-confidence and motivation to learn. (Hannula, 2006a)

Advances in our understanding of the neuropsychological basis of affect (e.g. Damasio 1995, LeDoux, 1998) have radically changed the old view of the relationship between emotion and cognition. Emotions are no longer seen as peripheral to cognitive processes or as 'noise' to impede rationality. Emotions have been accepted as necessary for rational behaviour. Moreover, research has also shown – although not yet fully understood – that certain emotions facilitate certain type of cognitive processing (Linnenbrink & Pintrich, 2004).

Focusing on motivation we may find ways to influence what the subjects want to do, not only how they try to achieve it. In the existing literature, psychological needs that are often emphasised in educational settings are autonomy, competence and social belonging (e.g. Boekaerts, 1999). These all can be met in a classroom that emphasises exploration, understanding and communication instead of rules, routines and rote learning. However, this requires that all feel safe and perceive that they can contribute to the process. A possible approach to meet all these conditions would be the open approach, and more generally focusing on mathematical processes rather than products. (Hannula, 2006a)

Humour

Already Kant (1952) considered the nature of humour. He stated "Laughter is the result of expectation which suddenly ends in nothing" (p. 199). His classical statement has started considering humour as a mental mechanism resulting in laughter. As another early scientific approach to humour, Freud (1991) divided comic into wit, humour and actually comic. Many kinds of activity, including wit, are directed on reception of pleasure from intellectual processes. A person feels pleasure from suddenly released energy, which is splashed out in the form of laughter. From this perspective already, we can perceive how a good joke can generate a joyful atmosphere and create a positive emotional background of activity.

The comic, humorous contents can be reached in various ways and techniques. For example, Veatch (1998) suggests a list of types that are funny: finishing to the point of irrationality, satire, literal understanding of metaphors, irony, ambiguity, word-play, contradiction, discrepancy, excessive rationality and a deviation from the usual.

Each of these types of the comic can be expressed as a joke or a problem in math context. As an example of a math contradiction we take a joke, here framed whithin the world of Winnie the Pooh:

> Pooh and Piglet sit on a small bench and talk. Pooh: "To us the parcel from Eeyore has come. In a box are ten sweets and a note from Eeyore as them to divide, to me seven and to you seven". Piglet: "Is it so? I do not understand. You have thought of it? ”Pooh: "I do not even want to think. But I have already eaten my seven sweets".

Our experience in teaching shows that information, when presented in humoristic form, is said to sound more convincing and to be more easily acquired. Humour can also act as means of removal of psychological pressure, a psychological discharge, and promote efficiency of pedagogical activity. Suhomlinsky (1975) wrote:

> I would name laughter as a back side of thinking. To develop ability to laugh in the child, to enhance his sense of humour - means to strengthen his intellectual forces, abilities, to teach him to think and to see the world wisely.

Grecu (2008) has considered use of humour in teaching. She sees it as a means of making educational process active and highlights seven basic functions of humour in pedagogical activity:

1) informatively-cognitive (Opens essential features and properties of subjects and the phenomena. Rejecting standard approaches, the humour bears in itself any discovery),

2) emotional (the Humour can act as means of creation of creative state of health and as means of emotional support)

3) motivational (The humour can serve as a stimulator of volitional processes)

4) communicative (the Person with humour is attractive for people)

5) developing (Humour promotes development of critical thinking, a sharpness of vision of the world, observation and consequently intellect)

6) diagnostic (by the laughter maintenance - at what the person laughs, it is possible to judge about his merits and demerits) and

7) regulative (the humour gives the chance to look at itself from an unexpected side, forming an adequate self-estimation).

In CheCha-method most of these are relevant, the most important functions being on top of the list. Grecu suggest the following techniques for designing of humour for educational tasks. These pedagogical techniques are paradox, finishing to the point of irrationality, comparison by the remote or casual attribute, return comparison, wit of absurd, pseudo-contrast or false opposition, a hint, a self-exposure of own faults, intentional ignoring of things that might cause laughter, and exaggeration of the certain features of behaviour.

Grecu has offered also classification of means of the comic: 1) "word-play" based on violation of language norm (carrying of terminology over to a context unusual to it).

Consider, for example, the following riddle: "*I am it while I do not know that I am. But I am not it when I know that I am. What am I?*" 2) Comparison, author's original neologisms, - based on artistic expressive means (double entendre, an ambiguity). Examples of such problems and riddles are easy for finding in L. Carroll's (1865) books about Alice. 3) Paradox, a simple example being the claim "I am lying now".

Also Dzemidok (1993) distinguishes several humoristic methods: modification and deformation of the phenomena, unexpected effects and amazing comparisons, disproportion in attitudes and communications between the phenomena, imaginary association of absolutely diverse phenomena, creation of the phenomena which deviate from logic. As an example of the latter method consider the following:

There were only 3 students attending a professor's lecture in University. Suddenly 5 students left the room. The professor said: "If 2 students enter this room, there is nobody attending."

Most types of humour and their techniques could be used at mathematics lessons. Thanks to entertaining tasks and comical contents of the problems the classroom climate promotes a positive interaction between the teacher and pupils. However, one must be aware that opportunities of humour as pedagogical means have their limits. Grecu (2008) gives several suggestions regarding these limite. She suggests that one should use humour gently and support humour of pupils. She also warns not to ridicule pupil's person, laugh at what the pupil is not able to correct or change or laugh at an involuntary mistake of the pupil. Rough joking would indicate lack of customs and disrespect of the pupil and hence is absolutely unacceptable for the teacher. Moreover, the teacher should avoid being the first to laugh at one's own joke, as it can cause the reaction opposite to expected.

Problem solving and open problems

Problems are said to be open, if their starting or goal situation is not exactly given and they usually have several correct answers (cf. Pehkonen 2004, 2008). Open-ended problems emphasize understanding and creativity (e.g. Nohda, 2000, Stacey 1995). This would not mean lowering the expectations, quite the contrary. If an open task allows the solver to gain deeper and deeper insights (a "chain of discovery"; Liljedahl, 2005) it can facilitate a state of sustained engagement. This would also lead to more intensive working.

Research has shown that problem solving can be engaging and enjoyable for many students, but it does not attract everyone. Schoenfeld (1985) defined an individual's beliefs or "mathematical world view" as shaping how one engages in problem solving. For example, those who believe that maths is no more than repetition of learned routines would be more likely to give up on a novel task than those who believe that inventing is an essential aspect of maths. Unfortunately, there are students who do not see the potential for engagement and enjoyment in a math problem. We see humour as a means to engage also those students who do not perceive math problems enjoyable to begin with.

THE FEATURES OF CHECHA MATHS

This research is more about creating tools for teaching than about analysing the reality of classrooms. The work has been started based on the first author's pedagogical intuition as a teacher and his will to engage pupils with maths. This research falls within didactical engineering (Artigue, 1994) or design research paradigm (Cobb, Confrey, diSessa, Lehrer, & Schauble 2003) and it has a clear practitioner approach: "How can the teacher use humour to engage pupils with maths?" This approach has developed gradually over a few years into a teaching approach that assumes:

* in the same assignment entertainment is combined with a set of difficulty levels;
* in the course of problems' solving there are conditions for emotions to rise;
* all pupils can participate actively in solving the assignment regardless of their abilities.

The educational space is constructed in such a manner that teamwork of the teacher and pupils accepts dialogue character and interest in mathematics is favored. While using CheCha method, we separate the following basic constructs: a) *entertainment in learning process*, b) *level of the problem's difficulty*, c*) plurality of problem solutions*. We refer to as *entertainment in learning process* the affective components which excite the interest, draw attention and/or create a joyful atmosphere. For example, as entertainment we assume appeal, extraordinary content, intriguing title and/or amusing formulations. *Level of the problem's difficulty* we define as the variable degree of solution's complexity, beginning from the "obvious", achievable for many children, proceeding to a more complicated. It is important that the simplest way not always guides to the right solution. *Plurality of problem solutions* is a construct that consists of variety of means and ways of solving problem on the same level of abstractness, understanding and complexity. Various approaches are possible in one problem and it is supposed to have both a set of ways of solving and sets of different solutions as a whole. For example, to create a problematic math situation such parameters, as incomplete condition, the overloaded contents, or introduction of "not existing in reality" factors are used.

RESEARCH METHODS

In this paper, we shall describe the method of creating mathematical assignments (CheCha-problems) and evaluate the practice of CheCha-maths teaching. We explore

1. What mathematical problems are entertaining from the pupils' point of view?
2. How CheCha-method influences the atmosphere in mathematics lessons?

The construction of CheCha-problems

The technique of construction of such problems consists of certain stages. At the initial stage there is a search of "matrix" of a condition or its author's creation. Useful sources to find problems that can be developed into CheCha-problems have been math jokes, E. Lir's and L. Carroll's books and collections of problems from math Olympiads. Chessboard has also been a good setting for such problems. The original problem is typically open or can be modified into an open problem, meaning that it has no unique and final solution.

The next principle is to consider age-typical interests of pupils, their specific personalities and personal preferences. Substantial richness of a context of a problem is carried out at a following stage. There is a transformation into a context that bears in it entertainment, extraordinary and comic flavour or lively situations. At the same time, level of difficulty and plurality of the solutions is considered, allowing a wide range of different levels of solutions and approaches.

Then the problem is introduced to pupils and there is the opportunity for feedback, which is stirring up cognitive activity through questions, solutions and discussions. The teacher observes and reflects upon pupils' thinking during problem solving, focussing on: the perception of a problem by pupils (acceptance or non-acceptance); questions asked by them (depth and breadth); a degree of understanding of the context. These help the teacher to find direction for task's development.

It is important to notice that for every area of math teaching and learning one can find or construct such CheCha-problems. This may lead to creation of a new problem, or changing of the task. For example: "*3 tortoises go one after another along the road. The tortoise says, "2 tortoises follow my rear". The second says, "One tortoise goes ahead", "One goes back of me". The third says, "Two are ahead", "One creeps behind". How can this be?"* One should note that this problem is more attractive than something about moving material points along a straight line, with particular coordinates. The most common answer here is that it is impossible. But, in fact, there can be the solutions. "3 tortoises go…": the words of the third tortoise contradict each other. The solution might be that the last tortoise is lying! …One tortoise is riding on another. …There is a time lapse between the phrases, allowing one tortoise to run ahead. …The fourth tortoise stays near or behind the last turtle, and begins moving after the first phrase of the third turtle… The road is circular… The road is triangular… There is a mirror behind the last turtle. When it looks at its back, it can see one more turtle. Progressing from considered examples, and, instead of tortoise, we turn to another object, e.g., cows. One more possible solution is the birth of a calf!

Using CheCha-problems in teaching and feedback from pupils

Research was carried out in 2 Finnish schools (Espoo 2007-2008 and Helsinki 2008-2009), in 7 classes with different level of acquaintance with CheCha-math method and various educational atmospheres. The first author was teaching in these schools.

1. In December, 2007 the first author surveyed pupils' preferences of entertaining features in maths. The questionnaire consisted of 5 questions of open and closed types, e.g. *"What in a math problem can be entertaining?"* Two of the questions were multiple choice questions concerning the respondents' view of entertaining maths. Respondents were 40 pupils from two seventh classes and one eighth class.

2. In February 2008 a second questionnaire was given in the same school (Espoo) to the pupils, where they were asked *which kind of problems they preferred*. In this survey 40 seventh graders from the same three classes responded.

3. In September, 2008 another questionnaire was administered in a school in Helsinki. The data were collected in two 7th grade classes (40 pupils) within the first month of employment of first author as the teacher in this school. Pupils were asked to fill in a questionnaire and draw a picture of a topic "Me at a math lesson".

a. In the first class (19 respondents) there was a favourable educational atmosphere and relations the teacher-pupil were built at dialogue level. It was promoted by game methods applied at a lesson with use of chess, go, katamino and other intellectual games. The given activity of the teacher was a basis for the future introduction in educational activity of a CheCha-method.

b. A comparison group (for the same survey) was a seventh grade class (21 respondents) of another teacher, in which CheCha-method was not applied.

RESULTS

1. When responding what can be entertaining in maths, the frequency of choises were humour (55 %), "something else" (27 %), "cutting and drawing" (25 %), "unusual names and properties" (12.5 %), “plurality of answers” (12.5 %), and “fabulousness of a plot” (10 %.) A negative answer had not been incorporated as an alternative, and 5 % of children had written "nothing" in their specification of what the 'something else' could be. Altogether 87.5 % of pupils mentioned reasons why maths can be entertaining.

2. The pupils' task preferences has shown, that tasks of comic character were most popular (51 %), then were the tasks that could be solved using Lego or Chessboard (33 %), cutting and drawing (30 %), a fantastic plot (15 %) and unusual names and properties (12 %), (Figure 1).

3. a. In this other survey the results were slightly different. This time the most popular choice was cutting and drawing (58 %), then the comical character (47 %), the tasks solved with the help Lego and Chess of (26 %), further a fantastic plot (21 %) and unusual names and properties (11 %).

The results of this survey have shown that 74 % of the respondents mention reasons why mathematics can be entertaining. Half of the students mention chess, and 29 % the personality of the teacher as the defining factor.

In the drawing task, 63 % have drawn a joyful image of a math lesson, 10.5 % of respondents drew themselves thinking or pondering, 15 % represented subjects of maths presented in a positive light (for example a notebook with the tasks solved correctly).

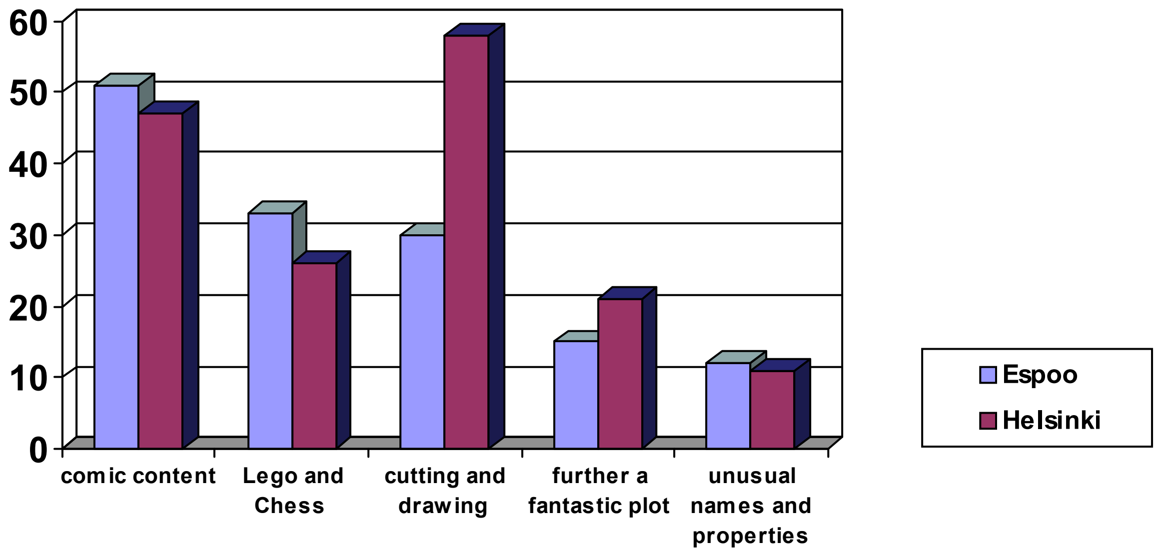

Figure 1. Pupils' responses to which types of tasks they prefer

When asked to continue the sentence "The CheCha-maths is ..." the most frequent answers were "Great!" (21 %) and "fun" (21 %), while 16 % of pupils noted that it is simultaneously a game and study. There were also individual answers of such a character as "creative and interesting", "many-sided", "various" and "laughter".

3. b. The other survey in the class where CheCha-maths was not applied produced somewhat different responses. For the question "It is possible to take pleasure at math lessons" only 26 % have positively answered, mostly responding utility of maths, instead any reference to its enjoyable nature. Also the drawing test did not show joyful atmosphere at a lesson. The priorities chosen by these respondents were cutting and drawing (67 %), the comical character (43 %), a fantastic plot (33 %), unusual names and properties (24 %) and the tasks solved with the help Lego and chessboard (10 %). On the offer to make definition "The entertaining maths is ..." the most frequent response was that such maths "is impossible" (29 %). Then was "drawing" (24 %) and there was a fair amount (29 %) of other positive characterisations (e.g. "games", "humour", "a funny nature", "easy").

CONCLUSIONS

One growing branch in mass media is 'edutainment' where EDUcational purposes are combined with enterTAINING qualities and interaction possibilities (e.g. computer games). Could math education learn something from the edutainment business in order to deepen the students' engagement with maths? We strongly believe that it is possible to develop suitable (open and multilevel) math tasks with attractive humorous flavouring, that make learning of maths very close to matter of laughter.

During the study in different schools developed methods, how can the teacher use humour to engage pupils with maths? The study showed:

1. From the pupils' point of view, entertaining tasks associated largely with humorous content. The longer pupils are working with humorous tasks, the higher the percentage of the preferences of those problems to others.

2. CheCha-math method influences the atmosphere in the lesson. The use of intellectual games (or creating a favourable atmosphere in other ways) prepares the ground for the use of humour in the lesson. In an unfavourable atmosphere, comical assignments can lead to undesirable results. The importance of the overall receptive atmosphere was observed in fall 2008. In one of the 7th grade classes taught a part of pupils responded negatively to use of comic tasks, speaking about "irrelevance" of jokes. When math problems were not understood, the comic presentation of problems caused negative reaction in a part of children. However, tasks with fantasy characteristics did not cause negative reaction. Pupils were distracted into conversations among themselves, and they moaned about the inconvenient arrangement in a class (the uncomfortably big group was placed in a computer class, not suitable for math lessons). After a replacement into an ordinary classroom the atmosphere had changed into more positive. Playful statements and problems began to be perceived positively, raising motivation on educational activity.

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EFFICACY BELIEFS AND ABILITY TO SOLVE VOLUME MEASUREMENT TASKS IN DIFFERENT REPRESENTATIONS

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The aim of this study was to investigate the relationship between students' efficacy beliefs and their performance in volume measurement tasks which were given in different representations. A group of sixth grade students (N=173) completed a four-part self-report questionnaire and solved six volume measurement tasks in different representations format: text, diagram of 3-D cube array and net diagram. Perceived efficacy to solve volume measurement tasks was found to be a significant predictor of students' general performance. Furthermore, high-ability students had stronger and more accurate efficacy beliefs towards tasks with net diagram which were unfamiliar, whereas low-ability students had more accurate efficacy beliefs towards verbal tasks which were familiar.

Key words: efficacy beliefs, volume, 3-D cube arrays, net.

INTRODUCTION

The affective domain has in recent years attracted much attention from mathematics research community (Philippou & Christou, 2002). A number of researchers who have examined thoroughly the connections and the relationship among affect and mathematical learning found that affect plays a decisive role in the progress of cognitive development (Bandura, 1997; Ma & Kishor, 1997; Philippou & Christou, 2002). One of the components of affective domain are self-efficacy beliefs (Goldin, 2002), which were found to have significant correlations and direct effects on various math-related variables (Pajares, 1996). However, although much work has been done in this area, little attention has been given to the relationship between self-efficacy beliefs and the use multiple representations in mathematics (e.g. Patterson & Norwood, 2004).

In this paper we try to investigate the relationship between efficacy to solve volume measurement tasks and performance in volume measurement of cuboid tasks which are given in different modes of representations.

THEORETICAL BACKGROUND

Self-efficacy beliefs and mathematics performance

Students' perceived self-efficacy for a task, are defined as their judgments about their ability to complete a task successfully (Bandura, 1997).

A number of studies have found a positive relationship between students' self-efficacy beliefs and mathematics performance (Pajares, 1996). More specifically, Pajares and Miller (1994) reported that self-efficacy in solving math problems was more predictive of that performance than sex, math background, math anxiety, math self-concept and perceived usefulness of mathematics. Additionally to this, Pajares and Kranzler (1995) found that self-efficacy made as strong a contribution to the prediction of problem-solving as did general mental ability, an acknowledged powerful predictor and determinant of academic outcomes. In this line, Mayer (1998) stressed that students who improve their self-efficacy will improve their success in learning to solve problems.

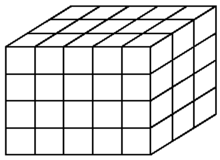
Researchers have also indicated that high-ability students have stronger self-efficacy and have more accurate self-perceptions (e.g. Pajares & Kranzler, 1995; Zimmerman, Bandura, & Martinez-Pons, 1992). Schunk and Hanson (1985) found that students who expected to be able to learn how to solve the problems tended to learn more than students who expected to have difficulty. In other words, students understand mathematics better when they have high self-efficacy than when they have low self-efficacy.

Self-efficacy beliefs have already been studied in relation to a lot of aspects of mathematics learning, such as arithmetical operations, problem solving and problem posing (e.g. Pajares & Miller, 1994; Pajares, 1996; Nicolaou & Philippou, 2007). However, these beliefs haven't been examined in relation to volume measurement tasks and this study tries to investigate this relationship.

Students' understanding of 3-D rectangular arrays of cubes

A number of researchers investigated students understanding of three dimensional rectangular arrays (3-D) of cubes, using interviews or tests (Ben – Chaim, Lappan & Houang, 1985; Battista & Clements, 1996). In particular, Ben – Chaim et al. (1985) indicated four types of errors that students in grades 5-8 made on the volume measurement tasks with three dimensional cube arrays. The first error was to count only the number of faces of cubes shown in a given diagram, while the second error was doubling that number. The third error was counting the number of cubes shown in the diagram and the forth error was doubling that number (see for example figure 1). In this study, when researchers asked students to determine how many cubes it would take to build such prisms, they found that only 46% of the students gave the correct answer, while most of them made the errors of type 1 or 2 (Ben-Chaim et al., 1985). These results are in line with those from a recent work by Battista and Clements (1996) where they found that 64% of the third graders and 21% of the fifth graders double-counted cubes. These types of errors made by students are clearly related to some aspects of spatial visualization (Ben-Chaim et al., 1985). In addition to this explanation, Battista and Clements (1996) stressed that many students are unable to correctly enumerate the cubes in such an array, because their own spatial structuring of the array is incorrect. In particular, they found that for some students the root of such errant spatial structuring seemed to be attributed to their inability to coordinate and integrate the views of an array to form a single coherent mental model of the array. However, Hirstein (1981) believes that these errors are caused by their confused notions of volume and surface area.

How many unit cubes does it take to make this rectangular solid? (Clements & Battista, 1996)



Four types of errors that students make on this problem:

Error type 1: Counting the cube faces shown in the diagram, e.g. 20+12+15=47

Error type 2: Counting the cube faces shown in the diagram and doubling that number, e.g. 47 x 2= 94

Error type 3: Counting the numbers of cubes showing in the diagram, e.g. 20+8+8=36

Error type 4: Counting the numbers of cubes showing in the diagram and doubling that number, e.g. 36 x 2=72

Figure 1: Four types of errors that students make on volume measurement problems.

THE PRESENT STUDY

The purpose of the study

The purpose of this study was to explore the relationship between students' efficacy beliefs to solve volume measurement tasks and their ability to solve volume measurement cuboids tasks; these were given in different modes of representations, namely text, diagram of 3-D cube array and net diagram. More specifically, the present study addresses the following questions: (a) Are students' efficacy beliefs to solve volume measurement tasks strong predictor of their performance in these tasks? (b) What is the relationship between students' efficacy beliefs to solve volume measurement tasks and their errors in dealing with 3-D cube arrays and net diagrams? (c) Are there differences in the efficacy beliefs and the accuracy of these beliefs among students of varied abilities?

Participants and Test

In the present study data were collected from 173 sixth grade students (84 females and 89 males) ranging from 11 to 11.5 years of age. These students were from 10 primary schools in Cyprus from rural and urban areas.

All participants completed a five-part test which was developed on the basis of previous studies (e.g. Ben-Chaim et al., 1985; Battista & Clements, 1996; Nicolaou & Philippou, 2007). For the purpose of this paper, we did not use students' answers from the first part of the test. The first four-parts of the test measured efficacy beliefs towards mathematical problems and volume measurement tasks and the fifth part measured students' ability to solve volume measurement tasks in different representations. Specifically, in the second part, students were asked to read each of the three volume measurement tasks: verbal task (SEiA), task with 3-D cube array (SEiB) and task with net diagram (SEiC) and state their sense of certainty to solve these tasks, without solving them. Responses were recorded on a 4 point Likert scale with 1 indicating not at all certain and 4 very much certain. In the third part, students were asked to state which one of the tasks from the second part was easy to solve (Es), was difficult to solve (Df), liked to solve (Lk) and did not find interesting to solve (Lint). The forth part comprised of five cartoon-type pictures and statements explaining the situation presented by each picture; the students were requested to select the picture that best expressed their efficacy beliefs (very high-SEI, high-SEII, medium-SEIII, low-SEIV and very low-SEV) to solve volume measurement tasks. The fifth part of the test had six volume measurement cuboids tasks which were given in different modes of representations: text, diagram of 3-D cube array and net diagram (see figure 2).

Verbal tasks

1. Mary tries to put 28 unit-sided cubes (1 cm edge) in a rectangular box with dimensions 2 cm x 5 cm x 3 cm. Is this possible? Explain your answer. (VPr1)

4. Four friends went to the cinema. They decided to buy some bags of nuts during the movie. The vendor said to them that there were two size bags of nuts, where:

• The prize of small bag was €1.

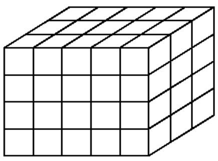
• The large bag's dimensions were two times the small bag's dimensions and its prize was €6.

The dimensions of small bag were 20 cm, 10 cm and 5 cm.

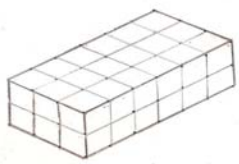
One child suggested to his friends that it was better to buy and share one large size bag, instead of buying four small bags. Are you agree? Explain your answer. (VPr4)

Tasks with diagram of three dimensional cube array

Find the volume (the number of cubes) of the following cuboids:



(SPr2a)

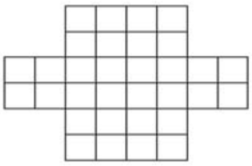


(SPr2b)

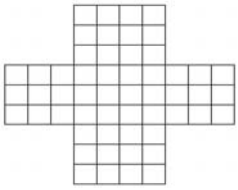
Which one of these cuboids has the greatest number of cubes? Explain your answer. (SPr2Ans)

Tasks with net diagram

The figures below show the nets of cuboids with one of its sides missing. Find the volume (number of cubes) of this net when folded:



(NPr3a)



(NPr3b)

Which one of these nets when folded can carry the least number of cubes? Explain your answer. (NPr3Ans)

Figure 2: Volume measurement tasks.

The coefficient of reliability Gronbach's Alpha of the five-part of test was very high (a=0.794). Specifically, we found that the reliability of answers of students in the first four-part of questionnaire was $\alpha=0.782$ and the reliability of answers in volume measurement tasks was $\alpha=0.810$.

Data Analysis

Students correct responses in volume measurement tasks were marked with 1 and incorrect response with 0. However, the marks to responses of the questions: "Which one of these cuboids has the greatest number of cubes? Explain your answer." and "Which one of these nets when folded can carry the least number of cubes? Explain your answer." were: 1 for fully correct response, 0.5 for partly correct response (wrong explanation) and 0 for incorrect answer. We used the classification of errors made in previous studies (Ben Chaim et al., 1985; Battista & Clements, 1996) to code the students' errors while solving the volume tasks with 3-D cube array diagram and net diagram.

To answer the research questions of this study, four different analyses were conducted: a Regression Analysis, an Implicative Statistical Analysis with the use of the computer software CHIC (Bodin, Coutourier, & Gras, 2000), an Analysis of Variance one way and a Crosstabs Analysis. The implicative statistical analysis is a method of analysis that determines the similarity connections and the implicative relations of factors.

RESULTS

We used regression analysis with independent variable students' efficacy beliefs to solve volume measurement tasks (answers of students in forth part of test) and dependent variable their general volume measurement performance in the test. We found that students' efficacy beliefs to solve volume measurement tasks can be a statistically significant predictor of their performance in the test (10,1%). Furthermore, we examined the predictive role of students' efficacy to solve verbal volume measurement tasks to their performance in these tasks and regression analysis confirmed that (6%). Additionally, students' efficacy to solve volume measurement tasks with 3-D diagram can be a statistically significant predictor of their performance in one of these tasks (3%). We also found that students' efficacy to solve volume measurement tasks in net diagram predicted only 4% of their performance in these tasks.

To examine the relationships between students' efficacy beliefs to solve volume measurement tasks, their performance in these tasks which were given in different representations and their errors in dealing with 3-D cube arrays and net diagrams, we employed the statistical implicative analysis for the data of this study and gave us the similarity diagram (see figure 3), which allowed for the grouping of the tasks and the statements based on the homogeneity by which they were handled by students.

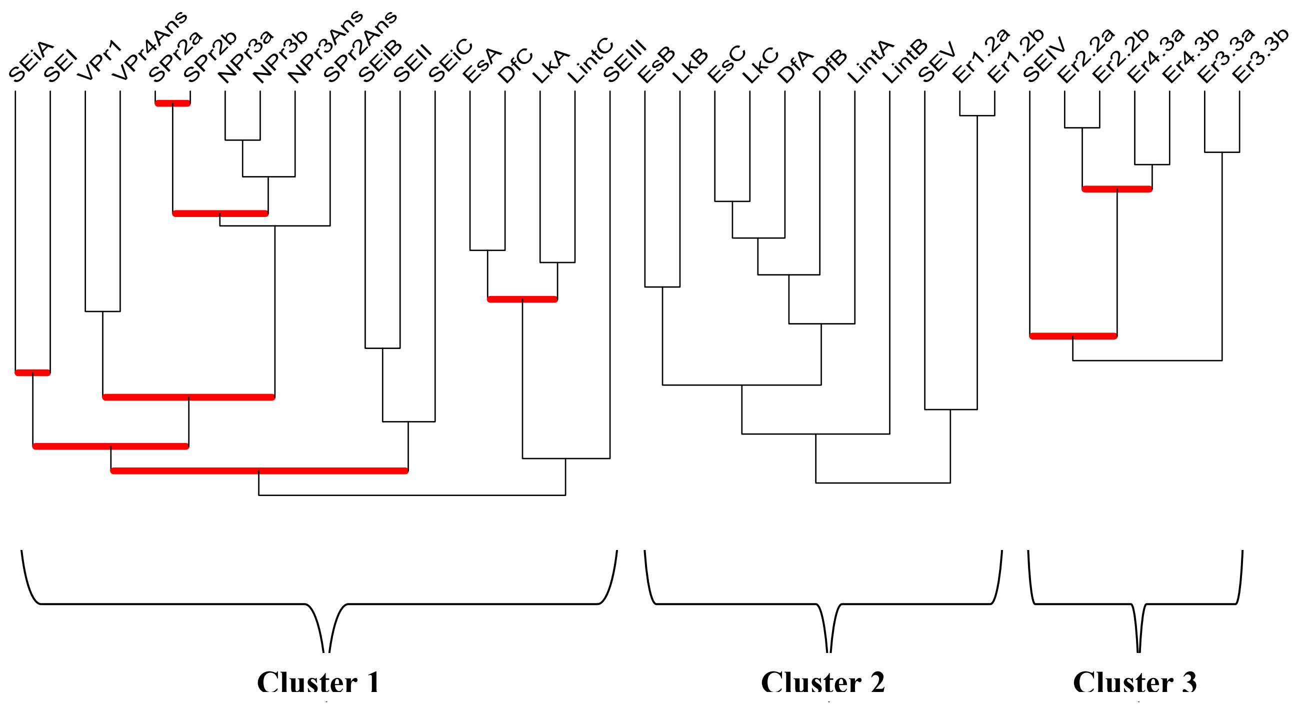


Figure 3: Similarity diagram of students' responses to the four-part of test.

Note: The similarities in bold color are important at level of significance 99%.

In figure 3, three distinct clusters of variables were formed. The first cluster consists of correct responses of students to volume measurement tasks and high efficacy beliefs, while the second and the third cluster consist students' errors and low efficacy beliefs. More specifically, the first cluster involved five similarity groups. The first group included the two statements of high efficacy beliefs to solve all volume measurement tasks and verbal tasks. The second group involved the verbal volume measurement tasks, while volume measurement tasks with 3-D cube array diagram and net diagram formed the third similarity group. These groups provided further support that different cognitive processes were required in order to solve verbal volume measurement tasks and volume measurement tasks with diagram.

However, their similarity connection indicated that equivalent content knowledge was needed to develop volume measurement ability in different representations. The forth group included the three statements of high efficacy beliefs to solve all volume measurement tasks, tasks with 3-D cube array diagram and tasks with net diagram. Finally, the fifth group of the first cluster involved mainly four statements which referred to students' evaluation for verbal tasks as easy and interesting and for tasks with net diagram as difficult and less interesting. All above groups of similarity of the first cluster show that students with high efficacy beliefs to solve volume measurement tasks in different representations solved these tasks in a similar way. Furthermore, these students assessed the verbal tasks as easy and interesting, while the task with net diagram as difficult and less interesting. It is hypothesised that students solved mainly verbal volume measurement tasks in their textbooks and so they had more experiences to solve these tasks than tasks with net diagram. Therefore, they felt more certain to solve familiar tasks than unfamiliar ones.

The second cluster involved two similarity groups. The first group mainly included four statements which referred to students' evaluation for tasks with net diagram as easy and interesting and for verbal tasks as difficult and less interesting. The second group involved the statement of low efficacy beliefs to solve volume measurement tasks and the wrong strategy: count the number of faces of cubes shown in diagram, which used from students to solve tasks with 3-D cube array diagram. The third cluster involved the statement of lowest efficacy beliefs to solve volume measurement tasks and errors to tasks with diagram. From the second and third cluster indicated that different cognitive processes were required to calculate the number of faces of cubes shown in 3-D cube array diagram and in net diagram. However, in the case of errors: count the number of faces of cubes shown in diagram and double that number, similar cognitive processes were required to apply it in 3-D cube array diagram and in net diagram.

The sample of this study was clustered into three groups according to their volume measurement performance in the tasks of the fifth part of the test. The performance of the three clusters of students was examined in respect to their efficacy beliefs to solve volume measurement tasks. The comparison of the means by one way ANOVA indicated statistically significant differences between these groups ($F_{(2,169)}=6.240$, $p=0.002$) at efficacy beliefs towards volume measurement tasks. Using Bonferroni procedure, we found only statistical significant differences at efficacy beliefs between students with the lowest performance ($\overline{X}= 3.10$) and highest performance ($\overline{X}=4.18$) in volume measurement tasks. Therefore, high-ability students have stronger efficacy beliefs towards volume measurement tasks than low-ability students.

However, at the same time, according to the results of the crosstabs analysis, students who solved the tasks of test correctly or wrongly indicated both very high efficacy beliefs and very low efficacy beliefs. We found that students who solved the tasks of the test correctly had more accurate self-efficacy than students who solved the tasks of the test wrongly. More specifically, high-ability students were more accurate in their efficacy beliefs towards tasks with net diagram in relation to their performance in these tasks (73% of students who solved the tasks with net diagram correctly indicated very high and high efficacy beliefs and only 7.5% of them indicated very low and low efficacy beliefs). The tasks with net diagram considered as an unfamiliar form of the volume measurement tasks for the students, because they did not solve any similar tasks in their mathematics textbooks. Also, crosstabs analysis showed that low ability students were more accurate in their efficacy beliefs towards verbal tasks in relation to their performance in these tasks (37% of students who solved verbal tasks wrongly indicated very high and high efficacy beliefs and 35% of them indicated very low and low efficacy beliefs). The verbal tasks are more familiar to the students, since their mathematics textbooks have a number of these tasks.

Additionally, the sample of this study was clustered into five groups according to their efficacy beliefs towards volume measurement tasks. The efficacy beliefs to solve volume measurement tasks of the five clusters of students were examined in respect to their general volume measurement performance. The comparison of the means by one way ANOVA indicated statistically significant differences between these groups ($F_{(5,166)}=3.697$, $p=0.003$) on volume measurement performance. Using Bonferroni procedure, students with very high efficacy beliefs ($\overline{X}=2.43$) and students with very low efficacy beliefs ($\overline{X}=0.55$) differed significantly in their general volume measurement performance.

DISCUSSION

The purpose of the present study was to investigate the relationship between students' efficacy beliefs to solve volume measurement tasks in different representations and their performance in these tasks. We found that students' efficacy beliefs to solve volume measurement tasks was a statistically significant predictor of the general volume measurement performance of students. The predictive role of efficacy beliefs was indicated from various studies in different concepts of mathematics (Pajares & Miller, 1994; Pajares & Kranzler, 1995; Nicolaou & Philippou, 2007).

In the similarity diagram three distinct clusters of variables were formed. The first cluster included students who solved correctly the tasks of the test and indicated very high and high efficacy beliefs towards volume measurement tasks, whereas the second and the third group involved students who used wrong strategies to solve volume measurement tasks with 3-D cube array diagram and net diagram and indicated very low and low efficacy beliefs towards volume measurement tasks. Specifically, these different similarity groups which were formed show that the confidence with which students approached volume measurement problems connected and had direct effects on their volume measurement performance.

We found, also, that high-ability students had stronger and more accurate efficacy beliefs towards volume measurement tasks in comparison to low-ability students. These findings confirm the earlier results by Pajares and Kranzler (1995) and Zimmerman et al. (1992). Furthermore, high ability students had more accurate efficacy beliefs towards volume measurement tasks with net diagram which were unfamiliar, whereas low-ability students had more accurate efficacy beliefs towards verbal volume measurement tasks which are more familiar to them.

Moreover, students who had high efficacy understand the volume measurement tasks better that the students who have low efficacy beliefs. This finding confirms the results of the study of Schunk and Hanson (1985). Also, students with high efficacy beliefs tend to assess the verbal tasks as easy and interesting, whereas the tasks with net diagram as difficult and less interesting. Therefore, these students' perceptions probably play an important role to their volume measurement performance and/or the development of their efficacy beliefs. This finding needs to be further explored.

In conclusion, the above findings about the predictive role of efficacy beliefs towards volume measurement tasks in different representations are very important in mathematics teaching and learning. Efficacy beliefs is an important component of motivation and behaviour (Pajares, 1996) and thus teachers need to develop ways to enhance efficacy beliefs of students of varied abilities. More specifically, high ability students need to solve "new" and creative tasks in which they will give the necessary attention and low ability students need to solve more easy and familiar tasks in which they can succeed.

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"AFTER I DO MORE EXERCISE, I WON'T FEEL SCARED ANYMORE" – AN EXAMPLE OF PERSONAL MEANING FROM HONG KONG

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What kind of meaning do students relate with mathematics education? To answer this question, the concept of personal meaning *is developed and integrated in an interplay with context and culture. Personal meaning hereby denotes the personal relevance students relate with a certain action or object. Finally, the concept is illustrated with an example of personal meaning constructed by a 15-year-old student from Hong Kong. Along this example, the relation of personal meaning and (learning) culture is disclosed.*

Key words: personal meaning, context, learning culture

INTRODUCTION

The demand for meaning in the context of mathematics education and education in general has been noted for many years. Hurrelmann stated in the early 1980ies that students are in the need of meaning when dealing with learning contents at school. This need, however, is way too often neglected. Therefore students do critically ask what meaning there is that might be relevant for them. In fact, students do not question the importance and necessity of school as being essential to prepare them for later life. They endure the time needed for their school education although school attendance is often judged as being unpleasant or even unmanageable. But still, they do miss a personal meaning which is relevant for them when they attend class. (Hurrelmann, 1983)

But what exactly is understood by the term *meaning* when thinking about school education? Do educators and students denote the same concept when using the term *meaning*? To be more precise: What kinds of meaning are there? And which meaning do students see when dealing with mathematics in school context? To shed some light on the obscurity of this realm, this paper starts with briefly presenting different understandings of *meaning* before the focus is put on the perspective of the students. Then, the concept of *personal meaning* is related to the notions of context and culture. The discussion shows in what way personal experiences and perspectives are important for the student to construct meaning. Finally, an example of personal meaning constructed by a 15-year-old Hong Kong student is presented to illustrate the concept and to show its relations to the (learning) culture in which the student has been socialised.

FROM MEANING TO PERSONAL MEANING

Meaning: A blurred concept

A review of the relevant literature shows that very different understandings of *meaning* are used. The notion may refer for instance to the act of leading the schema of an unconscious sensori-motor or mental activity to consciousness (Thom, 1973), to the development of a certain mathematical concept over time (Bartolini-Bussi, 2005), or to the collectively shared understanding and application of mathematical concepts (Biehler, 2005). These kinds of meaning deal primarily with mathematical concepts and develop a theory about its referents.

On the other hand, meaning can also be understood as a condition for students to engage in the action of learning (Alrø, Skovsmose & Valero, 2007), i.e. as an integrated aspect of acting (Lange, 2007) and the educational situation (Skovsmose, 2005), or as the personal relevance an object or action has for a certain student (Vollstedt, 2007). These interpretations move the focus from the meaning of concepts to the meaning of action, i.e. the educational process and the perspective of the students. The term *meaning* is therefore used here in a personal sense (Kilpatrick, Hoyles & Skovsmose, 2005).

Quite important differences between the understandings of *meaning* as described in the last two paragraphs can be detected. Howson therefore points out that

> one must distinguish between two different aspects of meaning, namely, those relating to relevance and personal significance (e.g., 'What is the point of this for me?') and those referring to the objective sense intended (i.e., signification and referents). (Howson, 2005, p. 18)

To sharpen the terminology used, I will use the more specific terms *personal meaning* when denoting the personal relevance of an object or action for a certain person, and *objective* or *collective meaning* when denoting a collectively shared meaning of an object or action (Vollstedt, 2007; Vollstedt & Vorhölter, 2008 [1]).

Characteristics of personal meaning and its construction

As described in Vollstedt (2007), some assumptions can be made concerning personal meaning. It is characterized by the following traits:

- Personal meaning is subjective and individual. This means that every person constructs his/her own meaning with respect to a certain object or action. As the construction of meaning is not collective but individual, different students sitting in the same lesson can also construct different meanings relating to the same object or action.
- The construction of personal meaning is also context bound. Here, context denotes on the one hand the subject context as well as the situation in the classroom. On the other hand, it also embraces the personal context of the students (see below).

- Personal meanings can be reflected on but normally do not have to. This means that the process of the construction of personal meaning can in some parts be dominant in the situation so that one is aware of it (e.g. in an Aha-experience); the meaning enters consciousness. On the other hand, meaning does not have to be conscious but can be constructed implicitly.

The student's perspective

Bearing in mind that there are different understandings of meaning in relation with mathematics education, one has to decide which perspective to put the focus on: collective or personal meaning? This means one has to ask whether rather mathematical concepts or the students are in the centre of attention.

My dissertation project reported on in this paper (see below) evolves from the context of the Graduate Research Group of Educational Experience and Learner Development located at the University of Hamburg. In this research group we investigate processes of learning and *Bildung* from the learner's perspective. Special attention is paid to the individually experienced tensions resulting from societal or institutional demands on the one hand, and the learner's individual responses being rooted in his/her biography on the other hand. The main emphasis is thereby put on the way how students acquire knowledge and skills as well as how they develop the ability to come to decisions and to responsibly act in an increasingly complex and difficult world (Graduiertenkolleg Bildungsgangforschung, 2006).

Due to the connection to the field of Educational Experience and Leaner Development, the focus of my study lies clearly on the learner's perspective. The study seeks to find out what kinds of personal meanings students construct in the context of mathematics education. Personal meaning is therefore understood as the personal relevance students see in an object or action (Vollstedt, 2007; Vollstedt & Vorhölter, 2008). Like Lange I therefore want to "look *with* children" (Lange, 2007, p. 271) instead of looking at them.

Personal meaning, context, and culture

Personal meaning cannot be constructed in a vacuum but is related to context. Context is here used as a cover term for both, situational context (i.e. context of the learning situation in terms of topic as well as classroom situation) and personal context. The personal context of a student then may consist of his/her personal traits (i.e. aspects which concern the student's self like his/her self-concept, motivation, or beliefs) and his/her personal background (i.e. aspects which concern the world around the student like his/her socio-economic status, migration background, or surrounding (learning) culture) (Vollstedt & Vorhölter, 2008).

Mercer describes context from the student's perspective in the following way:

> What counts as context for learners […] is *whatever they consider relevant*. Pupils accomplish educational activities by using what they know to make sense of what they are asked to do. As best they can, they create a meaningful context for an activity, and the

context they create consists of whatever knowledge they invoke to make sense of the task situation. (Mercer, 1993, pp. 31–32, italics in original)

Therefore the student decides which information and experiences are relevant for him/her to deal with the given task. I interpret Mercer's description in a broad way as not only knowledge but also for instance beliefs, goals or other kinds of personal traits or background may be relevant for the student in a learning situation. These are, however, object to cultural influence as culture has a strong impact on the way how learning takes place in any learning situation (Leung et al., 2006).

This understanding goes along with Mercer, who states that learning in the classroom depends both on culture and context as learning is,

> (a) culturally saturated in both its content and structure; and (b) accomplished through dialogue which is heavily dependent on an implicit context constructed by participants from current and past shared experience. (Mercer, 1993, p. 43).

When we take for instance the East Asian and the Western traditions, both, culture and context of a learning situation are very different as they are based on Chinese/Confucian and Greek/Latin/Christian traditions respectively (Leung, 2001). In how far culture also has an impact on the construction of personal meaning will be shown in the following section with the help of an example from Hong Kong.

PERSONAL MEANING CONSTRUCTED BY A HONG KONG STUDENT

So how does personal meaning finally look like when investigated in detail? To illustrate the concept, I will present some findings from a qualitative study which seeks to find out similarities and differences between the personal meanings constructed by students in two different learning cultures, namely Germany and Hong Kong (see Vollstedt, 2007). Due to the shortness of this paper I will restrict myself to Hong Kong data and results.

The study

In total, the study is based on 33 interviews with 15- and 16-year-old students in Germany (form 9 and 10) and Hong Kong (Secondary 2 and 3) [2]. In Germany I interviewed 16 students attending a grammar school; the 17 Hong Kong students attended band one EMI-schools (schools with the highest academic standards and English as medium of instruction [3]). The interviews began with a phase of stimulated recall (Gass & Mackey, 2000) based on a video-sequence of five to ten minutes from the last mathematics lesson the interviewee attended. The student was asked to utter and reflect on his/her thoughts he/she had when having attended the lesson. The stimulated recall was followed by a guided interview about various topics like the student's beliefs about and attitudes towards mathematics (lessons), his/her connotations of mathematics (lessons), or the feelings he/she associates with mathematics (lessons), i.e. personal traits. Aspects of personal background were not explicitly asked for [4]. In average, the interviews lasted about 35 to 45 minutes.

Grounded theory (Strauss & Corbin, 1996) was used to evaluate the data and to reconstruct different types of personal meaning. This means that some life is put into the theory of personal meaning, which has rather been developed abstractly. These types then are reflected on from a cultural perspective.

***Studying hard is soothing preparation for important exams*: A personal meaning constructed in the context of mathematics education in Hong Kong**

Emma, a 15-year-old girl from Hong Kong, attends a highly selective band one school in which the classes are divided into academic achievement. She is a member of class *Secondary 3C*, which is the class of the top 40 students of her year. Although she attends this class, she explains that she has difficulties with mathematics and shows a low mathematical self-concept (Marsh, 1986). This low self-perceived ability in mathematics, being part of her personal traits (i.e. personal context), is an important precondition for the personal meaning she constructs in relation with learning of mathematics at school. The following extract from the interview ([5]) may help to illustrate this point:

99	Interviewer:	First of all, what comes to your mind when you hear the word *mathematics*?
100	Emma:	First, at the beginning I feel, I'm afraid of mathematics. Because it is difficult for me to think. Think is the main problem for me. When I saw the mathematics sentence questions, I will feel scared. I think I don't understand, whether I understand that question or not, so that I feel scared. But after I do more exercise, I won't feel scared anymore and I feel I am safe.
101	Interviewer:	So is it because of the language, the problem is given in or is it because it's something unknown, or do you know why you are scared?
102	Emma:	I think it's not the language problem. I think is my problem because I think very slow. So I'm afraid I can't catch up with the other classmates.
103	Interviewer:	But you are in C class and C class is the best, isn't it?
104	Emma:	It is very difficult for me to go into this class because there is many pressure. There are many students are get high marks. So, there will be against students and students. So I need to study hard.

We can see that Emma comes to her low mathematics self-concept by means of internal and external references (Marsh, 1986). On the one hand she negates that her difficulties in mathematics are due to the fact that the mathematical problems and lessons are given in English (101-102), which is not her first language. The internal comparison of her self-perceived verbal ability with her self-perceived mathematics ability (Marsh, 1986; Marsh, Kong & Hau, 2001) make her come to this conclusion. She also, on the other hand, compares her abilities in mathematics with those of her classmates (102, 104), i.e. significant others in her frame of reference (Marsh, 1986). Due to the selective process, there are lots of very good students in her class so that it is not astonishing that Emma experiences high pressure when she compares her achievement with the one of her classmates. Especially as she mentions that there is quite some competition going on between the students (104).

The reason Emma gives for her difficulties with mathematics is that she has problems to think fast enough (100, 102). Therefore she stresses that actively doing mathematics can help “train us our mind and the logic” (66). Also, practice can help her to overcome her difficulties (100) as well as meet the pressure experienced between the students (104). She also refers to this point in another sequence of the interview in which she explains the importance of good grades with relation to the pressure caused by the *Hong Kong Certificate of Education Examination* (HKCEE):

198	Interviewer:	How important is it for you to achieve the mark you want to achieve in quizzes, or tests, or examinations, or whatever?
199	Emma:	Do more exercise. And when you see the questions, you should not feel afraid of them. Just like homework or worksheets, not a quiz or exams. So that we can relax and we won't feel more pressure.
200	Interviewer:	Is it important for you to get good marks?
201	Emma:	Yes, because we need to study in form four. And when we study in form five, there is Hong Kong CEE. It is very important because if we got a pass in a Hong Kong CEE we can study in form six and form seven. And if we are not pass in a Hong Kong CEE, maybe we can't study in form six, form seven and so that at that time maybe we need to find a job. But it is very difficult to find a job with form five level because many companies needs a person who got a university level. So the competition is very big.

We can see that Emma describes how practice can help to overcome anxiety and pressure as quizzes and exams may lose their threatening power when having done enough exercises beforehand (199). Therefore she is of the opinion that “it is not enough for us to do the school work. We should do more, so we find more practice exercise” (230). Her aim is to “remember all the steps” (230) necessary to solve a question. As a consequence she can relax and does not feel more pressure (199). On the other hand, she explains that the results of the HKCEE are so important for Hong Kong students as their future depends on them (201). This means that Emma reflects here on her future opportunities or foreground (Skovsmose, 2005).

Taken together we can describe Emma as a girl with low mathematical self-concept who suffers from fear of mathematics and examinations – especially the HKCEE. Personal traits are therefore highly influential for the personal meaning Emma constructs: she wants to learn and practice mathematics to overcome the pressure resulting from the examinations. Therefore she perceives practice as a possibility to soothe herself and work against her low mathematical self-concept.

Discussion from a cultural perspective

Emma's personal context as described in the last section can be explained with reference to the culture she was socialised in: the Chinese (a Confucian Heritage Culture (CHC) (Wong, 2004)). Leung shows that the CHC does have influence on how mathematics is taught in schools because “there exist distinctive features of mathematics education in East Asia and [...] those features are expressions of distinctive underlying cultural values” (Leung, 2001, p. 48). He identifies six features of mathematics education in East Asia and contrasts them with features in Western countries. To provoke discussion, he formulates these features in the form of the following six dichotomies (East Asia vs. West): product (content) vs. process; rote learning vs. meaningful learning; studying hard vs. pleasurable learning; extrinsic vs. intrinsic motivations; whole class teaching vs. individualised learning; and concerning the competence of teachers: subject matter vs. pedagogy (Leung, 2001). Leung, however, stresses the point that

> [i]t does not mean that all East Asian societies are on one side of the dichotomies and all Western countries are on the other side. Very often, it is a matter of the relative positions of the two cultures on a continuum between two extremes rather than two incompatible standpoints. (Leung, 2001, p. 38)

Emma is certainly not the only student with a low mathematical self-concept who studies hard and practices as much as possible to pass the HKCEE. This behaviour is, as far as I can judge from observation and data evaluation, somehow typical for Hong Kong students. It seems to be culturally determined and can be related to the three features of East Asian mathematics education that refer to students' behaviour, namely rote learning, studying hard, and extrinsic motivation.

Emma's attitude to practice as many tasks as possible can be explained by the Chinese belief that practice makes perfect (Li, 2006). It is closely linked with the feature of rote learning which Leung describes to be rooted in the East Asian view on the nature of mathematics learning. In East Asia, rote learning or memorization are not negatively connoted but, on the contrary, accepted and necessary steps of learning (Leung, 2001). Also, memorization and understanding are not necessarily separated (as a Western view might presume) but may be intertwined to lead to higher quality outcomes (Dahlin & Watkins, 2000).

Closely linked to the belief that practice makes perfect is the belief that studying hard is necessary to gain deep knowledge of the subject. This belief comes from the East Asian view that learning is necessarily accompanied by hard work (Leung, 2001). How deeply rooted this belief is in China can be deduced from the Chinese characters denoting education: 教育. They consist of different parts which mean 'young people' (lower left part of the first character), 'hard burden' (upper part of the first character), and 'development' (second character). So taken together the characters of 'education' confer the idea that “young people grow and develop under the condition in which they make every endeavor to tackle tough tasks” (Li, 2006, p. 131). Therefore, diligence and effort are needed to come to a deep level of pleasure and satisfaction as the outcome of study.

Finally, Emma studies hard to prepare herself for the HKCEE, which she has to sit in 2.5 years. Although the HKCEE is still fairly far in her future, it already has quite some power over Emma. This power comes, due to the large population, on the one hand from the serious competition between students for university admission. There is, however, also a historical argument of the big importance of exams in China or Hong Kong respectively. Throughout history, education has been a way for social advancement insofar as examinations had to be taken to be selected for important officer positions (Li, 2006). In addition, examinations are a warrantable source of motivation in the East Asian understanding. As Leung points out, "East Asians believe that, being human, we need some 'push' in our learning" (Leung, 2001, p. 43). Therefore, an optimal level of pressure is helpful to direct students' energy and attention to study and to learn.

From this illustration we can see that culture has an impact on the context of the individual in different ways. On the one hand, cultural beliefs seem to determine his/her actions and beliefs about learning. On the other hand, Leung (2001) shows that culture also shapes the identity of mathematics education, i.e. the learning situation.

CONCLUSION

The discussion of personal meaning has shown in what way the personal context is important for constructing personal meaning in the context of mathematics education. It is of special importance that personal meaning may be explained with reference to culture (the Confucian Heritage Culture in Emma's case). Her personal meaning (practising mathematics soothes and prepares for important exams) could be related to the CHC on three levels. Some of her personal traits (being diligent) as well as some of the actions she carries out in line with her personal meaning (working hard, practising as much as possible) seem to be rooted in cultural beliefs which are part of the CHC culture. So – as culture seemingly does matter for the construction of personal meaning – it is at near hand to support Leung, Graf & Lopez-Real, who assume that "the impact of cultural tradition is highly relevant to mathematics learning" (Leung et al., 2006).

NOTES

1. The German term for *personal meaning* we use in our research is *Sinnkonstruktion*. *Objective* or *collective meaning* on the other hand are equivalents of *Bedeutung*.

2. In Hong Kong, compulsory schooling starts with primary school, which lasts for 6 years (*Primary 1* to *Primary 6*). Subsequently students attend up to 7 years of secondary school. After *Secondary 5*, the *Hong Kong Certificate of Education Examination* (HKCEE; similar to GCSE in the United Kingdom) has to be sit.

3. Secondary schools in Hong Kong are divided in band one to three. This division is based on the achievement of their students in the HKCEE. After finishing primary school, Hong Kong students

are divided into different groups according to their achievement in relation to the standing of their school. Only high-achieving students are allowed to attend a band one school afterwards.

4. All students come from rather privileged and well-educated background. This can be argued by the kind of school they attend (private band one school/grammar school). For other aspects it was assumed that interviewees would give the information voluntarily or could be asked about it.

5. The transcripts of the interviews are simplified in language in the way that stuttering and breakups are left out, grammatical mistakes are not corrected but left unchanged. As Emma is very fluent in English, it was not necessary to mark hesitation etc. in the quoted sequences.

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MOTIVATION FOR LEARNING MATHEMATICS IN TERMS OF NEEDS AND GOALS

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This article suggests a framework for analysing students' motivation for learning mathematics. In the present paper, motivation is defined as a potential to direct behaviour. This potential is structured through needs and goals. The author examines students' motivation in terms of needs and goals, and the emphasis is on the psychological needs for competence and autonomy. The proposed theoretical framework as an analytical tool is useful in describing the students' goals and changes in goals in details. It could also contribute to increased insight into relations between different aspects of instructional designs and the students' motivation for learning mathematic. The usefulness of the theoretical framework will be illustrated with some findings from the study.

INTRODUCTION

In mathematics education there has not been done much work on people's motivation to date (Evans & Wedege, 2004; Hannula, 2006). Only a few researchers have distinguished between intrinsic and extrinsic motivation in mathematics (Goodchild, 2001; Holden, 2003; Middleton & Spanias, 1999), or between task orientation and ego orientation (Nicholls, Cobb, Wood, Yackel, & Patashnick, 1990; Yates, 2000). Some mathematics educators have discussed students' motivation under the terms of motivational beliefs (Kloosterman, 1996; Op't Eynde, De Corte, & Verschaffel, 2002) and interest (Köller, Baumert, & Schnabel, 2001; Schiefele & Csikszentmihalyi, 1995). Evans and Wedege (2004; , 2006) consider people's motivation and resistance to learn mathematics as interrelated phenomena.

Hannula (2006) points out that many of the above approaches fail to describe the quality of the individual's motivation for learning mathematics in sufficient detail. He suggests that the reason for this is that the authors' approaches aim to measure predefined aspects of motivation, not to describe it (p. 166). Hannula developed a theoretical foundation of motivation as a structure of needs and goals, and his study shows that the students' goals vary a lot from person to person. The aim of this article is to present (develop) a theoretical framework for analysing the students' motivation for learning mathematics, in terms of needs and goals. The article reports on a particular aspect of a study where the focus is the development of Norwegian upper secondary school students' motivation for learning mathematics when they experience an inquiry mathematics teaching approach. The study followed a design-research approach in that it involved both instructional design and classroom based research (Cobb, 2001). I collected a large and varied pool of data (participant observation, semi-structured interviews, videotapes of students working, conversations with the teacher, students' diaries, collection of material, assessment) on seven of the students. The focus of this article is the development of theory. Some findings from the study will be presented, mainly to illustrate the usefulness of the theoretical framework. Due to space constraint, the original data and analyses cannot be included. The interested reader should return to original papers.

MOTIVATION

Motivation is defined in different ways in the literature of (achievement) motivation, and I have chosen to use the following definition:

> Motivation is a potential to direct behaviour that is built into the system that controls emotion. This potential may be manifested in cognition, emotion and/or behaviour. (Hannula, 2004, p. 3)

Motivation is considered as a potential to direct behaviour, and therefore, my focus is on the orientation of motivation. According to the definition, students' motivation may be manifested in cognition, emotion and/or behaviour. For example, a student's motivation to get a good grade in mathematics may be manifested in happiness (emotion) if he or she scores high on a test. It may also be manifested in studying for a test (behaviour) and in new conceptual learning (cognition) when studying for the test. Needs are specified instances of the potential to direct behaviour (Hannula, 2004). Psychological needs that are often emphasised in educational settings are competence, relatedness (or social belonging) and autonomy (e.g. Boekaerts, 1999; Ryan & Deci, 2000). I have chosen to define motivation as a potential to direct behaviour and therefore the orientation of motivation becomes central. Thus it is necessary to add a more fine grained conceptualization of motivation focusing on needs and goals.

Self Determination Theory and needs

Self Determination Theory (SDT) is a general theory of motivation that focuses on psychological needs, and I have chosen to use Ryan and Deci's (2002) definition of needs. Before presenting the definition, I will give a short presentation of the theory. Most contemporary theories of motivation assume that people engage in activities to the extent that they believe the behaviours will lead to desired goals or outcomes (Deci & Ryan, 2000). Within Self determination theory one is concerned about the goals of the behaviour and what energizes this behaviour. SDT is founded on three assumptions. The first assumption is that human beings have an innate tendency to integrate. *Integrating* means to forge interconnections among aspects of one own psyches as well as with other individuals and groups in his or her social world:

> …all individuals have natural, innate and constructive tendencies to develop an even more elaborated and unified sense of self. (Ryan & Deci, 2002, p. 5)

This assumption of active, integrative tendencies in development is not unique to SDT. However, specific to this theory is that this evolved integrative tendency cannot be taken for granted. The second assumption in SDT is that social-contextual factors may facilitate and enable the integration tendency, or they may undermine this fundamental process of the human nature:

> …SDT posits that there are clear and specifiable social-contextual factors that support this innate tendency, and that there are other specifiable factors that thwart or hinder this fundamental process of human nature. (Ryan & Deci, 2002, p. 5)

In other words, there is a dialectic between an active organism and a dynamic environment (social context) such that the environment act on the individual, and is shaped by the individual. To describe and organize the environment as supporting versus thwarting the integrative process, the concepts of needs are used. *Needs* are defined through optimal functioning (growth and well-being), and I have chosen to use the following definition:

> There are necessary conditions for the growth and well-being of people's personalities and cognitive structures, just as there are for their physical development and functioning. These nutriments are referred to within SDT as basic psychological needs. (Ryan & Deci, 2002, p. 7)

Looking back at Hannula's definition, psychological needs are specified instances of the general potential to direct behaviour. The third assumption in SDT is that human beings have three basic psychological needs, the needs for competence, relatedness and autonomy (Deci & Ryan, 2000; Ryan & Deci, 2002). Within SDT, competence, relatedness and autonomy are defined in the following way:

> *Competence* refers to feeling effective in one's ongoing interactions with the social environment and experiencing opportunities to exercise and express one's capacities (Ryan & Deci, 2002, p. 7). *Relatedness* refers to feeling connected to others, to caring for and being cared for by others, to having a sense of belongingness both with other individuals and with one's community (Ryan & Deci, 2002, p. 7). *Autonomy* refers to being the perceived origin or source of one's own behaviour (Ryan & Deci, 2002, p. 8). (My italics in the three quotations)

According to the definition, competence is not an attained skill or capacity, but it is a felt sense of confidence and effectiveness in action. The individual feels and experiences competence in the specific situation, it is not a product that shall be used (Wæge, 2007). In that case it is different from the way it is used by Hannula (2002). Hannula defines competence as the individual's functional understanding and skills. He considers competence to be a product, something the individual could use. Relatedness, in the definition above, refers to the psychological feeling of being together with other persons in a secure community or unity. In a similar way as for the construct of competence, Hannula considers social belonging (or relatedness) to be a target to attain. It also includes a goal of social status in the group. Within SDT relatedness refers to the students' feelings of belongingness with others. When individuals are autonomous they experience themselves as volitional initiators of their own actions. Cobb and colleagues (Cobb, 2000; Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; diSessa & Cobb, 2004) use the concept of intellectual autonomy as a characteristic of a student's way of participating in the practices of a classroom community. They speak of the students' awareness and willingness to draw on their own intellectually capabilities when making mathematical decisions and judgments as they participate in mathematics activities. Hannula define autonomy as "the need to have control over own actions and to feel self-determining" (Hannula, 2002, p. 74). His definition differs from Ryan and Deci's definition in that it adds an aspect of having control over own actions.

The concept of needs is useful because it allows the specification of the social-contextual conditions that will facilitate motivation. According to SDT, students' motivation will be maximized within social contexts that provide them with the opportunity to satisfy their basic psychological needs for competence, autonomy and relatedness. I have chosen to use Ryan and Deci's definitions of the three psychological needs [1]. The data in the study did not give a basis for detailed analyses of the student's needs for relatedness and the goals the students' have in relation to this need. Therefore, the need for relatedness was not a focus in my study. In my study I focused on the students' needs for competence and autonomy. In his study, Hannula focuses on the three psychological needs for competence, relatedness and autonomy, but as I pointed out above, his definitions of the constructs differ from Ryan and Deci's definitions, which are the ones I have chosen to use.

Needs and goals structures

Hannula's definition of motivation (above) purports the potential to direct behaviour is structured through needs and goals. Needs and goals are specified instances of the potential to direct behaviour. According to Hannula, goals are derived from needs, and the difference between needs and goals is their different level of specificity. A need may be directed toward a relatively large category of objects, while a goal is directed toward a specific object (Hannula, 2004). For example, in my study, Berit realised her need for competence as a more specific goal of gaining a good grade. She translated her need for autonomy into the more specific goal of developing her own ideas, independently of the teacher. Another student, David, realised his need for relatedness as a goal to gain the mathematics teacher's confidence and respect.

According to Boekaerts, the students' goal structures are complex, and they tend to pursue multiple goals. The goals are hierarchically arranged, and some students are pursuing multiple goals at the same time, navigating elegantly between them. Other students place their goals in serial positions, devoting attention to the goals that have temporarily gained priority in their hierarchy of goals (Boekaerts, 1999). The goals are related to each other, and pursuing one goal might be necessary to attain another goal or different goals may be seen as contradictory (Boekaerts, 1999; Shah & Kruglanski, 2000). Learning goals and performance goals are usually considered as contradictory to each other (Lemos, 1999; Linnenbrink & Pintrich, 2000), but Hannula's (2004) and my own findings (Wæge, 2007) indicate that these goals should not be seen as mutually exclusive goals in mathematics education. To exemplify this I present an utterance of a student [2]:

Berit: […] I think it has been pretty enjoyable. In the beginning I thought it was a bit difficult (Interviewer: Mm) because I was not used to this kind of teaching approach. And maybe it got kind of, we didn't have any homework and {inaudible} then you didn't do any homework either. They only said "You can do what you want to do", and generally you are busy. And it becomes like "No, you don't have to do the mathematics". So, I think it is good that we have got more homework now (I: mm). Things like that. […] I think this mathematical approach is much better. The full-day test [3] was pretty special this time, because usually I didn't quite understand what I was doing {inaudible}. Do this, follow rules and things like that. This time I thought that I understood everything and I thought the test went very well. And then I get a 4[4] and when I didn't understand it I used to get a 5. But I almost think it's better to try to understand a little more and nevertheless get a lower grade. Anyhow, I think it is possible to increase the grade. It's only a new way of thinking. It's quite interesting, I think {laughing} strange, yes.

My analysis of Berit shows that she has a specific goal of relational understanding in mathematics (Skemp, 1976). Her sense of mastery and her feeling of succeeding in mathematics are higher when she experiences that she understands the mathematics problems, than when she uses rules without understanding. Another important goal for Berit is to get good grades on the mathematics tests. Her goals of relational understanding in mathematics and good grades in mathematics support each other mutually. Getting good grades are important to Berit, but relational understanding in mathematics is the most important goal for her.

FIVE MOTIVATION VARIABLES

There is a serious methodological problem with research on a mental construct like motivation. Students' motivation cannot directly be observed, and thus measured, and it needs to be reconstructed through interpretation of the observable. I have developed an instrument to assess students' motivation for learning mathematics in terms of cognition, emotion and behaviour. In doing this I focus on the five sets of motivational variables that Stipek, Salmon, Givvin & Kazemi (1998) used in their study entitled: "The value (and convergence) of practices suggested by motivation research and promoted by mathematics education reformers" [2]. These are the students'

1. focus on learning and understanding mathematics concepts as well as on getting right answers;
2. enjoyment in engaging in mathematics activities;
3. related positive (or negative) feelings about mathematics.
4. willingness to take risks and to approach challenging tasks;
5. self-confidence as mathematics learners;

All these motivation variables figure prominently in the achievement motivation literature and in the mathematics reform literature. In analysing the data, I assess these five motivation variables and I analyse the needs and goals of the students in relation to these specific motivational orientations. More specifically, the analysis is divided into two parts. First I analyse the data according to the five motivation variables. In the second part, I analyse the student's needs and goals in relation to these five specific motivational orientations. Furthermore, my emphasis is on the students' need for autonomy and competence.

TEACHING APPROACH

The teaching approach in the study was intended to give more space for the students to satisfy their needs for competence and autonomy, than teacher-centred and teacher-controlled teaching approaches. In the study attention was given to the development of students' mathematical thinking and reasoning. Our (the teacher and I) task was to create instructional activities that supported the development of both collective mathematical meanings evolving in the classroom community and the mathematical understanding of the individual student. We tried to support

> …the collective learning of the classroom community, during which taken-as-shared mathematical meanings emerge as the teacher and students negotiate interpretations and solutions (Gravemeijer, Cobb, Bowers, & Whitenack, 2000, p. 226).

The teacher always asked the students "What did you think when you solved this problem? What strategies did you use?" In the written tasks we developed, the students were frequently asked to explain their solutions and strategies, and the students were invited to find several solution strategies to a problem. The teacher tried to promote a classroom microculture (Cobb, Boufi, McClain, & Whitenack, 1997) where active participation and encouragement to understand were emphasised. In some of the instructional activities the students had to develop their own ideas, apply the mathematics in realistic situations and draw their own conclusions. Collaboration was important in our teaching approach. When the student's were given problems they were not familiar with, we wanted the students to collaborate. The students had an opportunity to experience themselves and their peers as active participants in creating mathematical insight. Every student brought a personal contribution at his or her level. These elements of our design study were suitable for meeting the students need for competence, autonomy and relatedness.

THE THEORETICAL FRAMEWORK – SOME KEY POINTS

The proposed theoretical framework for analysing students' motivation is useful in describing students' goals and changes in goals in detail. The framework is useful in clarifying students' notion of what it might mean to understand in mathematics. For example, the analysis of Berit shows that for her, to understand means to know what to do and why. We may also understand the relations between different goals through the use of such a framework. The complete analysis of Berit shows that there was a strong connection between her goal of relational understanding and her goal of finding her own solutions. She believes that finding own strategies for solving problems helps her in learning and understanding mathematics. As I described above, her goal of getting a good grades in mathematics and mastery goal, in this case a goal of relational understanding in mathematics, mutually supported each other.

The study shows that students' motivation for learning mathematics, although it is considered relatively stable, can be influenced by changes in the teaching approach. The case of Berit shows that students' motivation for learning mathematics might change in a relatively short time. Within the first semester of the school year, Berit changed her goal of instrumental understanding (Skemp, 1976) to a goal of relational understanding in mathematics.

We may also understand the relations between different aspects of the instructional designs developed in the study and the students' motivation for learning mathematics in terms of needs and goals through this framework. The analysis of Berit indicate that a combination of working with mathematics problems and routine tasks from the textbook, and the fact that the students were given opportunities to find their own solutions and rules for solving the problems, in collaboration with peer students and with guidance from the teacher, contributed to a sense of understanding and mastery with Berit.

I perceive that the theoretical framework as an analytical tool captured the complexity and the richness of the students' motivation in detail, and the tool made it possible for me to present detailed descriptions of the students' motivation for learning mathematics.

NOTES

1. See Wæge (2007) for a detailed description of my interpretation of the definitions.

2. Key to transcripts: […] extracts edited out of transcript for sake of clarity; {inaudible} unclear words; {text} comments about context or emotional behaviour like laughing; {.} 1 sec pause, {..} 2 sec pause, and so on.

The interviews took place in Norwegian. I have tried to translate from colloquial Norwegian to colloquial English, but it does not give an exact word for word translation. My analysis took place without any translation, that is, I analysed the transcripts in the original language.

3. At the end of each semester, the students have an all-day test in mathematics.

4. 1 is the lowest grade and 6 is the highest.

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