CERME 6 WORKING GROUP 7

TECHNOLOGIES AND RESOURCES IN MATHEMATICAL EDUCATION

WORKING GROUP 7 TEAM

Rosa Maria Bottino bottino@itd.cnr.it

Ghislaine Gueudet (Chair) Ghislaine.Gueudet.1@univ-rennes1.fr

Stephen Hegedus shegedus@umassd.edu

Hans-Georg Weigand weigand@mathematik.uni-wuerzburg.de

TABLE OF CONTENTS

WG7 call for papers

Technologies in mathematical education has been a theme present at CERME from the first edition. The available technologies evolved a lot during these years, the research followed, sometimes anticipated these evolutions; successive CERME proceedings give a clear picture of this process. At CERME 5 conference, the conclusions of the technology working group (Kynigos *et al.* 2008), as well as Artigue's and Ruthven's interventions, signal perspective evolutions towards more comprehensive studies, in several respects, and CERME6 invites to go further in the directions they indicate.

A first direction consists in the consideration of technologies within a range of resources available for the students, the teachers, teachers trainers etc. These agents can draw on software, computers, interactive whiteboards, online resources, but also more traditional geometry tools, textbooks etc. Various kinds of digital material are now extensively used, and they can be viewed as belonging to a wider set of curriculum material (Remillard 2005) and teaching resources (Adler 2000). This intention to take into account technology, but also all possible resources is indicated by the new working group's title. This does not mean that studies considering a specific technology are not welcome. Nevertheless, we invite authors studying a specific tool to question the articulation between this tool and other materials. The purpose is to open the working group to research on diverse kinds of resources, even if the work still keeps a specific focus on digital material.

A second direction corresponds to the theoretical approaches. Design issues need to focus on integration and impact, especially in the use of innovative technology. It entails the development of approaches framing research on fidelity, efficacy, and effective integration (Hegedus&Lesh 2008). Moreover, analysing the complex phenomena linked with use of resources (their design, their use by students and teachers, the associated learning and professional development) require holistic approaches.

These two statements also meet the key challenges identified by the recent ICMI 17 study, "Technology revisited" (Hoyles and Lagrange, to appear), whose conclusions will contribute to frame the work done within the group.

We propose three main themes for WG7 contributions. Their orientation, like in previous CERME conferences (Drijvers *et al.* 2006, Kynigos *et al.* 2008), are basically the tools, the teachers, and the students. The need to account for recent evolutions in the available technologies and resources, as well as in research, leads us though to emphasize specific aspects: design, articulation between design and use; interaction between resources and teachers' professional practice; technologies, tools and students mathematical activity. The work on each of these themes will take into account the two directions developed above: evolution of the resources considered, evolution of the theoretical perspectives; more details are given below.

1) Design and articulation of design and use

Studying design issues obviously includes questioning the characteristics of the tools, their affordances as intended by the designers. What are these characteristics? Which are their implications for mathematics epistemology? Recent evolutions, from material designed for students, and intended to speak through the teacher, to materials speaking to the teacher are recorded by research about curriculum material (Remillard 2005). Do the resources integrate recommendations, assistance for their use in class, and which kind of assistance? But digital materials can also be modified by their users; more generally the appropriation processes induce modifications of the tools. How do teachers, and students, modify the characteristics of the technologies and resources they draw on within their appropriation processes? Reports that focus on fidelity, efficacy, and assess the potential impact of the technologies and resources are welcome.

2) Interaction between resources and teachers' professional practice

The appropriation processes modify the teachers' professional practice. Which are the relations between the teachers' work with resources and their professional development? This question includes the studies concerned with integration of technology, a prerequisite for appropriation. It naturally goes further, considering lifelong professional development, and teacher education issues. Which kinds of training, to assist the integration of technology, of other materials? A specific interest will be devoted to collective issues: collective teachers' work, especially as permitted by networking means; communities of practice, and link between communities and professional development. How is it possible to assist the emergence of communities, what is produced in the course of teachers' collective work?

3) Technologies, tools and students mathematical activity

The central question in this theme has been extensively studied: how do students learn with tools and technology? The recent evolution of digital materials leads to devote a specific interest to the change of activity induced by complex materials like virtual learning environments; by portable technologies like USB keys; by networking means, allowing new forms of collaboration, between students, between teachers and students. What modifications of the students mathematical activity, collective activity in particular, do these materials induce? How do digital materials articulate with other resources in the students' activity? What are the consequences of these new combined materials on the students' relationship with mathematical knowledge?

The working group is concerned with the use of technology and resources in mathematics education at all levels, from primary school to university. Studies related with out of school mathematics learning will also be considered, studies about another academic field, and proposing comparisons with mathematics are also welcome.

References

Adler, J. (2000). Conceptualising resources as a theme for teacher education, *Journal of Mathematics Teacher Education* 3, 205–224.

Artigue, M. (2008). Digital technologies: a window on theoretical issues in mathematics education, in Pitta-Pantazi, D. and Philippou, G. (eds.) *Proceedings of the fifth congress of the European Society for Research in Mathematics Education*, CERME 5, Larnaca, Chypre, http://ermeweb.free.fr/CERME5b/, 68-82.

Drijvers, P., Barzel, B., Maschietto, M., Trouche, L. (2006). Tools and technologies in mathematical didactics. In Bosch, M (eds.), *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* , Sant Feliu de Guíxols, Spain, , http://ermeweb.free.fr/CERME4/, 927-938.

Hegedus, S., Lesh, R. (2008) *Democratizing Access to Mathematics through Technology: Issues of Design, Theory and Implementation- In memory of Jim Kaput's work*. Special issue of *Educational Studies in Mathematics,* 68(2).

Hoyles, C., Lagrange, J.-B. (to appear). *Digital technologies and mathematics teaching and learning: Rethinking the terrain*, Springer.

Kynigos, C., Bardini, C., Barzel,B., Maschietto, M. (2008). Tools and technologies in mathematical didactics, in Pitta-Pantazi, D. and Philippou, G. (eds.) *Proceedings of the fifth congress of the European Society for Research in Mathematics Education*, CERME 5, Larnaca, Chypre, http://ermeweb.free.fr/CERME5b/, 1332-1338.

Remillard, J.T. (2005). Examining key concepts in research on teachers' use of mathematics curricula, *Review of Educational Research* 75(2), 211–246.

Ruthven, K. (2008) Teachers, technologies and the structures of schooling, in Pitta-Pantazi, D. and Philippou, G. *Proceedings of the fifth congress of the European Society for Research in Mathematics Education*, CERME 5, Larnaca, Chypre, http://ermeweb.free.fr/CERME5b/, 52-67.

A RESOURCE TO SPREAD MATH RESEARCH PROBLEMS IN THE CLASSROOM

Gilles Aldon, Viviane Durand-Guerrier.

INRP, IREM de Lyon, IUFM de Lyon, LEPS (Université de Lyon)

Abstract : in this communication we intend to present a digital resource the aim of which is to give aid to teachers to use research problems in their classes; in a first part we are going to present the theoretical framework which was used by the team in the conception of the resource and the consequences on its model; we will present the results of a study dealing with the role and the impact of the resource used by teachers preparing lessons.

INTRODUCTION

Different works have shown the benefits of the use of research's problems [Polya, 1945, Schoenfeld, 1999, Brown and Walter, 2005, Harskamp and Suhre, 2007, Arsac et al., 1991, Arsac and Mante, 2007], in the construction of knowledge and both the interest of teachers and the difficulties to deal with in the classrooms; moreover, the institutional injunctions of using research problem are important in France and are going to take part in the final evaluation of the secondary school [Fort, 2007].

As far as we are concerned, and in the framework of the Piagetian psychological theory, we assume that the construction of knowledge has to go through an adjustment to the milieu as we will define it in the next section, and in this context, research problems are elements of the "material milieu" that teachers offer to learners.

We also assume that amongst all hindrances of generalization of research problems in the classroom, the following points are decisive:

- the important part of the experimental dimension in problem solving clashes with the main representation of mathematics amongst maths teachers but also in the society;
- the focus on heuristics and reasoning skills in maths research problem is in contradiction with the institutional constraints of teaching maths notions, particularly regarding French maths curricula;
- difficulties for teachers to pick out in the students' activity the mathematics part of their work, and, as a result the notions which can be institutionalized;
- the difficulties teachers have to assess such a work, the usual assessment modalities being not appropriate.

In this context, a team of teachers and researchers[1] from different institutions (IREM de Lyon, IUFM de Lyon, INRP and LEPS[2]), has worked on the construction of a numerical resource the aim of which is to give aid to maths teachers in order to use research problems in their teaching. In this paper, we will present the main theoretical frameworks used in the construction of this resource and will show, through the results of a particular study, the role this resource can play in the activity of teachers from the preparation of a lesson to the implementation in the classroom.

THE THEORETICAL CHOICES

This resource was written to be a part of the milieu of the teachers in the meaning Brousseau [Brousseau, 1986,Brousseau, 1997, Brousseau, 2004] and after him [Margolinas, 1995, Bloch, 1999, Bloch, 2005, Houdement, 2004] give to this concept. More precisely, learners learn through regulations of their links with their milieu. Going a bit deeper in this concept, Margolinas [Margolinas, 1995] described the structure of the milieu as a set of interlocked levels which can be described as follow:

Level	Teacher	Pupil	Situations	Milieux
3	Noosphere-T		Noospherian situation	Construction milieu
2	Builder-T		Construction situation	Milieu of project
1	Project-T	Reflexive pupil	Project situation	Didactical milieu
0	Teacher	Pupil	Didactical situation	Learning milieu
-1	Teacher in action	Learning pupil	Learning situation	Reference milieu
-2	Teacher observing	Pupil in action	Reference situation	Objective milieu
-3	Teacher organising	Objective pupil	Objective situation	Material milieu

Table 1 Structuring of the milieu

[1] Gilles Aldon, Pierre-Yves Cahuet, Viviane Durand-Guerrier, Mathias Front, Michel Mizony, Didier Krieger, Claire Tardy

[2] IREM : Institut de Recherche sur l'Enseignement des mathématiques ; IUFM : Institut Universitaire de Formation des Maîtres ; INRP : Institut National de Recherche Pédagogique ; LEPS : Laboratoire d'Etude du Phénomène Scientifique, Université de Lyon.

In this table, the milieu of level *n* is the situation of level *n-1* and is made up of the existing relationships between *M*, *P* and *T*. Using the symmetry of the table and, in our case, proposing to the teachers a situation, (in the acceptation of the didactical theory of situations) in which the a-didactical situations of action had as aim to allow teachers to construct, by themselves, the knowledge necessary to conduct a situation of problem research in the classroom [Peix and Tisseron, 1998] we speak of the material and objective milieu of the teachers. In this study, the resource appears to be a part of the material milieu of the teacher and the question is: is it possible, for a teacher, to use the resource to facilitate his tasks:

organizing the material milieu of the pupils,

understanding the objective milieu of the pupils and the links between their knowledge and conceptions

choosing the pertinent notions to be institutionalized in the reference milieu of the pupils, and anticipating the conflicts between misconceptions and tools to solve…

Moreover, the theoretical framework of cognitive ergonomics through its concepts and methods allows us to study the competencies of the teacher in his interaction with the work system, and more particularly in the relationship between the prescribed tasks and his activity. Lastly, and in the field of using a numerical resource in professional tasks, the concepts of utility, usability and acceptability [Tricot et al., 2003] have been sounded out in two different ways:

by an evaluation by inspection in order to construct and organize the resource,

by an empirical evaluation in a professional situation.

Utility is "the question of whether the functionality of the system in principle can do what is needed" [Nielsen, 1993]

Usability can be defined as: "the capability to be used by humans easily and effectively" [Schackel, 1991], but also "the question of how well the users can use that functionality" [Nielsen, 1993]

Acceptability refers to the decision to use the artefact, and answers the questions: is this artefact compatible with the culture, the social values, global organisation in which the artefact has to be included.

PRESENTATION OF THE RESOURCE

Structure

It is possible to use this resource in different ways; theoretical texts about the experimental dimension in mathematics [Dias and Durand-Guerrier, 2005, Kuntz, 2007] can be read as well as different presentations made in conferences [Aldon, 2007, Aldon and Durand-Guerrier, 2007]. It is also possible to understand the sense of the resource by reading a curriculum-vitae [Trouche and Guin, 2008] of the different steps and reflections of its building. The different situations are outlined using a common structure:

- Maths situation out of the classical literature on open problems developed in particular in IREM de Lyon (nowadays, there are seven maths situation):
  - Egyptian fractions: break down 1 into the sum of fractions of numerator 1.
  - Trapezoidal numbers: study of the sum of consecutive whole numbers.
  - The river: study of the shortest distance between two points with constraints.
  - The number of zeros of n!: study of the digits of n! in different numeration systems.
  - The greatest product: study of the product of integers of fixed sums.
  - Polya's urns: study of the dynamic of the composition of an urn in a repeated experience.
  - Inaccessible intersection: find a line going through an inaccessible point.
- Maths objects that may be used to solve the given problem: for each of the situations, the a-priori analysis allows to extract the mathematics objects that are part of the mathematics situation and can be used in the process of resolution.
- Learning situation: how the maths situation has been transformed into a didactical situation? In this part of the resource, reports of real experiments can be read.
- References
- Synthesis: a ten pages synthesis of the situation allows teacher to familiarize themselves with the content of the section.
- Connected situations: how is it possible to protract the situation and what are the extensions in the maths researches nowadays?

Introduction of the resource

In order to confirm the hypothesis and to evaluate the utility, usability and acceptability of the resource, we built an experimentation with teachers from the first handover of the resource to the real experiment of a research problem in the classroom. In this section we are going to focus on the first handover in order to evaluate the usability of the resource.

The methodology of this part of the experimentation was built using an observation of teachers faced to a professional problem (preparing a lesson using research problem); the context was a training course with sixteen teachers involved. They discovered the resource during this course as an artefact in the sense that the functioning of the resource has not been explained; the observer (the first author of this paper) recorded dialogues of two teachers and in the same time recorded the computer screen.

There is a confrontation, for the same person, between the position of expert (a teacher preparing a lesson, hence choosing objectives, a problem linked to these objectives, organising time of the lesson …) and the position of beginner in two different ways: using research problem in his (her) preparation and using a new tool. The theoretical framework of the didactic situation theory gave us the possibility to observe the position of the resource in the milieu of the teacher and to observe why this resource gives a possibility to the teacher to have a look into the pupils' objective milieu as described above. The cognitive ergonomics framework gives us keys to analyse the activity of teachers in this professional situation. Moreover, the concept which is tested was the usability of the resource, using the following criterions [Tricot et al., 2003]:

Possibility of learning the system

Control of the errors

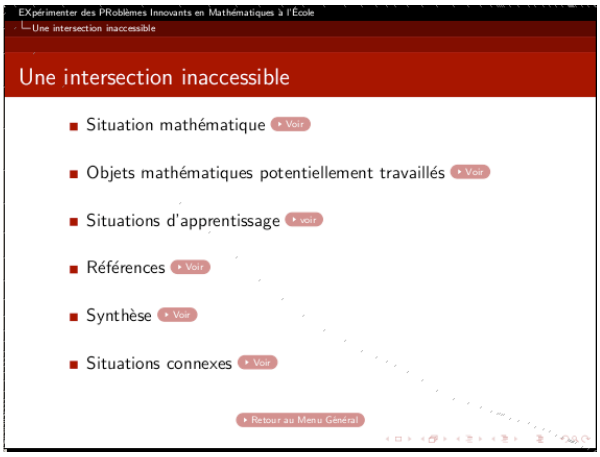
Memorization of the functioning

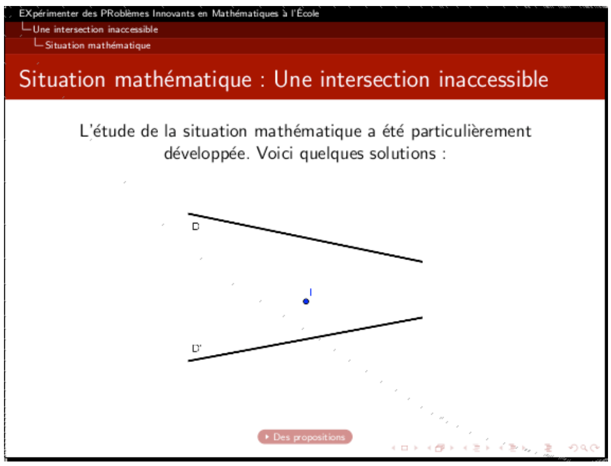
Efficiency

Satisfaction

But also, and we will see why later, its acceptability, that is to say the degree of confidence the teachers have.

The first result that we can highlight is the very quick adaptability of the observed teachers in front of the resource. After à three minutes wandering, the teachers used the different path in the resource to find exactly what they want as it can be possible





to see when teachers changed from one situation to another. In the first time, the mouse hesitated on the screen, going from one button to the others before the click, and progressively, the structure became clearer and the adequacy between the given objective and the browsing into the resource became safer:

After nine minutes:

Are you interested?

Yes

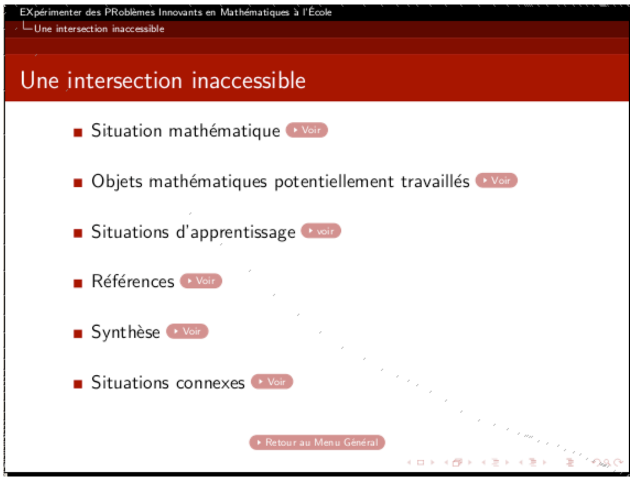
(*click on "situation mathématique*[3]")

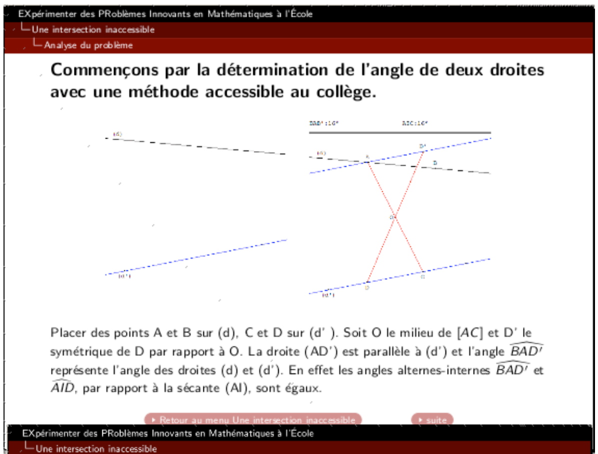
(*two clicks and two screens in one second*)

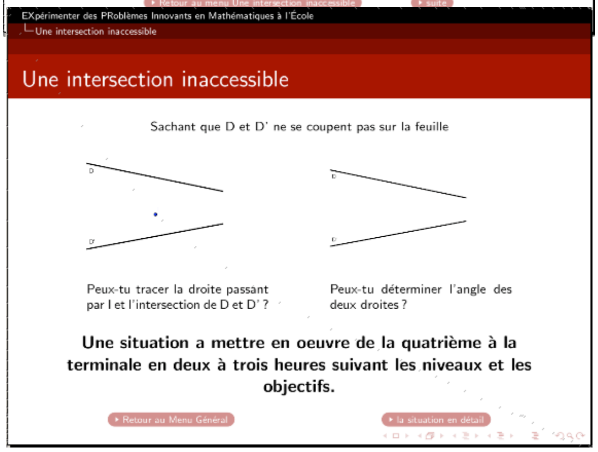
The mathematical situation… (*they read*)

Possible for our pupils (*click, click*)

I would like to see that (*the mouse turn over the menu "possible maths objects…*")







The second important observation, linked to the concept of acceptability can be seen by the feeling of trust in the authors of the resource; at the beginning of the exploration, the two teachers click on the menu: theoretical framework, and after some seconds says:

[3] Mathematical situation

"We are not going to read the whole text…"

And, later, in front of a situation, one of the teachers said:

"We are going to read what they say…"

These two brief sentences show us the growing of the confidence during the use of the resource and can be considered as a clue of the acceptability of the resource. The other observations and particularly the use of the resource to construct a real lesson confirm us in the feeling of the acceptability of the resource.

Realisation

In order to go on in the evaluation of the resource, a second experimentation has been built with the goal of testing the utility and the acceptability of the resource; we observed a research problem lesson focusing more particularly on the interactions between pupils and teacher during the situation of action. The teacher who was observed and interviewed, participated to the training course described above.

The chosen mathematical situation was the trapezoidal numbers and the question given to the pupils of a scientific eleventh class[4] was:

What are the whole numbers which are sum of at least two consecutive integers?

Interrogating the two theoretical frameworks, the interview with the teacher allows us to bring to light utility and acceptability of the resource, but also to understand the position of this resource in the teacher's milieu.

Utility of the resource is in this case obvious, the teacher having prepared the lesson with the resource:

"Yes, yes I use it… I read all you wrote about this problem. Oh, yes, without the resource, I think I should not give this problem to my pupils, because I would have spent too much time to do this work… I would not do that!"

Regarding acceptability, a lot of clues allow us to consider, for this teacher and in this experimentation, the resource as acceptable, for example the feeling that the lesson created using the resource brought a new dimension to her course:

"I think I'll do that earlier next year, to create something in the class, precisely, this dynamic which makes the pupils actors, as I said to you, a pupil was speaking from the board, and I was at the back of the class, and the other pupils ask questions… I think it's a good way to involve pupils in the maths lessons, to put a lot of them in maths… For me, it's very confident, visibly they enjoy this time, and I think it's something important to insert pleasure in maths lessons, it's something which questions me, because it's so easy to do maths without pleasure!"

[4] Pupils are sisteen-seventeen years old

Moreover, the interview confirms the position of the resource in the objective milieu of the teacher in a posture of preparation of a lesson including a research problem.

In an other hand the observation of a group of pupils gives us interesting feedback about the mathematical objects students deal with and shows that the *a-priori* analysis of the resource corresponds to the reality of the class; for example, one of the mathematical object which was highlighted by the authors of the resource related to this problem was the powers of two. In other words, the hypothesis was that powers of two belong to the objective milieu of the pupils and, consequently are a field of experiencing; the confrontation of pupils with these objects allows them to change their position in the milieu and to bring with the help of the institutionalisation these objects in the reference milieu of the pupils:

F2: (using her calculator) two to the power five gives thirty two… Yes ; two to the power seven gives one hundred twenty eight

G: two hundred and fifty six, five hundred and twelve, thousand and twenty four, two thousands and twenty eight …

F2: how do you calculate to obtain the results so quickly?

G: you multiply by two

F2: Ah yeah right!

In this small excerpt, the two definitions of the powers of two as an iterative or recursive process are called up and the link between these definitions is made by F2; it is possible to think that the recursive definition belongs now to her objective milieu and a necessary work must be done to institutionalize it in her reference milieu. The fact that this object was present in the resource allows the teacher to pay attention to this dialogue and to use it in her lesson:

CONCLUSION

The described engineering and the results of observations and interviews show the place of the resource in the milieu of the particular teachers involved in this experiment, and clearly show the utility, usability and acceptability of this resource. Regarding the didactical theory of situations, this experimentation shows that the resource emplaced in the material milieu of the teachers can be mobilised in their objective milieu and used in the setting up of research problem lessons in the classroom. The resource also allows teachers to launch themselves in the different milieu of the students and to understand the position of mathematical objects in these milieus, and consequently it facilitates the institutionalization.

However, new questions appear, in particular linked to the genesis of this resource and its transformation from an external resource possibly used by a teacher to a document available in his/her environment.

BIBLIOGRAPHY

[Aldon, 2007] Aldon, G. (2007). La place des tice dans une démarche expérimentale en mathématiques. In *Académie de Clermont, en ligne*, http://www3.ac-clermont.fr/pedago/maths/pages/UE2007/texte/Texte_11.doc.

[Aldon and Durand-Guerrier, 2007] Aldon, G. and Durand-Guerrier, V. (2007). The experimental dimension in mathematical research problems. In *Actes de la CIEAEM59*, http://educmath.inrp.fr/Educmath/partenariat/partenariat-inrp-07-08/exprime/1presentation.pdf.

[Arsac et al., 1991]Arsac, G., Germain, G., and Mante, M. (1991). *Problème ouvert et situation-problème*. IREM de Lyon.

[Arsac and Mante, 2007] Arsac, G. and Mante, M. (2007). *Les pratiques du problème ouvert*. Scéren CRDP de Lyon.

[Bloch, 1999] Bloch, I. (1999). L'articulation du travail mathématique du professeur et de l'élève dans l'enseignement de l'analyse en première scientifique. *Recherche en Didactique des Mathématiques*, 19/2:135–194.

[Bloch, 2005] Bloch, I. (2005). *Quelques apports de la théorie des situations à la didactique des mathématiques dans l'enseignement secondaire et supérieur*. PhD thesis, IUFM d'Aquitaine.

[Brousseau, 1986] Brousseau, G. (1986). Fondements et méthodes de la didactique des mathématiques. *Recherches en Didactique des mathématiques*, 7/2.

[Brousseau, 1997] Brousseau, G. (1997). La théorie des situations didactiques. In *Cours à l'Université de Montréal*, http://pagesperso-orange.fr/daest/guy-brousseau/textes/TDS_Montreal.pdf.

[Brousseau, 2004] Brousseau, G. (2004). *Théorie des situations didactiques*. La pensée sauvage éditions.

[Brown and Walter, 2005] Brown, S. and Walter, M. (2005). *The art of problem posing*. Lawrence Erlbaum Associates, Inc.

[Dias and Durand-Guerrier, 2005] Dias, T. and Durand-Guerrier, V. (2005). Expérimenter pour apprendre en mathématiques. *Repères IREM*, N°60:p. 61–78.

[Fort, 2007] Fort, M. (2007). Expérimentation d'une épreuve pratique de mathématiques au baccalauréat scientifique. *http://media.education.gouv.fr/file/98/4983.pdf*.

[Harskamp and Suhre, 2007] Harskamp, E. and Suhre, C. (2007). Schoenfeld's problem solving theory in a student controlled learning environment. *Comput. Educ.*, 49(3):822–839.

[Houdement, 2004] Houdement, C. (2004). Mathématiques, didactique et découpages : la richesse d'un problème. *Actes des journées de formation IREM Montpellier*, pages 43–52.

[Kuntz, 2007] Kuntz, G. (2007). *Démarche expérimentale et apprentissages mathématiques*.
http://www.inrp.fr/vst/Dossiers/Demarcheexperimentale/sommaire.htm.

[Margolinas, 1995] Margolinas, C. (1995). La structuration du milieu et ses apports dans l'analyse a posteriori des situations. *Les débats de didactique des mathématiques*, annales 1993-1994.

[Nielsen, 1993] Nielsen, J. (1993). *Usability engineering*. Academic Press Inc. Boston.

[Peix and Tisseron, 1998] Peix, A. and Tisseron, C. (1998). Le problème ouvert comme moyen de réconcilier les futurs professeurs d'école avec les mathématiques. *Petit x*, 48:5–21.

[Polya, 1945] Polya, G. (1945). *How to solve it ? A New Aspect of Mathematical Method*. Princeton University Press.

[Schackel, 1991] Schackel, B. (1991). *Human factors for informatics usability*, chapter Usability - context, framework, design and evaluation, pages 21–38. Cambridge university press.

[Schoenfeld, 1999]Schoenfeld, A. (1999). Looking toward the 21st century: Challenges of educational theory and practice. *Educational Researcher*, 28(7):4–14.

[Tricot et al., 2003] Tricot, A., Plégat-Soutjis, F., Camps, J.-F., Lutz, A. A. G., and Morcillo, A. (2003). Utilité, utilisabilité, acceptabilité : interpréter les relations entre trois dimensions de l'évaluation des eiah. *Archive EIAH*.

[Trouche and Guin, 2008] Trouche, L. and Guin, D. (2008). Un assistant méthodologique pour étayer le travail documentaire des professeurs : le cédérom sfodem 2008. *Repères IREM*, 72.

NEW DIDACTICAL PHENOMENA PROMPTED BY TI-NSPIRE SPECIFICITIES – THE MATHEMATICAL COMPONENT OF THE INSTRUMENTATION PROCESS

Michèle ARTIGUE, Caroline BARDINI

Université Paris Diderot - Paris 7, Université Montpellier 2

Relying on the collective work carried out within the e-CoLab project concerning a pilot experiment on the new calculator TI-nspire, we address the issue of the relationships that can arise between the development of mathematical knowledge and instrumental genesis. By analyzing the design of some resources, we first show the importance teachers involved in the project gave to such relationships. We then approach this issue from the student's perspective, using examples for illustrating the intertwined development of mathematical and instrumental knowledge.

INTRODUCTION

Educational research focusing on the way digital technologies impact, could or should impact on learning and teaching processes in mathematics has accumulated along the last two decades as attested for instance by the on-going ICMI Study on this theme. Questions and approaches have evolved as research understood better: the ways in which the computer transposition of knowledge (Balacheff, 1994) acts on mathematical objects, the impact of digital technologies on semiotic activities, and the influence of such impact on learning processes (Arzarello, 2007). Questions and approaches have evolved also due to the technological evolution itself, e.g. for instance the increased potential that technology offered to access mathematical objects through a network of inter-connected and interactive representations, as well as to develop collaborative work (Borba & Villareal, 2004). Increased technological power, nevertheless, generally goes along with increased complexity and rising distance from usual teaching and learning environments. Researchers have hence become more and more sensitive to the processes of instrumentalization and instrumentation that drive the transformation of a given digital artefact into an instrument of the mathematical work (Guin, Ruthven, & Trouche, 2004). They have revealed the underestimated complexity, and the diversity of the facets of such instrumental genesis both on the student and teacher side (Vandebrouck, 2008).

This contribution situates within this global perspective. It emerges from a national project of experiment on the new TI-*n*spire in which the authors are involved. This artefact is quite innovative but also rather complex and remote from standard calculators, even from the symbolic ones. This makes the didactical phenomena and issues associated with both its instrumentalization and instrumentation particularly problematic and visible. In this contribution, we pay a particular attention to the interaction between the development of mathematical knowledge and of instrumental genesis, analyzing how the teachers involved in the project manage it and how students experience it. Through a few illustrative examples, we point out some phenomena which seem insightful from this point of view, before concluding by more general considerations.

PRELIMINARY CONSIDERATIONS

Let us first briefly present the TI-*n*spire and its main innovative characteristics, then the French project e-CoLab, the theoretical frame and methodology of the study.

A new tool

TI-*n*spire CAS (Computer Algebra System) is the latest symbolic 'calculator' from Texas Instrument. At first sight, it just looks like another calculator; however, it is a very novel machine for several reasons:

Its nature: the calculator exists as a "nomad" unit of the TI-*n*spire CAS software which can be installed on any computer station;

Its directory, file organiser activities and page structure, each file consisting of one or more activities containing one or more pages. Each page is linked to a workspace corresponding to an application: Calculations, Graphics & geometry, Spreadsheet and lists, Mathematics Editor, or Data and statistics;

The selection and navigation system allowing a directory to be reorganised, pages to be copied and/or removed and to be transferred from one activity to another, as well as moving between pages during the work on a given problem;

Connection between the graphical and geometrical environments via the Graphics & geometry application, the ability to animate points on geometrical objects and graphical representations, to move lines and parabolae and deform parabolae;

The dynamic connection between the Graphics & geometry and Spreadsheet & lists applications through the creation of variables and data capture and the ability to use the variables created in any of the pages and applications of an activity.

When presented to the TI-*n*spire, we assumed that these features could offer new possibilities for students' learning as well as teachers' actions. They could foster increased interactions between mathematical areas and/or semiotic representations. They could also enrich the experimentation and simulation methods, and enable storage of far more usable records of pupils' activity. However, we also hypothesized that the new nature of this calculator and its complexity would raise significant and partially new instrumentation problems both for students and teachers and that making use of the new potentials on offer would require specific constructions, and not only an adaptation of the strategies proved successful with other calculators.

Excerpts both from students' interviews and teacher's questionnaires carried out/ handed out at the end of the first year of experiment support our hypotheses:

> "At first it was difficult, honestly, I couldn't use it… now it's OK, but at first it was hard to understand… the teacher, other students helped us and the sheet we got helped us out… how to save, use the spreadsheet, things like that…" (Student's interview)

"In my opinion the richness of mathematical activities thanks to the connection between the several registers is the key benefit […] The difficulty will be the teacher's workload to prepare such activities so to render students autonomous." (Teacher's questionnaire)

"There are still a few students for whom mathematics poses a big problem and for whom the apprenticeship of the calculator still remains arduous. These students find hard to dissociate things and tend to think that the obstacles they face are inherent to the tool rather than to the mathematics themselves." (Teacher's questionnaire)

Context of the research

This study took place in the frame of a two-year French project: e-CoLab (Collaborative mathematics Laboratory experiment), based on a partnership between the INRP and three IREM: Lyon, Montpellier and Paris [1]. It involved six 10th grade classes. Each student was provided with a TI-*n*spire CAS calculator, kept it throughout the school year and was allowed to take it home. The groups on the 3 sites were composed of the pilot class teachers, IREM facilitators and university researchers. They met regularly on site although the exchange also continued distantly through a common workspace on the *EducMath* site, which allowed work memories to be shared and common tools (questionnaires, resources) to be designed.

All pilot teachers had strong mathematical background but the expertise in using ICT varied from one to another. In the 1st year of the project, teachers and students were equipped with a prototype of the TI-*n*spire they had never worked with before. However, the willing of articulating mathematical with instrumental knowledge was shared by all teachers, despite the work they later on admitted it required:

"We have to devote an important amount of time to the instrumentation. This requires teachers to invest quite some time in order to design the activities, especially if they want to associate the teaching of mathematical concepts." (Teacher's questionnaire)

Theoretical framework

Two theoretical streams guide our analyses.

The first one is related to the *instrumental approach* introduced by Rabardel (1997). For Rabardel, the human being plays a key role in the process of conceiving, creating, modifying and using instruments. Throughout this process, he also personally evolves as he acclimatises the instruments, both in what regards his behaviour as well as his knowledge. In this sense, an instrument does not emerge spontaneously; it is rather the outcome of a twofold process involved when one "meets" a technological artifact: the instrumentation and the instrumentalization. Rabardel's ideas have been widely used in mathematics education in the last decade, first in the context of CAS (cf. (Guin, Ruthven & Trouche, 2004) for a first synthesis) then extended to other technologies as spreadsheets and dynamic geometry software, and more recently to on-line resources. Recent works such as the French GUPTEN project have also used the concept of *instrumental genesis* for making sense of the teachers' uses of ICT (Bueno-Ravel & Gueudet, 2008).

We are also sensitive to the semiotic aspects of students' activities. Not only are we taking into account Duval's theory of semiotic representation (Duval, 1995) and the notions attached to it (semiotic registers of representation and conversion between registers for instance), but more globally the diversity of semiotic systems highly intertwined that are involved in mathematical activity including gestures, glances, speech and signs (*i.e.* the "semiotic bundle" (Arzarello, 2007)). In particular, when examining student's activity, we pay specific attention to the embodied and kinesthetic dimension of it (Nemirovsky & Borba, 2004) via the pointer movement or students' gestures.

Methodology

We are interested in the students' instrumental genesis of the TI-*n*spire and in particular in considering the role mathematical knowledge plays in such genesis. This analysis cannot be carried out without taking into account the characteristics of the tasks proposed to students and the underlying didactical intentions. Our methodology thus combines the analysis of task design as it appears in the resources produced by the e-CoLab group, and the unfolding of students' activity.

The analysis of students' activity relies on screen captures of students' activities made with the software Hypercam. HyperCam, already used in other research involving the study of student's use of computer technology (see for e.g. Casyopée, (Gélis & Lagrange, 2007)), enables to capture the action from a Windows screen (e.g. 10 frames/sec) and saves it to AVI movie file. Sound from a system microphone has also been recorded and some of the activities have been video-taped.

When relevant, we also back up our analysis by relying on students or teachers' interviews/questionnaires carried out independently from the activities.

TEACHERS' INSTRUMENTATION – DIDACTICAL INTENTIONS

Didactical intentions

The pilot teachers involved in the experiment cannot be said to be "ordinary teachers". All of them have been involved, in one way or another, in the IREM network; thus they are all somehow sensitive to didactical considerations and share a fairly common pedagogical background. The relative success of the project was in part due to this familiarity, as one teacher acknowledged: "It is easier to communalize if we share the same pedagogical principles."

In particular, the willingness of intertwining mathematical content with instrumental knowledge was commonly held, and despite the hard work that it meant, the joint work was perceived as a true added value as teachers seemed to work in harmony:

> "We have to carry the instrumentalization and the mathematical learning in parallel. Activities are not evident to think of and take time to design. The help from others make us gain time and provide us with new ideas." (Teacher's questionnaire)

Imprint on resources

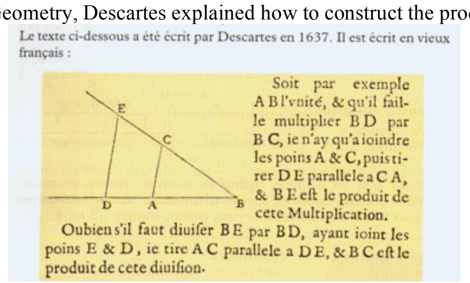
Around 25 resources were designed during the two years of the project. There are two kind of resources: those created essentially to familiarize pupils with the new calculator (presentation of the artifact and introduction of some of its potentials), and those constructed around (and we should add "for") the mathematics activity itself [3]. In what follows, we mainly focus on the resources that support the teaching/learning of mathematical concepts and examine how teachers managed to articulate mathematical concepts with instrumental constituents.

The didactical intentions previously mentioned are clearly visible when examining the resources teachers designed, showing that these were built taking into account both the instrumental knowledge and mathematical content. In fact, the activities are anchored on the mathematical component yet they also reflect that a progressive instrumentation was carefully planned as new mathematical concepts unfold.

The *Descartes* resource is very enlightening in this sense. It has been adapted by the Paris team from a resource built by the Montpellier team, having in mind an introduction into the dynamic geometry of the calculator associated with an application of the main geometrical notions and theorems introduced in Junior High School. It also offered the advantage of linking the prior work which had just been performed on numbers and geometry.

In this resource, several geometrical constructions are involved, enabling products and quotients of lengths to be produced and also the square root of a given length to be constructed. For the first construction proposed, the geometrical figure is given to the students together with displays of the measurements required to confirm experimentally that it does provide the stated product (fig. 1). The students simply have to use the pointer to move the mobile points and test the validity of the construction. Secondly, for the quotient, the figure provided only contains the support for the rays [BD) and [BE). The students are required to complete the construction and are guided stepwise in the successive use of basic tools as "point on", "segment", "intersection point", "measurement" and "calculation". Thirdly, they are asked to adapt the construction to calculate the inverse of a length. Finally for the square root they have the Descartes figure and text, and are required to organise the construction themselves. Instructions are simply given for the two new tools: "midpoint" and "circle".

In his treatise on Geometry, Descartes explained how to construct the product of 2 numbers


Le texte ci-dessous a été écrit par Descartes en 1637. Il est écrit en vieux français :

Soit par exemple A B l'vnité, & qu'il faille multiplier B D par B C, ie n'ay qu'a ioindre les poins A & C, puis tirer D E parallele a C A, & B E est le produit de cete Multiplication.

Oubien s'il faut diuiser B E par B D, ayant ioint les poins E & D, ie tire A C parallele a D E, & B C est le produit de cete diuision.

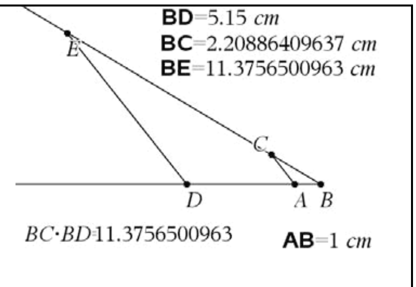


Figure 1. *First part of the Descartes resource (extracted from the pupil sheet and the associated tns file)*

In what concerns the resource *Equal areas*, the mathematical support is an algebraic problem with geometrical roots; it consists in finding a length OM such that the areas of two surfaces are equal (fig. 2). The expression of the two areas as functions of OM are 1st and 2nd degree polynomial expressions, and the problem has a single solution with an irrational value. This therefore falls outside of the scope of the equations which the students observed are able to solve independently. In the first version of the resource, their work was guided by a worksheet and included the following stages: geometrical exploration and 1st estimate of the solution, refining the exploration with spreadsheet to end in a interval for the solution of length 0.01, use of CAS to obtain an exact solution, and production of the corresponding algebraic proof in paper/pencil.

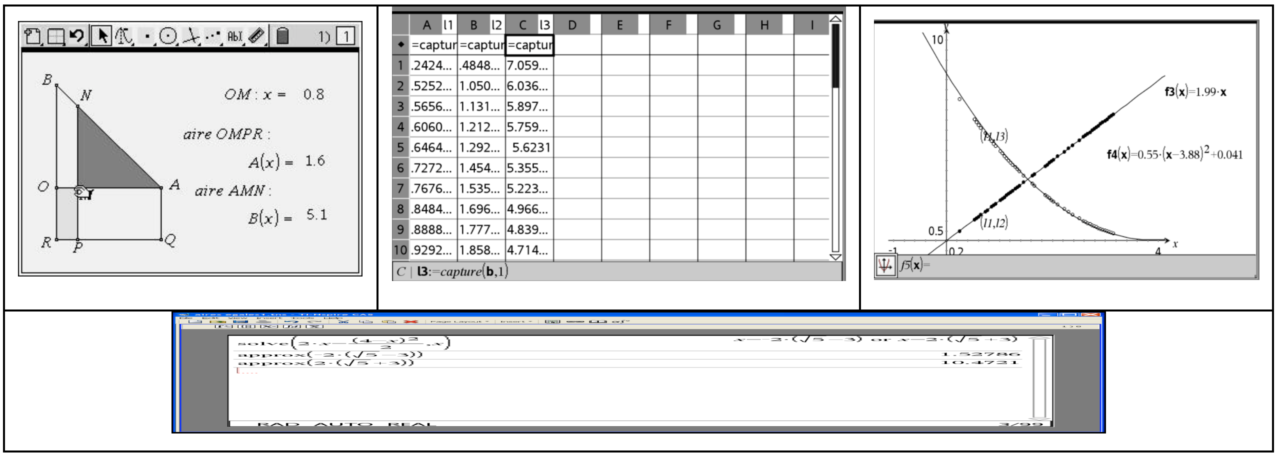


Figure 2. *Exploring progressively the problem of Equal Areas using different applications*

Experimentations led to the development of successive scenarios where more and more autonomy was given to the students for solving this problem; yet they still imposed students to use several applications, to discuss the exact or approximate nature of the solutions obtained, and to judge the global coherence of the work.

MERGING MATHEMATICS AND INSTRUMENT – STUDENTS' VIEWPOINT

Our analysis will rely on the experimentation of the two particular resources already mentioned (*Descartes* and *Equal areas*) for the following reasons: they have been designed with a clear focus on both mathematical and instrumental issues, but take place at different moments of students' learning path and have different mathematical and instrumental aims. *Descartes* has been proposed early in the school year; it aims at introducing the dynamic geometry of TI-*n*spire while revisiting some main geometrical notions of junior high school, and connecting these with numbers and arithmetic operations. *Equal areas* has been proposed several months later, at the end of the teaching of generalities about functions. It aims at the solving of a functional problem from diverse perspectives, and at discussing the coherence of the results that these complementary perspectives provide. It also aims at providing us with some evidence on students' instrumental genesis after 6 months of use of TI-*n*spire.

Students and the *Descartes* resource

Two sessions were used for this resource in the experimentation, and an interesting contrast was observed between the two sessions. The smooth running of the first session evidenced that a first level of instrumentalization of the dynamic geometry of TI-*n*spire was easily achieved in this precise context. The successive difficulties met in the second session revealed both the limits of this first instrumentalization and the tight interaction existing between mathematics and instrumentation. In what concerns the instrumentalization, we could mention for instance students who inadvertently created points that could superimpose on the points of the construction and invalidate measurements; the fact that they could not handle short segments on the calculator, or that they had not understood how to "seize" length variables in the geometry window for computing with them…

Regarding the interaction between mathematics and instrumentation, one difficulty appears to be particularly perceptible in this situation: measures and computations in the geometry application are dealt with in approximate mode. Thus, when testing the validity of the construction proposed by Descartes for the quotient for instance, the students did not get exactly what they expected and were puzzled. Very interesting classroom discussions which attest the intertwining of mathematical and instrumental issues emerged from this situation. Students had limited familiarity with the tool, and had to understand that exact calculations are reserved to the Calculation application. The problem nevertheless was not solved just by giving this technical information, showing that this was not enough for making sense of such information. It also required students to reflect upon the idea of number itself, to be able to discreminate a number from its diverse possible representations, and to consider the notions of exact and approximate calculations.

Students and the *Equal area* resource

As already explained, this resource is quite different from the previous one and students had been using the TI-*n*spire for more than 6 months when it was proposed to them. It has been experimented several times with different scenarii, and the analysis of the data collected is still on-going [2]. Some instrumentalization difficulties were still observed, even when students worked with an improved version of the artifact. These often concerned the spreadsheet application, less used, but the main difficulties tightly intertwined mathematics and instrumental issues as in the previous example. We will illustrate this point by examining the use of spreadsheet for finding and refining intervals that include the solution.

Students used the spreadsheet application after a geometrical exploration of the problem. This prior exploration had convinced them of the existence and uniqueness of the solution, provided an approximate value and showed that the geometrical application could not provide exactly equal values for the two areas. The use of the spreadsheet application generally raised a lot of difficulties linked to the syntax for defining the content of the successive columns, and for refining the step taking into account the existing limitation in the number of lines available. Students often tried to refer to spreadsheet files used in previous problems to solve them. Some could be helpful (another functional problem), some were problematic (a probabilistic situation recently studied). Choosing an appropriate file required to see the similarities and differences between the mathematical problems at stake. Benefiting from an adequate file required to match the two mathematical situations, establishing correspondences between the data and variables involved, and understanding how these reflected in the syntax of the commands. The use of the generated tables, once obtained, also raised many difficulties. Students tried to get the same values for the two areas or to find the closest ones. This was not easy at all, and very few of them were spontaneously able to create a new column for the difference. Moreover, when asked to find an interval for the solution, they were unable to exploit the table in a successful way. The idea that the solution of the problem corresponded to an inversion in the order of the two areas, and that they had thus to look at the two successive lines showing this inversion for getting the limits of the interval asked for was not a natural idea. The screen copies and discussions between students or/and with the teacher of this episode clearly illustrate to what point mathematics and instrumentation are intertwined.

In these two examples, we have focused on the mathematical/instrumental connection through the analysis of students' difficulties but the observations also show episodes where an original mathematical/instrumental synergy is at stake. We will illustrate this by examining the activity of a group of students working on the same problem, but with greater autonomy. This group begun by a geometrical exploration, then defined the two functions expressing the areas and moved to a graphical exploration, selecting an appropriate window for the problem ($0 \leq x \leq 4$). They carried out cleverly this exploration, created the intersection point of the two curves to get its coordinates and found numerical values with only 6 decimals. This fact associated with the visual evidence of the intersection point convinced them that they had got the exact solution. They came back to the geometry page and checked that this solution was coherent with the approximate value with 2 decimals they had already got. They then moved to the calculation application (exact mode) and asked for the solution of the equation. They obtained 2 irrational values and were puzzled. The screen captures show several quick shifts between the graphic and calculation pages, before one of the boys decided to ask for an approximate value of the two solutions. Once obtained, they came back to the graphic page, changed the window so to visualize the 2^{nd} intersection point, seemed satisfied, went back to the geometry page and discarded the 2^{nd} solution as non relevant. Once more, we cannot enter into more details, but the productive interplay is here evident. Let us just add that an interesting collective discussion took then place about the conviction of obtaining an exact solution in the graphic page, and the rationale underlying it. Linked with a deep mathematics discussion, the way TI-*n*spire manages approximations in the different applications and the way the user can pilot the number of decimals was clarified.

For making sense of such synergies and instrumented practices, there is no doubt in our opinion that a semiotic approach that solely takes into account the treatments inside a given semiotic register of representation or conversions between such registers is not fully adequate. What we observe indeed is a sophisticated interplay between different instruments belonging to the students' mathematical working space, and a swing between these certainly supported by technological practices developed out of school. These are efficiently put at the service of mathematical activity, and part of their efficiency also results from their kinesthetic characteristics.

Moreover, when we consider the diversity of perspectives students elaborated for approaching the same mathematical problem, and the small group and collective discussion that rose about the potential and limits of these different perspectives as well as their global coherence, there is not doubt that the work performed by the students in this task corresponds to a quality of mathematical activity hardly observed in most grade 10 classes in France.

CONCLUSION AND PERSPECTIVES

Due to its specific features which distinguish TI-*n*spire from other calculators and as it had been envisaged *a priori,* the introduction of this new tool was not without difficulty. It required considerable initial work from the teachers, both to allow their own rapid familiarisation as well as the students' one but also to actualize the potentials offered by this new tool in mathematics activities. When examining both the design of the resources created by the pilot teachers and the work performed by students, as we have tried to show in this contribution, we seize how delicate and somehow frail the harmony between the mathematical and instrumental activity is, and how the semiotic games underlying it are complex. We also see the impact of new kind of instrumental distance (Haspekian & Artigue, 2007) and closeness that shaped teachers' and students' activities: on the one side, distance with more familiar mathematical tools and especially graphic and even symbolic calculators, on the other side closeness with technological artifacts on offer out of school (computers, IPods, etc…). These characteristics affect differently teachers and students, and differently individuals belonging to the same category according to their personal characteristics and experience. They can have both positive and negative influence on teaching and learning processes and these needs to be better understood. For that purpose, beyond the theoretical constructs we have used in this study, we think interesting to extend the tool/object dialectics (Douady, 1986) to the instrumental component of the activities. By choosing to closely articulate mathematical and instrumental knowledge, the later is inevitably introduced within a specific mathematical context. Reinvesting instrumental knowledge also requires, even implicitly, students to decontextualise and to a certain extend generalize what has been acquired.

NOTES

1. A more general overview of the project as well as other findings can be found elsewhere (see Aldon et al., 2008).

2. More will be accessible at the time of CERME6 we hope in (Hérault, to appear).

3. Some resources can be found at: http://educmath.inrp.fr/Educmath/partenariat/partenariat-inrp-07-08/e-colab/

ACKNOWLEDGEMENTS

We would like to thank all e-CoLab team and in particular D. Baroux, M. C. Combes, F. Hérault and J. Salles who have elaborated the resources mentioned in this text, experimented them in their classroom, and contributed to the analysis developed here.

REFERENCES

Aldon et al. (2008). Nouvel environnement technologique, nouvelles ressources, nouveaux modes de travail : le projet e-CoLab, *Repères IREM* 72, pp. 51-78 and EducMath http://educmath.inrp.fr/Educmath/lectures/dossier_mutualisation/

Arzarello, F. (2007). Mathematical landscapes and their inhabitants: Perceptions, languages, theories. In E. Emborg & M.Niss (Eds), *Proceedings of ICME 10*, pp. 158-181. Copenhaguen: Roskhilde University Press.

Balacheff, N. (1994). Didactique et Intelligence Artificielle, *Recherches en didactique des mathématiques*, 14/1-2, pp. 9-42.

Borba, M.C. & Villareal, M.E. (2004). *Humans-with-Media and the Reorganization of Mathematical Thinking.* New York: Springer.

Bueno-Ravel, L. and Gueudet, G. (2008). Online resources in mathematics: teachers' genesis of use. In Pitta-Pantazi, D. and Philippou, G. *Proceedings of CERME 5*. pp. 1369-1378. Larnaca: University of Cyprus.

Douady, R. (1986). Jeux de cadres et dialectique outil/objet, *Recherches en didactique des mathématiques*, Vol. 7.2, pp. 5-32.

Duval, R. (1995). *Sémiosis et pensée humaine, Registres sémiotiques et apprentissages intellectuels*. Peter Lang SA.

Gélis, J.M., Lagrange, J.B. (2007, to appear). The Casyopée project: computer symbolic computation for students' better access to algebraic notation and rich mathematics. *8th ICTMT*.

Guin, D., Ruthven, K. and Trouche, L. (Eds.) (2004). *The didactical challenge of symbolic calculators: turning a computational device into a mathematical instrument*. New York: Springer.

Haspekian M., Artigue M. (2007). L'intégration de technologies professionnelles à l'enseignement dans une perspective instrumentale : le cas des tableurs. In, Baron, Guin et Trouche (Eds), *Environnements informatisés et ressources numériques pour l'apprentissage*, pp. 37-63. Paris: Hermes.

Nemirovsky, R. & Borba, M. (Eds.) (2004). Bodily Activity and Imagination in Mathematics Learning. *Educational Studies in Mathematics,* 57(3), pp. 303-321.

Rabardel, P. (1997). Activités avec instruments et dynamique cognitive du sujet. In *Outils et Signes* (pp. 35-49). Paris : Editions scientifiques européennes.

Vandebrouck, F. (Ed.) (2008). *La classe de mathématiques : activités des élèves et pratiques des enseignants*. Toulouse: Octares Editions.

DESIGNING A SIMULATOR IN BUILDING TRADES AND USING IT IN VOCATIONAL EDUCATION

Annie Bessot, Colette Laborde

DIAM – LIG, University of Grenoble, France

Abstract. This paper deals with the design, the production and the uses of a simulator for the activity of marking out on building sites from reading a marked plan. The main design principle of this simulator lies in that it is not meant for reproducing accurately the real context of the activity but it should offer the possibility of posing problems of the work situation through a prior conceptual analysis of the professional activity.

What is a reading-marking out activity in a building work? Most of building tasks are based on reading plans for marking out on the building site. We call this kind of tasks, reading-marking out tasks. In a building site, setting out elements takes into account what will be set out later. For example, when a floor is to be laid down, the marking out of the floor must leave holes for water pipes and electric cables. Setting out a wall must plan location for windows and doors by marking out their contour. Such marking out is called “boxing out”. Generally speaking, a boxing out is a formwork placed in the middle of a structure before casting concrete, used to set aside an area in which additional equipment can be added at a later date. This task of reading information from a plan to mark out contours and boxing out on the building site is usual for workers in building trades.

Two types of controls can be distinguished in the marking out of boxing out:

- controls coming from reading information on the plan
- subsequent and effective controls at the moment of putting the additional elements (pragmatic controls),

the first type of controls being oriented towards the second type of controls.

The first type of controls is the focus of our attention. In absence of pragmatic control, only controls guided by knowledge about space and instruments can take place. The activity of setting out boxing out can allow researchers to observe conceptualisation and help them answer questions such as: what is the nature of knowledge involved in this activity? How is such knowledge organized and what relationship does it have with the artefacts available on the building site?

The observation of students of a vocational school gave evidence of a discrepancy between procedures of students and of professionals in this reading-marking out activity on building site from reading a plan. Two types of analysis were carried out in order to better know this discrepancy and to understand the reasons: an analysis of the geometry in action underlying the students’ activity in reading marking out tasks in workshop and an analysis of the transposition of the professional activity in vocational education was needed. The first analysis is presented in Bessot & Laborde (2005). The second analysis focused on the place and status of reading-marking out activities in vocational education, in particular when preparing students to a certification of qualified workers for building trades (in French : Brevet d'Enseignement Professionnel). It was carried out and showed that the reading marking out situation that constitutes an indivisible entity in the professional practice is divided or almost absent from the vocational education institution (Metzler 2006). A simulator is for us a means of designing situations restoring the unity of reading marking out activity in the three teaching places of French vocational education in which knowledge about space is part of the learning aim: in the mathematics teaching (in particular geometry), in the teaching of construction, in the teaching of practice in workshop.

According to a key design choice, the simulator was meant as an *open-ended environment offering the possibility of constructing didactic situations* based on problems previously identified in the analysis of professional situations.

FONDAMENTAL PROBLEMS INVOLVED IN READING-MARKING OUT PROFESSIONAL SITUATIONS

Previous research on different types of space (Bessot & Vérillon 1993, Brousseau 1983, Berthelot & Salin 1992, Samurcay 1984) as well as the analysis of professional practices (Bessot & Laborde 2005) allowed us to identify three types of problems related to the invariants specific to reading-marking out situation. The two first types are related to mesospace, the third type to the instruments of the building site.

The first type of problems is the problem of locating the local space in which marking out takes place within the mesospace of the building site. Two types of space are involved: the local spaces in which marking out the lines is achieved, and the global space of moves that allows the worker to move from one local working space to another one.

Locating the local space requires coordinating three frames of reference (Samurçay ibid.):

- the frame of reference attached to the subject (egocentric reference frame)
- the frame of reference of the lines marked on the building site (allocentric reference frame) to construct from fixed existing objects of the mesospace that may also be lines already marked on the building site
- the frame of reference of the plan that is the dimension system.

The second problem related to mesospace deals with the coordination of local spaces (Brousseau *ibid.*) that may be distant from each other. This coordination is needed in the process of obtaining the expected global set of marked lines of mesospace.

The third problem is related to the use of instruments: transferring measures requires taking into account the features of the instruments.

2. CHOICES FOR SIMULATING MESOSPACE

In order to decouple the problem of local marking out from the one of moving and orienting were created two different windows: the first window allows the worker to have access to various local spaces but never to the entire space; the second one provides access to the visual field of the worker within the global space and his/her move in this global space. In the second window (global space) one can only move, in the first one, one can mark out by means of instruments and one can move without a general view (through the scrolling bars). Here are presented the features of these two windows.

- **Window simulating the local space for marking out**

This window simulating the visual field of the worker with real dimensions 1,50 m by 1,10 m is the screen of the computer providing a representation of the real visual field on a scale of 1 to 5 (Fig. 3).

One can perform measurement and marking out with the simulated instruments (see below). This window is located within the global space for marking out which is not visually totally accessible. One can move in the global space from one local space to another one by using the scrolling bars of the window (Fig. 1) but with only a partial view at each moment making difficult the linking up of local spaces.

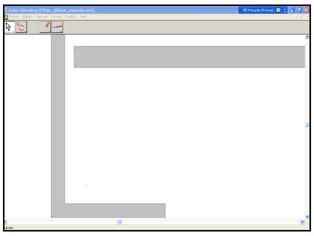


Fig. 1: Window simulating the marking out local space

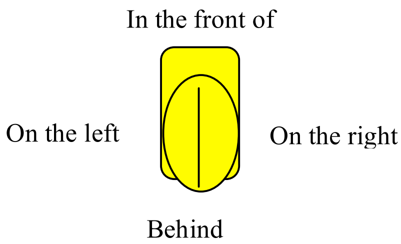
We wanted to simulate the change of viewpoint when the worker is moving away from or closer to the lines marked on the site. Zoom out (Zoom-) and zoom in (Zoom+) possibilities have been set up to simulate these moves, moving away and moving closer. Zoom facilities are limited in order to avoid a global view of the space for marking out. In addition, it is not possible to perform marking out when the zoom tool is active but it is possible to move the instruments. At any time, it is possible to come back to marking out by pressing the key "Zoom 0". This zooming possibility makes easier an accurate reading of the marks of the measuring tape and the move from one marking out local space to another one at a small distance.

- **Window simulating the global space**

In order to locate the current marking out local space within the whole space, it is possible at any time to have access to the simulation of the global space by pressing F9 key. The window global space is simulated by a squared vignette with a 7,5 cm long side representing a real squared space with a 5m long side (Fig. 3 et 4).

When opening the window, a yellow hard hat appears that represents the worker with its visual field represented by a rectangle. This rectangle is the image at scale of the screen (marking out local space). When opening the window, the yellow hard hat is always oriented vertically below the rectangle (Fig. 3, 4 et 7).

It was chosen to simulate the moves of the worker (yellow hard hat) and not its position (Fig. 6 et 7). Two moves are possible: shifts and rotations which are multiples of a quarter turn. Shifts are performed by directly moving the rectangle through the mouse. Rotations are egocentric and are performed by pressing one of the three buttons « > », « < », « □ »: to get the marking out local space on the right of the worker press button « > », on the left of the worker press button « < », behind the worker press button « □ ». When back to the local space (Fig. 6), the worker sees the lines oriented as resulting from the move performed in the global space window. In this way the decision of moving and the effect of the move on the visual field are decoupled. If from the marking out local space one comes back to the global view (F9 key), when opening the window, the yellow hard hat is always below the rectangle representing the local space (Fig. 7). Without a fixed frame of reference, the change of position cannot be inferred from the position of the yellow hard hat with respect to the fixed border of the screen.



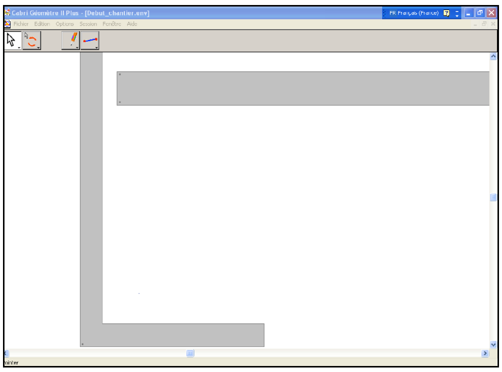


Fig. 2: Window «marking out local space»

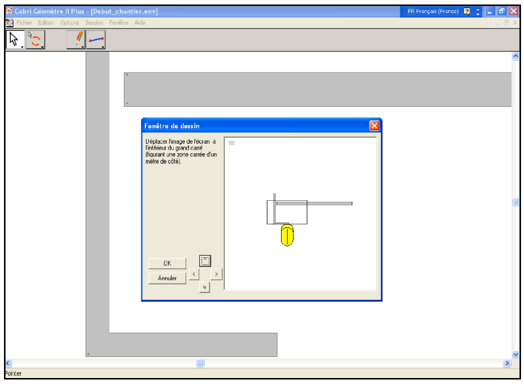


Fig. 3: Window global space in the screen (after pressing F9 key)

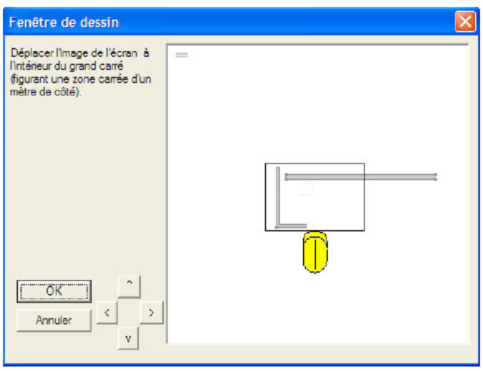


Fig. 4: Local space in the global space window

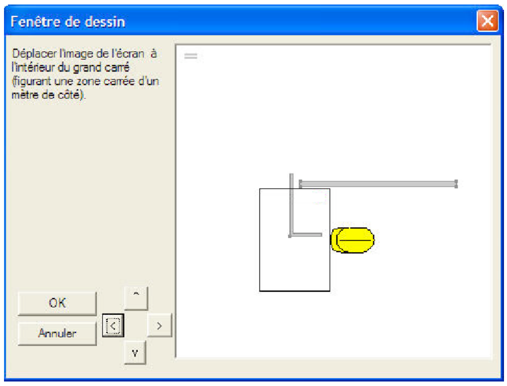
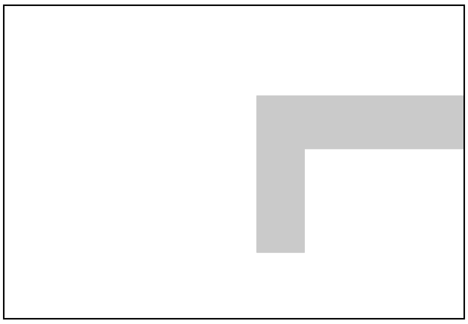


Fig. 5: After pressing button « < »



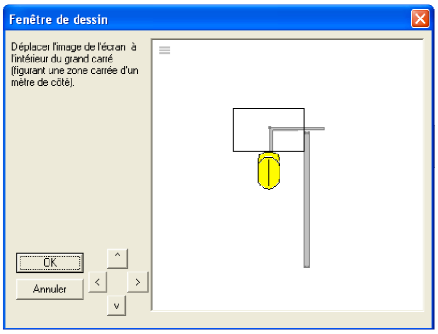


Fig. 6: After pressing « OK » back to local space

Fig. 7: After pressing « F9 » back to global space

3. CHOICES FOR SIMULATING OBJECTS

□ Choices for simulating the prefabrication table

The prefabrication table in which the slab is poured, is simulated by three rectangles with same width 0,05m joined in an U shape: the table is 4m long and 2,5m wide. When opening the simulator, the borders of the table may have various directions with respect the borders of the screen: parallel to the screen borders (see Fig. 8) or not (see Fig. 10). The U shape can be oriented in various directions (see Fig. 8 and 9).

Fig. 8: Prefabrication table parallel to the screen borders

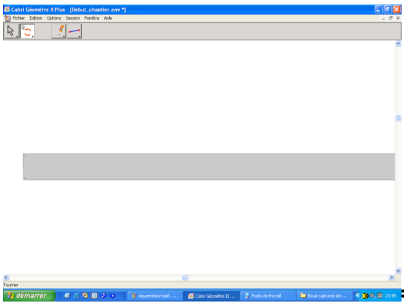
Fig. 9: Prefabrication table parallel to the screen borders, in another orientation

Fig. 10: Prefabrication table non parallel to the screen borders

The table is not totally visible in the local space although as fixed object of this space, it can serve as frame of reference of the mesospace for locating lines in coordination with the plan. The table is only totally visible in the global space window (key F9).

□ Choices for simulating the use of instruments

The choices for simulating instruments deal with their aspect, their accessibility, their moves and their use. We decided that all instruments should look like real instruments. In particular their dimensions are proportional to real dimensions. The 2,5m long ruler and the 3m long tape even partly unwound stick out beyond the visual field (see Fig. 11 and 12).



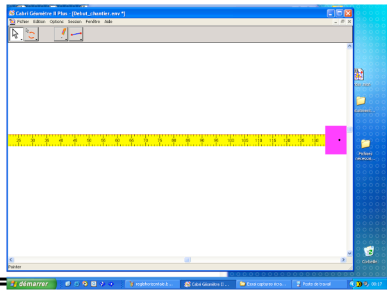


Fig. 11: The ruler cannot be totally seen

Fig. 12: Apart of the measuring tape

Marking out instruments, namely the pen and the blue line are permanently visible as icons at the top of the screen.

Instruments for measuring and transferring geometric properties read from the plan (setsquare, ruler and tape) are put at the beginning in three boxes labelled with their names, which are simulated by rectangles located in a corner of the global space accessible by moving in this space. Once an instrument is out by clicking on its box, the worker may have to move to find it again in his/her visual field (resorting to the global space window or to zoom) and to shift it in the screen (local space) to the adequate location in order to perform a marking out.

The materiality of the instruments was not preserved in that simulated instruments can overlap. However seeking to make the edge of an instrument coinciding with the prefabrication table or with the edge of another instrument partly replaces this materiality. However note that the simulated tape is also retractable as in reality in a pink squared case.

4. CONCLUSION ABOUT THE DESIGN OF THE SIMULATOR

One of the important contributions of simulators lies in the possibility of being freed of the constraints of reality, like the irreversibility of some actions or the time passage.

It is clear that the simulator transforms the relationships of the worker with space. But what is lost in fidelity can be gained in terms of problems and control. Indeed, in the use of the simulator, separating local and global spaces requires from the subject to make the decision of seeking information in the global space. To this end the subject leaves the local space in order to be and move in the global space, and then must come back in order to perform marking out. These conscious back and forth moves do not occur in reality. As a result of this separation, the subject is certainly faced with a coordination problem of frames of reference of the two spaces.

The additional action of back and forth moves between the two spaces is tedious, it transforms the reading marking out strategies and favours predictions to decrease the number of back and forth moves. But it gives rise to observations for the subject and the educator and consequently can become an object of a reflexive work analysing strategies in real and simulated situations.

Another contribution of the simulator is the possibility of controlled variation offered to the educator. The same simulator can give rise to different uses in vocational education. The educator has the command of the type of use and of tasks given to the students. An example of a didactical situation is briefly presented below.

5. EXAMPLE OF A DIDACTICAL SITUATION MAKING USE OF THE SIMULATOR

The situation reported here raises the problem of continuing a marking out already done without transmitting to the worker information on what has been set out. This situation simulates a usual professional problem. Solving this problem requires that the worker identify the local space within the global space by coordinating various frames of reference including the frame of reference of the plan.

- **Instructions of the situation**

The plan of slab 1 with three boxings out.is given (Fig.13) to the students.

1) Open the file “slab 1”
2) As visible, the contour of slab 1 and one boxing out have already been marked.
3) Mark out the two other boxings out of slab 1.

Here below is given the plan of slab 1 provided to the students as well as the windows local space and global space.

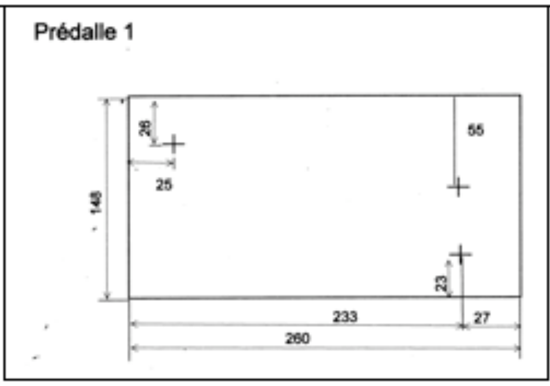


Fig. 13: Plan of slab 1

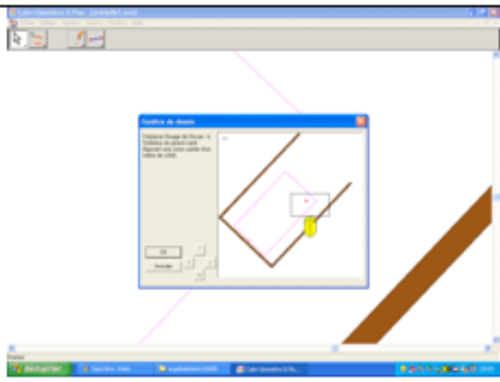


Fig. 14: The two windows

The plan is oriented by the orientation of the writing (from left to right and from top to bottom) and consequently imposes a position for reading. It is represented in this position on Figure 13. When opening the file “slab 1”, part of the prefabrication table, part of the lines and the boxing out R(25, 26) are visible in the local space (Fig.15).

In figure 14, it is visible that the slab is rotated through 180° with respect to the frame of reference of the plan.

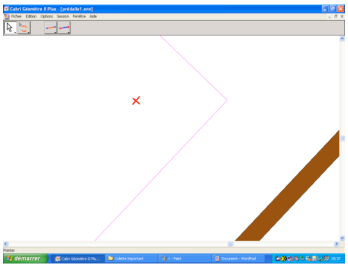


Fig.15: At the opening of file slab 1

A priori analysis of the situation

In the marking out activity, the worker's aim is to reproduce in the mesospace the image of the drawing of the fabrication plan. The continuation of the marking out requires interpreting the boxing out already marked in mesospace as the image of a boxing out of the plan.

Two cases are possible:

- Either the plan and its (*unfinished*) image in the working local space have a similar orientation and the boxing out is erroneously considered as R(27,23)
- Or measures are taken in order to identify the already drawn boxing out with a boxing out of the plan.

The choice of the dimensions of boxings out in slab 1 is deliberate. The distances to the border of the two boxings out R(25 ; 26) and R(27 ; 23) are visually close, favouring thus the mistake of the first case in absence of the professional gesture of taking information on what has already set out.

Incorrect interpretation of the already marked boxing out without measuring : R(27 ; 23)

Two other boxings out must be marked. Here is only considered the case of boxing out R(27 ; 55) as the only one likely to lead to feedback. Two procedures for marking out R(27 ; 55) are possible:

- Either through an alignment with R(27 ; 23) by resorting to the only measure 55 : *no feedback*.
- Or by resorting to two measures 27 and 55 without making use of the alignment. Once the marking out is done, *the absence of alignment of the two marked boxings out provides feedback that leads to reject the interpretation of the existing boxing out a*s R(27 ; 23). This leads to the second case which is analyzed below.

Correct interpretation of the already marked boxing out through checking by measuring: R(25 ; 26)

The coordination between the plan and its unfinished image can be achieved in two ways.

- *Real or mental half turn of the plan of slab 1*

The plan is rotated through 180 ° effectively or in thought to superimpose the image on the screen with the rotated plan: the marking out is performed with a prefabrication table in the position “open on the right, closed on the left”.

- *Move in the mesospace through resorting to the global space window*.

To keep the prefabrication plan in its privileged position and make it coinciding with its image on the screen, it is possible to use F9 key to get access to the global space in order to simulate a half turn in this space: the table is then in the position “open on the left, closed on the right”. When back in the local space, the boxing out already marked is the image of R(25 ; 26). Boxings out can be marked in the same position as they are on the plan.

The situation is aimed to provide multiple opportunities in which checking measures of marked objects in mesospace (prefabrication table, lines) lead to an economy in marking out. Checking is a critical gesture of building trade as claimed by the educators in vocational education.

- ***A posteriori* analysis of the situation**

As displayed in table 1, only 3 pairs out of 5 resort to measuring on the marking out, in order to identify the boxing out.

Interpreting the already marked boxing out	without measuring	with measuring	
	R(27,23) *Pairs 1 and 2*	R(20,21) then R(27,23) *Pair 6*	R(25,26) *Pairs 4 and 5*

Table 1: Checking procedures of already marked boxing out

Let us analyze the checking procedures of the three pairs 4, 5 and 6.

Pair 4 made two checks by measuring the dimensions of the slab and the dimension of the already marked boxing out (26 cm) which is sufficient for identifying the boxing out.

Pair 5 checked only one measure (26 cm) and did a half turn of the plan to make the screen matching the plan.

Pair 6 drew surprising conclusions: the already marked boxing out is first considered as not in the plan, then as the erroneous boxing out R (27, 23). Verbal interactions among students V and N of this pair allow us to understand those successive conclusions. As pairs 1 and 2, V immediately identifies the already marked boxing out as R(27,23). But N insists on measuring. Then he measures one of the dimensions of the boxing out and obtains 20 cm as a result of a wrong use of the measuring tape: the distance is measured by making coinciding the centre of the boxing out with the border of the case of the measuring tape (with width 5 cm in real size). He then measures the second dimension in the same way and obtains 21cm. Surprised not to find any boxing out of the plan, he resumes each measuring twice or three times.

V: it fits nothing. It means that it is already marked, then we must mark out the three others. We make one more, that's it.

N doubts that there can exist 4 boxings out and asks questions about the use of the measuring tape to observer O. He admits that he never used a measuring tape!

N: the end of the tape, is it at the black mark (corresponding to the clip of the real tape) or at the other end?

O: it is at the black mark as on a real tape… do you know, don't you?

N: No, I don't know, I never used a tape.

V: Didn't you?

The doubt about correct using of the tape as well as the cost of its use in the simulator lead them to give up checking the correspondence between measures and dimensions on the plan. They come back to the first opinion of V, i.e. identifying the already marked boxing out as R(27,23).

The simulator made possible to face the students with the usual professional problem of continuing a marking out, which is a fundamental issue of the professional activity, as claimed by the teachers. The simulator revealed that even at the end of the vocational training, almost half the students do not resort to checking and among those who checked, the use of instruments may cause difficulties. This checking professional gesture is not available to all students at the end of the school year.

REFERENCES

Berthelot R. & Salin M.-H. (1992). *L'enseignement de l'espace et de la géométrie dans la scolarité obligatoire*. Thèse de l'université Bordeaux 1.

Bessot, A. & Laborde, C. (2005). Vers une modélisation d'une géométrie en acte dans les activités de lecture-tracé du bâtiment, In C. Castela & C. Houdement (Eds.) *Actes du séminaire national de didactique des mathématiques* (pp. 39-76). Année 2005, Paris: Editions ARDM et IREM de Paris 7.

Bessot, A. & Vérillon, P. (1993). *Espaces graphiques et graphismes d'espaces*. Grenoble: Editions La Pensée Sauvage.

Brousseau, G. (1983). Etude des questions d'enseignement. Un exemple : la géométrie. *Séminaire de didactique des mathématiques et de l'informatique*, n°45, Grenoble: IMAG, 183-226.

Metzler, L. (2006) *Utilisation de Cabri Bâtiment pour poser des problèmes de lecture – tracé dans le bâtiment et bien d'autres encore* ... Mémoire de master 2 EIAHD, University Joseph Fourier.

Samurcay, R. (1984). *La coordination des points de vue dans l'espace chez l'enfant : analyse des référentiels et des calculs spatiaux*. Thèse, Paris, EHESS.

ESTABLISHING DIDACTICAL PRAXEOLOGIES:
TEACHERS USING DIGITAL TOOLS IN UPPER SECONDARY MATHEMATICS CLASSROOMS

Mary Billington

University of Agder

This paper discusses elements of the didactical work of ordinary mathematics teachers using digital tools. The upper secondary school in Norway where the data was collected has run an internal project to integrate the Personal Computer into the mathematics classroom. Using the Instrumental Approach as a framework this paper seeks to describe and interpret elements of teacher practice exploring also the notion of instrumental genesis from a teacher perspective. From the analysis of classroom observations, interviews, meetings, and study of documents three main didactical practices were found to be linked to the introduction of the digital tools: the digital notebook, the digital textbook, and the phenomenon of weaving between tools/instruments in the classroom.

INTRODUCTION

The recent school reform in Norway, Knowledge Promotion 2006, formally acknowledges digital competence as one of the five basic skills students should acquire and develop in their formal schooling[5]. This places on schools and individual teachers a responsibility to integrate these tools into classroom practice. This study looks at the practice of two teachers in a comprehensive upper secondary school in Norway who have been using digital tools over a period of five years. In 2007 the school joined the project "Learning Better Mathematics", hereafter LBM[6], a developmental project initiated by school authorities through a co-operation with University of Agder. Data used in this paper was collected at the school's point of entry to the project. The classrooms observed were equipped with a blackboard and a projector with screen and set up as "paperless" environments where all students had their own laptop PC and when observed rarely used paper and pencil in their mathematics lessons: all student work was done on the computer.

THEORETICAL FRAMEWORK

The theoretical approach employed emerged in the mid-nineties in France when researchers became aware that traditional constructivist frameworks were inadequate in the analysis of CAS environments (Artigue, 2002). Artigue claims that this

---

[5] Knowledge Promotion (Kunnskapsløftet 2006). These basic skills are given as the ability: to express oneself orally to read, to do arithmetic, to express oneself in writing, to make use of information and communication technology

[6] The project is supported by the Research Council of Norway

approach is less student centred but provides a wider systemic view also giving the instrumental dimension of teaching and learning more focus (Artigue, 2007). The approach uses notions both from the Theory of Instrumentation from the field of Cognitive Ergonomy, and from the Anthropological Theory of Didactics (ATD hereafter) in the field of Mathematics Education (Laborde, 2007).

Cognitive Ergonomy considers all situations where human activity is instrumented by some sort of technology. The theory of instrumentation employs the notion of "instrument" and the notion of "instrumental genesis" (Artigue, 2002). The instrument has a mixed identity, made up of part artefact and part cognitive scheme. It is seen as a mediator between subject and object but also as made up of both psychological structures, called schema which organise the activity, and physical artefact structures such as pencil, paper, or digital tools (Béguin & Rabardel, 2000). For the individual user, the artefact becomes an instrument through a process of instrumental genesis which involves the construction of personal schema or the appropriation of socially pre-existing schemes (Artigue, 2002). This process of instrumental genesis has two elements, *instrumentalisation* the process whereby the user acts on the tool shaping and personalising the tool, and *instrumentation* the process whereby the tool acts on the user shaping the psychological schema (Rabardel, 2003). Instrumental genesis is a process occurring through the user's activity through participation at the social plane. Guin and Trouche (1999) applied the Theory of Instrumentation in research in mathematics classrooms, studying the process by which the graphic calculator becomes an instrument for the students to learn mathematics. They term the teachers' role in guiding the students' instrumental genesis *instrumental orchestration*. This is defined as a plan of action having four components: a set of individuals, a set of objectives, a didactic configuration and a set of exploitation of this configuration (Guin & Trouche, 2002, p. 208).

ATD on the other hand aims at the construction of models of mathematical activity to study phenomena related to the diffusion of mathematics in social institutions, see for example (Barbé, Bosch, Espinoza, & Gascón, 2005). The theory analyses human action including mathematical activity by studying *praxeologies*:

> But what I shall call a praxeology is, in some way, the basic unit into which one can analyse human action at large. (Chevallard, 2005, p. 23)

Any human praxeology is constituted of a practical element (praxis) and a theoretical element (logos). The praxis has two components, the task and the technique to solve the task. The logos also has two components, the technology (or discourse) and the theory which provide a justification for the praxis.

Mathematical knowledge in an educational institution can be described in terms of two types of praxeologies: mathematical praxelologies and didactical praxeologies. The object of the didactical praxeologies is the setting up of and construction of the the mathematical praxeologies. It is these didactic praxeologies, representing teacher practice, that are of interest to me in my study. Questions arising are: What constitutes or defines the didactical task, technique, discourse and theory? How are the mathematical praxeology and the didactical praxeology entwined? How do the existing didactical praxeologies change when digital tools are introduced into the mathematics classroom? Laborde's conclusion that, "A tool is not transparent. It affects the way a user solves a task and thinks" (Laborde, 2007, p. 142) should apply equally to both teacher and student.

Research indicates that the interventions of the teacher are critical in relation to student learning of mathematical knowledge when digital tools are introduced (Guin & Trouche, 1999). The teacher's instrumental orchestration is part of the didactical praxeology. As new tools are introduced, the teacher must develop new didactical praxeologies to support the students' instrumental genesis for the particular tool (Trouche, 2004, p. 296). The teacher must also incorporate the new tool into an existing repertoire of tools and didactical techniques. Practically in the classroom, this involves for the teacher: (1) Organisation of space and time, (2) the choosing of the mathematical tasks and the techniques to solve these tasks, and (3) the steering of the mathematical activity in the classroom by discourse.

Aim and research question

This paper aims to identify features of didactical praxeologies that have been established in relation to the introduction of the digital tool and also to describe the process of introduction of the digital tool and changes to practice from the teacher perspective. The research questions are: What features of the teachers' didactical praxeologies can be identified as pertaining to/originating specifically from the introduction and use of the digital tool? Can these features be seen as evidence of a process of instrumental genesis for the teachers in relation to the digital tool? What factors influence this process?

This short paper allows for in depth discussion of only some of the features indicated above. I have therefore selected features that appear to be of significant importance to the teachers when they describe the changing practice in relation to the tool. The paper also seeks to describe only commonalities in teacher practice.

THE EMPIRICAL STUDY

The teachers, their classes and classrooms

The two teachers in this study very generously opened their classrooms and gave of their time to this researcher. Both were active in initiating the ICT project at the school. The ICT project had been established and operated entirely within the school and was not part of any external research, design or development project. It is therefore claimed that it is the practice of two "ordinary" teachers that is described in this paper. In 2005, the school was the only school in the country to conduct final examinations in mathematics entirely on the portable PC.

This part of the study involved classroom visits to two classes of approximately twenty five students. The students were studying the subject "Theoretical

Mathematics 1" (1T), which is allocated three double lessons a week, each of 90 minutes duration. These two classes were two of five classes at the school studying this subject. Each classroom was equipped with a blackboard and a projector with screen. The screen covered part of the blackboard but it was still possible to use the blackboard. The technical features of the environment functioned without difficulties in the observation period. The classrooms observed presented as "paperless" environments as all students had their own laptop PC, leased from the school, and when observed rarely used paper and pencil in their mathematics lessons though this was permitted. All student work including exercises, notes, rough work was done on the computer. I have chosen to refer to this practice as the "digital notebook". Standard paper textbooks were no longer in use as the teachers have developed their own digital textbooks, which are made available to the students through a Learning Management System (LMS). This practice I refer to as the "digital textbook". The classrooms appeared very orderly as there were no books, papers, rulers or other items littering the desks. Each student had a PC and perhaps a bag placed on the floor under the desk. The students started work quickly plugging in and turning on the PC, contrasting sharply with "normal" classrooms where students take some minutes to find notebooks, textbooks, pencils and so on. In the observed lessons only the teachers used the projector. Student work was not displayed using the projector.

Data collection and analysis

Data collection over a period of four months involved: audio recording of an introductory meeting between the school and the university where the two teachers, a school leader, two researchers and a project leader from the university were present; lesson observation with video recording of eight lessons; audio recording of three semi-structured interviews before and after lessons with the teachers; audio-recording of seven structured interviews with students (Billington, 2008); and audio data from LBM project meetings where the teachers were present and took part in discussions. The writer was present at all events, taking field notes. In the classroom observations, researchers were present as observers, taking no active part in the planning or carrying through of the lesson. Shortly after each event a preliminary data reduction using the notes and recordings was made. Passages were also transcribed. Later all data was again reviewed, coded and further transcribed. Each data episode renders different information helping to build a picture of teacher practice identifying didactical praxeologies that would not be there without the digital tool. The meetings and interviews tell of the temporal dimension and of the changing nature of the didactical praxeologies from the teacher perspective and also reveal the institutional influences. Classroom episodes record teacher activity in the classroom revealing techniques of instrumental orchestration. Student interviews tell of the students' instrumental genesis and the teachers' orchestration from the students' perspective.

Analysis of data from meetings and interviews

The teachers were very keen to discuss the introduction of the digital tool and there were clear indications in the data that the teachers saw a process of development in their teaching practice. Examples of such comments were as follows:

> Teacher 1: … and it, it has been, been of course, a long process to come this far, this software …
>
> Teacher 2: But …there is, as such, a remarkable difference from when we started, now…

Reviewing the data from the meetings and interviews, reoccurring themes emerged. These were first categorised under three headings, justification, implementation and evaluation. I then attempted to interpret these themes in the light of the theory as presented in the table below. In a didactical praxeology, implementation would pertain to the praxis while justification and evaluation would pertain to the logos.

Justification	**Implementation**	**Evaluation**
Teachers explained why "we do what we do and continue to do what we do "	Teachers explained how they organised and carried out the project	Teachers talked about what they identified as affordances and constraints of the tool
Didactical theory – justification of practice	**Didactical tasks and techniques**	**Didactical technology (discourse) – relating theory to tasks and techniques**

Table 1: Interview Themes

The most common reoccurring themes under implementation were: the digital textbook, the digital notebook and teaching techniques in the classroom. There was also some discussion input from a school leader, which is relevant to the discussion on orchestration.

Results and discussion of data from meetings and interviews

As stated above, the teachers referred constantly to the introduction of the digital textbook and the digital notebook. Discussion of these two innovative features of the implementation occupied much of meeting and interview time. The teachers referred to the digital textbook as "Learning Book"[7]. This digital textbook has replaced the usual paper textbook that students would normally buy. It is made available through the functioning LMS. Commenting on the digital textbook, the teachers explained that as the project progressed they found that the students preferred to read the notes that they had made rather than read the paper textbook. As a new syllabus came into force this year they decided to make their own digital textbook from scratch.

> Teacher 1: Yes. Totally from scratch, just from the syllabus. Not from any textbook ….We have taken the syllabus point by point …

[7] Here literally translated from the Norwegian "læringsbok"

> Teacher 2: Now we use the syllabus, and it has been extremely useful to go thoroughly into the plans and now we have to make the right choices … we feel we have to make a good deal of choices … that we make for the students …

The teachers have been provoked to return to the mathematical goals in the syllabus and build from these. This development is in line with that described by Monaghan (2004). The students save this textbook to their own PC and can write in memos, and notes. All problems and exercises are also made available through the LMS for the students. According to the teachers giving out solutions on the LMS saves time that can be used to other things, for example, "we can go around and help". The students also retain these files from year to year whereas previously they sold the textbook at the end of the year. In terms of the theoretical framework of ATD this could be interpreted as a transposition of mathematical knowledge (Balacheff & Kaput, 1996) from the syllabus to a form usable on the PC.

The second innovation, the digital notebook, a notebook kept by each individual student where s/he writes and stores all notes, exercises, and rough work on his/her own PC, was also clearly important to the teachers. In fact one teacher gave this aspect some credit for the increased enrolment of girls in these maths courses.

> Teacher 1: ...and they (girls) sat on the fence for a year or so. And then a few girls signalled to the others, see here, and then the girls joined in force, ….That was when the girls saw that this was not about playing games, but this was a way to make it very nice. They got everything very systematic, got a way to keep all their notes in order, and very, very nice presentation, and this, the girls thought was very ok, and the boys too, now they have all their notes from last year and can build on this.

Choosing supporting materials for the student is a didactical task for the teacher. In this case the production of a digital textbook and the promotion of a digital notebook are clearly identifiable as innovations in relation to normal practice and could be interpreted as an instance of instrumentalisation where the user shapes the tool to his/her purpose. Data from the student interviews confirmed that these two innovations were important in the students' instrumental genesis (Billington, 2008).

This leads to the reoccurring third theme in the meetings and interviews: reflection over teaching practice in the classroom. The teachers expanded on the teaching philosophy on which they have based the project claiming that they tried to avoid the standard structure of theory, example, exercises, and method.

> Teacher 2: We have had a main principle since we started with this. These textbooks are always alike, theory, examples, and then exercises exactly like the example, and then examples that are almost the same. As far as possible we try to avoid this. Our philosophy is fewer exercises and they can rather sit and struggle with the same exercise and if it takes the whole lesson that does not matter.

Interestingly the teachers did not expose on the wonders of the digital tool per se, but rather talked of the teaching possibilities with the tool as illustrated by these quotes.

> Teacher 1: I have much more influence on my own teaching before...
>
> Teacher 1: The role of the teacher is a bit …you have greater possibilities, that is what we have seen …
>
> Teacher 2: But, I must say, for my sake, that I have opportunities that I would never had had without the PC.

These possibilities can be interpreted as new didactical techniques. One teacher claimed that his teaching had changed since the students have now chosen not to use the standard paper textbooks. They discussed the need to focus on understanding rather than the reproduction of algorithms. They saw the creation of the digital textbook as allowing them more freedom to steer the activities of the classroom in line with their philosophy. These reflections I interpret as discourse justifying the praxis element of the didactical praxeologies.

Choosing for students the mathematical tasks, and the techniques and tools to solve these tasks, is a didactical task for the teacher. These tools include the textbook as well as the digital tools, the software and the hardware. The nature of this didactical task has changed for these teachers in the course of the project. They have explained how previously they just followed the book, a routine, but now because of the new situation they have been forced to make new choices. They now worked together to select mathematical tasks themselves rather than following a set up in a book.

Analysis of data from classroom observation

In looking at the data from classroom observations I attempted to identify didactical praxeologies that were a result of the introduction of the digital tool. In the classroom observation data I looked at the teachers' (1) Organisation of space and time, (2) Choice of mathematical tasks and mathematical techniques and physical tools, and (3) Steering of activity through discourse, considering these to be three practical moments of the didactical praxeologies.

In the lessons observed, neither the organisation of space or time nor the choice of mathematical tasks seemed to be dependent on or unique for a classroom where the digital tool of the PC has been introduced. For example, analysis of the time disposition in lessons showed a script with recapping, homework correction, new theory, and then exercises with approximately 50 – 60 % of the lesson time spent with students working alone or in pairs on exercises. Some time however was given to the explanation of the technical aspects of performing the mathematical techniques with the digital tool. This time allocation varied from lesson to lesson.

Deviation from a standard classroom environment without digital tools was observed in the type of tools used by the students and by the teacher and also in the public discourse of the teacher. Choosing the tools for use in the lesson, for the teacher and for the students to carry out mathematical tasks is a didactical task. This is an ongoing task as choices are made in the planning but also in the conduct of the lesson. Two aspects that stood out in the observations were the manner in which the teacher used both the digital tool and the blackboard to support his/her public discourse and the manner, which the teacher referred to and talked about using the digital tool when describing the mathematical techniques to solve the mathematical tasks. This second aspect involves a too broad discussion to take in this paper but will be discussed in the thesis of which this work forms a part.

Results and discussion of classroom observation

In the classroom observations the teachers used both the blackboard and the screen, which was connected to the PC to support their public discourse. One feature that emerged frequently in each observed lesson, I term "weaving". Weaving describes the manner in which the teacher moved between the available tools. Three physical tools were noted to be in use when the teacher was holding public discourse: the blackboard, the PC+screen, and gestures with own body such as tracing out a curve in the air. Each of these tools is used in conjunction with the voice and schemas (cognitive apparatus). It appeared that in prepared sequences of the lessons the digital tool was used but in spontaneous situations, for example when pressed for further explanation, the teacher turned to the blackboard or to gestures.

Discussing this weaving with the teachers, one teacher explained, that "we use what is appropriate in the situation". Teachers seemed to identify affordances and constraints of each tool. It appeared that an affordance of the blackboard was that it allowed more personal and spontaneous expression by the teacher. It may also be the case that such unplanned use of the digital tool requires a high level of skill and familiarity with the tool and as such this is a constraint of the tool. In a later instance one teacher began to draw a circle on the blackboard freehand but suddenly stopped saying; "I have an excellent tool to do this", and then drew the circle using the dynamic geometry software on PC screen instead. Also the mathematical tasks in use were standard tasks, which could be solved without the digital tool. Had these tasks been more complex or tasks that required the use of digital tools perhaps the response of the teacher would have been different.

CONCLUSIONS AND FURTHER DISCUSSION

Returning to the research questions, three features of the didactical praxeologies as specifically pertaining to and "provoked" by the introduction of the digital tool have been identified and discussed: the digital notebook, the digital textbook, and the phenomena of weaving between tools/instruments in the classroom. The two features that are seen as particularly important by the teachers are the digital textbook and the digital notebook. These could be interpreted as examples of instrumentalisation whereby the teacher as user has adapted the tool to his/her usage. In the classroom, the observation of patterns of inter-dependent mediation between physical tools that have been adapted by the teachers, where they weave between blackboard and the digital tool in response to the situation, could be interpreted as observations of schema or expression of instrumentation as in these cases the tool which is thought to be the most appropriate is used.

Can the project implementation described above be modelled as a process of instrumental genesis for the teachers and is such a modelling helpful in gaining an understanding of the situation? Further examination of teacher discourse will provide more information about this possible instrumental genesis process though tentative findings in this report seem to lead in this direction. Some issues to be discussed in relation to such a process are for example: the temporal dimension; if instrumental genesis is a process how is it possible to identify the different stages of this process for the teacher; and also as to which observations would indicate the formation of schema. The notion of instrumental orchestration has been discussed earlier. Is the process of instrumental genesis for the teacher also influenced by some constraining factors? Comments by the teachers indicated that, for the teachers, the process is steered in part on an organisational level by the schooling authorities at school, region, and national levels. Financial and policy support from schooling authorities is necessary for the survival of the ICT project. In the meetings, the school leader was highly supportive of the project and expressed the opinion that when students think it (mathematics) is fun, then they use more time on mathematics and so become better at it. Enrolment in mathematics has also increased dramatically. However, more important to the teachers seemed to be the response of the students. In the categories of justification and evaluation the majority of comments by the teachers concerned student learning and engagement as illustrated by the comment below.

> Teacher 1: Need to give students a challenge. Students are not educated to work in this way. Now they think it is fun. Looking for methods …

For the teachers in this study, the students' response to the new situation appears to influence the teachers' use and adaptation of the digital tool. Such comments as above also indicate that the teachers are aware of their role in orchestrating their teaching to support the instrumental genesis of the student.

REFERENCES

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. International Journal of Computers for Mathematical Learning, 7(3), 245-274.

Artigue, M. (2007). Digital technologies: A window on theoretical issues in mathematics education. In D. Pitta-Pantazi & G. Philippou (Eds.), European research in mathematics education V. Proceedings of CERME 5 (pp. 68-82).

Balacheff, N., & Kaput, J. (1996). Computer-based learning environments in mathematics. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), International handbook of mathematics education (pp. 469-501). Dordrecht: Kluwer Academic Publishers.

Barbé, J., Bosch, M., Espinoza, L., & Gascón, J. (2005). Didactic restrictions on the teacher's practice: The case of limits of functions in Spanish high schools.. Educational Studies in Mathematics, 59(1), 235-268.

Béguin, P., & Rabardel, P. (2000). Designing for instrument mediated activity Scandinavian Journal of Information Systems, 12(173-190).

Billington, M. (2008). An interview technique to explore the process of instrumental genesis. Paper presented at the NORMA08, Copenhagen. To appear in the proceedings.

Chevallard, Y. (2005). Steps towards a new epistemology in mathematics education In M. Bosch (Ed.), European research in mathematics education IV. Proceedings of CERME4 (pp. 21-30): Fundemi -IQS, Universitat Ramon Llull

Guin, D., & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: The case of calculators. International Journal of Computers for Mathematical Learning, 3(3), 195-227.

Guin, D., & Trouche, L. (2002). Mastering by the teacher of the instrumental genesis in CAS environments: necessity of instrumental orchestrations. ZDM, 34(5), 204-212.

Laborde, C. (2007). Towards theoretical foundations of mathematics education. ZDM, 39(1), 137-144.

Monaghan, J. (2004). Teachers' activities in technology-based mathematics lessons. International Journal of Computers for Mathematical Learning, 9(3), 327-357.

Rabardel, P. (2003). From artefact to instrument. Interacting with Computers, 15(5), 641-645.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. International Journal of Computers for Mathematical Learning, 9(3), 281-307.

METHODS AND TOOLS TO FACE RESEARCH FRAGMENTATION IN TECHNOLOGY ENHANCHED MATHEMATICS EDUCATION

Bottino Rosa Maria & Cerulli Michele

Institute for Educational Technology, C. N. R. of Genova, Italy

This paper, at large, addresses the issue of how to successfully bring into school practices the results in technology enhanced mathematics learning obtained at research level. The distance among different research teams and between researchers and teachers is addressed in terms of fragmentation of the research field. A methodology is presented to reduce such fragmentation illustrating a pathway followed at the European level in the EC co-funded projects TELMA and ReMath.

INTRODUCTION

In the last CERME conference (Cyprus, February 2007) two plenary sessions (Ruthven, 2007; Artigue, 2007), drawing from the discussions developed in different working groups, highlighted key issues concerning Technology Enhanced Learning (TEL) in mathematics.

According to Ruthven (ibid pg. 52), despite of a generalized advocacy for new technologies in education, these have had a limited success in school. As a matter of fact, he observes that, even if technologies had some positive impact on the instruction of teachers, they remain marginal in classroom practice. This is true, in particular, for mathematics, even if, from the beginning, a wide number of researchers have been concerned with the study of the opportunities brought about by new technologies to the teaching and learning of this discipline (Lagrange, Artigue, Laborde, & Trouche, 2003). As a matter of fact, despite the positive results produced in a number of experimental settings and the budget invested by many governments for equipping schools, actual use of ICT tools in real school environments is still having a limited impact. Recent studies witness difficulties encountered by teachers in implementing teaching and learning activities mediated by technologies due to variables such as working environment, resource system, activity format, curriculum script and time economy (Cuban, 2001; Sutherland, 2004). The coordination of such variables is necessary in order to develop a coherent use of technological tools and to form an effective system. According to Ruthven (ibid pg. 64), this challenge "involves moving from idealised aspiration to effective realisation through the development of practical theories and craft knowledge". Drawing from our own experience, we identify as a crucial issue the necessity to establish effective interactions among the different actors involved in the process, that is researchers, teachers, policy makers, curriculum developers, software designers, etc.

Such a view is coherent with what is reported in (Pratt, Winters, Cerulli & Leemkuil, in press) from the perspective of educational technology designers. Authors, making reference to the specific field of games for mathematics education, speak of the necessity of a multi-disciplinary approach to design and deployment of technologies as opposed to the frequently experienced design fragmentation. Such fragmentation is often due to the fact that the different communities involved are not fully cognisant of the structuring forces that impinge on each other's activities. From one hand, discontinuities between design and deployment of technological tools impede the effective use of such tools in school practice and, on the other hand, the development of isolated projects that often do not go beyond experimental settings, do not contribute to cumulative knowledge about the design process that could inform future work. Pratt et al. advocate the need to integrate key stakeholders in the creation of technology enhanced learning tools, as "the problem of design fragmentation remains a real impediment to widespread innovation in the field". They thus state the opportunity of creating multidisciplinary teams focusing on the design and deployment of educational technology that bring together the perspective of different stakeholders: designers, educators, researchers, etc.

Fragmentation, however, is not only a problem experienced among different communities of stakeholders, but it is a problem often experienced also within each community. In particular, as highlighted by Artigue (2007) during her plenary speech at CERME 6, this is one of the key issues of concern within the community of the researchers in mathematics education, and, in particular, within the community of researchers focusing on technology enhanced learning in mathematics. Such a fragmentation is rooted at theoretical level, as witnessed also by the work of the working group 11 of ERME that has been established to discuss such specific issue (Prediger, Arzarello, Bosch & Lenfant, 2008). As a matter of fact the theoretical background of a research team has an important bearing on the epistemological assumptions, the research methodologies, the way in which tools, and, in particular, technology enhanced tools, are perceived and used.

At the European level, where a great variety of different approaches and background is present, there is a specific sensibility to the problem of fragmentation and to the necessity to find feasible ways to overcome it, since, as observed in (Arzarello, Bosch, Gascón & Sabena, 2008) a too wide variety of poorly connected conceptual and methodological tools does not encourage consideration of the results obtained as convincing and valuable. Moreover, in the specific area of TEL, there is the need of designing and implementing tools and methodologies that have a wide scope of application and that are not restricted to a particular community or context. For these reasons, following the impulse given by projects funded by the European Community, efforts have been made to try to overcome such fragmentation.

Our Institute, together with other research teams, has been involved in European research projects concerned with Information Society Technologies (IST) for several years and, in particular in Networks of Excellence (NoE) and Specific Targeted Research Projects (STREPs). These are two instruments of the European Community 6^{th} and 7^{th} Research Framework Programmes that aim at promoting research integration and collaboration in several fields including technology enhanced learning (more information can be found, for example, at the following URL: http://cordis.europa.eu/fp7/home_en.html, accessed September 2008).

This paper presents some methods and tools, developed within the context of such European projects, which have been developed and tested to address the fragmentation issues discussed above.

Firstly we report on the work performed within the TELMA (Technology Enhanced Learning in Mathematics) initiative that explored the conditions for sharing experience and knowledge among different research teams interested in analysing mathematics learning environments integrating technologies, in spite of the differences in the theoretical frameworks and in the methodological approaches adopted. For this purpose, the notions of "*didactical functionality*" (Cerulli, Pedemonte & Robotti, 2006) and of "*key concerns*" - issues functionally important (see Artigue, Haspékian, Cazes, Bottino, Cerulli, Kynigos, Lagrange & Mariotti, 2006) - together with a methodology based on the idea of a "*cross experiments*" (Artigue, Bottino, Cerulli, Georget, Mariotti, Maracci, Pedemonte, Robotti & Trgalova, 2007; Bottino, Artigue & Noss, 2008) were defined and conceptualized as concrete methods to address the problem of fragmentation.

Secondly, we give account of some of the outcomes of the ReMath project that, building on the results of the TELMA project, has addressed the fragmentation problem from the perspective of the design, implementation, and in-depth experimentation of ICT-based interactive learning environments for mathematics, thus involving not only researchers but teachers and technology designers as well. In particular, within the ReMath project, the problem of how to effectively support collaboration in pedagogical planning has been faced. Efforts have been made to provide a solid basis for accommodating the different perspectives adopted, for analysing the factors at play, and also for understanding the initial assumptions and theoretical frameworks embraced. A web-based system, the Pedagogical Plan Manager (PPM), was developed to support researchers, tool designers and teachers to jointly design and/or deploy mathematics pedagogical plans involving the use of technological tools (Bottino, Earp, Olimpo, Ott, Pozzi & Tavella, 2008).

Summing up, in the following sections, we delineate the process that has brought us to afford the problem of the fragmentation of approaches and frameworks, in the field of mathematics teaching and learning mediated by technologies, from different but complementary perspectives.

A COLLABORATIVE METHODOLOGY FOR NETWORKING RESEARCH TEAMS IN TECHNOLOGY ENHANCED LEARNING IN MATHEMATICS

NoEs have been established by the European Commission within the last Framework Research Programmes as instruments to promote integration and collaborative work of key European research teams and stakeholders in given fields. In particular, the network of Excellence Kaleidoscope was established and funded with the aim of shaping the scientific evolution of technology enhanced learning (see also: http://www.noe-kaleidoscope.org, accessed September 2008). Since each knowledge domain raises specific issues either for learning or for the design of learning environments, within Kaleidoscope a number of different joint research initiatives, covering a wide range of domains, have been carried out. Among these, TELMA initiative was specifically focused on Technology Enhanced Learning in Mathematics. It involved six European teams[1] and had as its main aim that of building a shared view of key research topics in the area of digital technologies and mathematics education, proposing related research activities, and developing common research methodologies.

In TELMA, each team brought to the project particular focuses and theoretical frameworks, adopted and developed over a period of time. Most of these teams have also designed, implemented and experimented, in different classroom settings, computer-based systems for supporting teaching and learning processes in mathematics. It was clear from the beginning that, to connect the work of groups that have different traditions and frameworks it was necessary to develop a better mutual understanding and to find some common perspectives from which to look at the different approaches adopted. It was also necessary to develop a common language since the same words were sometime used with different meanings by each team, causing misunderstanding and hindering productive collaboration. Moreover, it became clear that the theoretical assumptions made by each team, were often implicit and thus not accessible to the others.

The notion of didactical functionalities

In order to overcome these difficulties it was decided to focus the work of TELMA on the theoretical frameworks within which the different research teams face research in mathematics education with technology. A first level of integration has been then pursued through the definition of the notion of *didactical functionality* for interpreting and comparing different research studies (Cerulli, Pedemonte & Robotti, 2006). Such notion has been used as a way to develop a common perspective among teams linking theoretical reflections to the real tasks that one has to face when designing or analysing effective uses of digital technologies in given contexts. The notion of didactical functionality is structured by three inter-related components:

- a set of features/characteristics of the considered ICT-tool;
- an educational aim;
- the modalities of employing the ICT-tool in a teaching/learning process to achieve the chosen educational aim.

The different didactical functionalities designed and experimented by each team have been compared trying to delineate how different theoretical backgrounds can influence the design of an ICT-based tool, the definition of the educational goals to be pursued, and the modalities of use of the tool to achieve such goals. At the beginning, this analysis was conducted on the basis of a selection of papers published by each team. This approach, even if useful, was considered not sufficient to enter the less explicit aspects of the research work of each team. For this reason, TELMA researchers decided to move toward a strategy that could allow them to gain more intimate insights into their respective research and design practices. This strategy is based on the idea of 'cross-experiments' and on the development of a methodological tool for systematic exploration of the role played by theoretical frames.

The cross-experiments methodology

The idea of cross-experiments was developed in order to provide a systematic way of gaining insight into theoretical and methodological similarities and differences in the work of the various TELMA teams. This is a new approach to collaboration that seeks to facilitate common understanding across teams with diverse practices and cultures and to elaborate integrated views that transcend individual team cultures. There are two principal characteristics of the cross-experiments project implemented within TELMA that distinguish it from other forms of collaborative research:

- the design and implementation by each research team of a teaching experiment making use of a ICT-based tool developed by one of the other team involved;
- the joint construction of a common set of questions to be answered by each team in order to frame the process of cross-team communication.

Each team was asked to select an ICT-tool among those developed by the other TELMA teams (Figure 1). This decision was expected to induce deep exchanges between the teams and to make visible the influence of theoretical frames through comparison of the didactical functionalities developed by the designers of given tools and those implemented by the teams experimenting the tools. Moreover, in order to facilitate the comparison between the different experimental settings, it was also agreed to address common knowledge domains (fractions and introduction to algebra), to carry out the teaching experiments with students between the 5th to 8th grade, and to perform them for about the same amount of time (one month).

Guidelines (Cerulli, Pedemonte & Robotti, 2007) were collectively built for monitoring the whole process: from the design and the a priori analysis of the experiments to their implementation, the collection of data and the a posteriori analysis. Beyond that, reflective interviews (using the technique of "interview for explicitation" (Vermesch & Maurel, 1997)) were a-posteriori organized in order to make clear the exact role that theoretical frames and contextual characteristics had played in the different phases of the experimental work, either explicitly or in a more naturalized and implicit way.

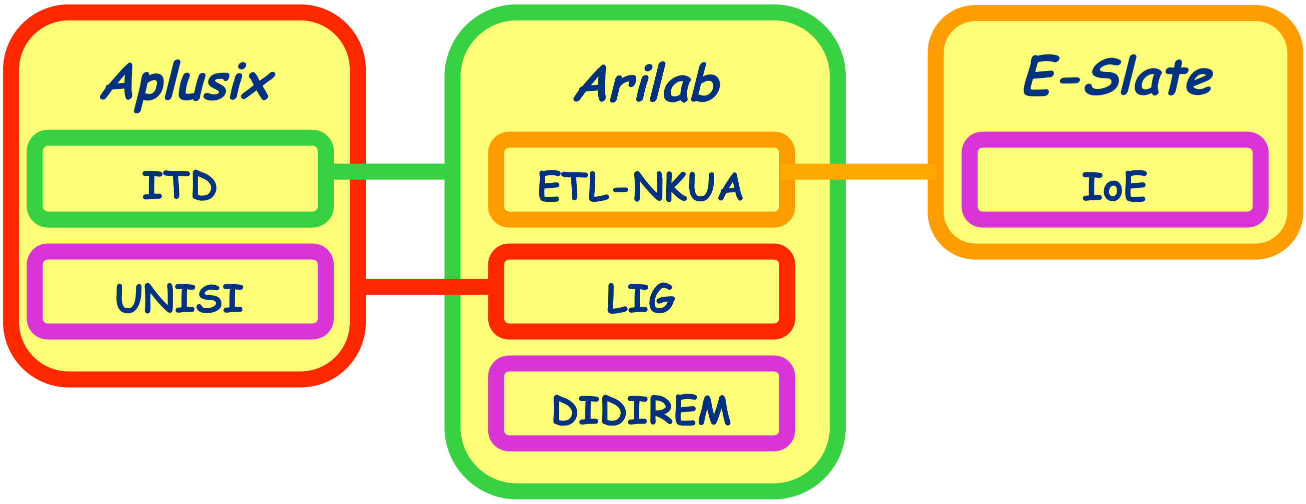


Figure 1: ***Aplusix*****, developed by Metah, was experimented by ITD and UNISI.** ***Arilab*****, developed by ITD, was experimented by LIG, ETL-NKUA and DIDIREM.** ***E-Slate*****, developed by ETL-NKUA was experimented by IoE.**

It was hypothesized that, for each team, the use of a non familiar (alien) tool would have made problematic, thus visible, design decisions and practices that generally remain implicit when one uses tools developed within his/her research and educational culture, and that this visibility would have been increased by the guidelines' request of making explicit the choices performed.

The cross experiments provided interesting insights on the complexities involved in designing and implementing mathematics learning environments integrating technology and allowed to make some reflections (see also: Bottino, Artigue & Noss, in press; Cerulli et al., 2008; Artigue et al. 2007).

The first reflection was on the conditions that can facilitate the sharing of experience and knowledge among researchers in spite of the differences in the theoretical frameworks adopted. Theoretical frameworks, while influencing design and analysis of a teaching experiment, were far from playing the role they are usually given in the literature. As a matter of fact, in the design of the cross-experiments, theoretical frameworks acted mainly as implicit and naturalized frames, and more in terms of general principles than of operational constructs. Even if some variations could be noticed, all the teams experienced a gap between the support offered by theoretical frames and the decisions to be taken in the design process. The acknowledgment of such a gap can be a starting point for establishing a better communication channel not only among researchers but also with teachers. As a matter of fact, a marked emphasis on theoretical assumptions is often too far from the practical needs of teachers. For this reason it is important to establish the exact role that theoretical frameworks play in the planning of an effective teaching experiment. In particular, it was found that researchers tend to overestimate such role, thus making the distance with teachers' needs even bigger. A methodology aimed at making explicit, and at justifying, the choices made, proved a useful tool for reducing communication disparities.

A second observation concerns the understanding of what it means to adapt an ICT based tool to a context different from the one it was designed for. In our work this was accomplished by experimenting in each country tools developed in other countries by different teams. Thanks to the adopted methodology and to the request of making explicit assumptions, choices and decisions taken, it was possible to individuate some variables that strongly affect the development of teaching experiments involving the use of technologies. For instance, the attention paid to different *research priorities* (e.g. the detailed organization of the milieu; the social construction of knowledge; the teacher's role) and to *local constrains* (e.g. curricular; institutional; cultural) appeared to be crucial. Such variables are to be deeply considered and made explicit in the communication with teachers to effectively support them to adapt research experiments to their teaching contexts. In other words, researchers should find ways to make explicit all the key assumptions at the basis of their experiments. Of course, this is not enough, since, as suggested in (Pratt, Winters, Cerulli & Leemkuil, in press), it is also necessary to promote a more strict collaboration between researchers, tool designers and teachers also at the level of the design and the implementation of ICT based tools, and in the planning of the experiments.

Taking into account these needs, and on the basis of the results obtained in TELMA, a new European project was thus developed, involving the same research teams: the ReMath project (IST - 4 – 26751 - STP). In this project the issue of collaboration between different stakeholders was addressed by developing a specific tool to be used to design teaching experiments involving ICT based tools.

A TOOL TO SUPPORT THE COMMUNICATION OF DIFFERENT STAKEHOLDERS IN THE PLANNING OF LEARNING ACTIVITIES INVOLVING TECHNOLOGY

The TELMA project provided a strategy for reducing the difficulties of communication among researchers; this strategy proved to be quite effective, thus it was decided to adapt it to the needs of the ReMath project where communication in a wider community, including software designers, researchers and teachers, has been addressed.

The Remath project, which is still in progress, has two main goals: the development of ICT-based tools for mathematics education at secondary school level and the design and experimentation, in different contexts, of learning activities for classroom practice involving the use of such tools (see: http://remath.cti.gr/default_remath.asp; accessed September 2008). In order to pursue this last goal, a cross-experiment methodology, widening the one developed by TELMA, has been adopted. A tool, the Pedagogical Plan Manager (PPM), has, thus, been developed to support pedagogical planning and the communication of the different stakeholders involved in the activity. In particular, such tool has been developed to support communication between researchers and teachers when planning learning activities involving ICT tools. The idea was originated by the analysis of some the difficulties pointed out by researchers in the wide field of learning design (Koper & Olivier, 2004). In this context some crucial questions had specifically called attention to: how ICT may be usefully employed in supporting the design phase of an innovative learning process? How is it possible to make explicit those pedagogical and contextual reflections that are at the heart of the design of a learning activity and which is the impact of the adopted theoretical frameworks? How is it possible to facilitate dialogue and transfer between teachers, researchers, and software designers?

The idea was that of developing a web-based tool to be used by researchers and teachers to design teaching experiments concerning the educational mathematics software developed in the project. Such tool had to be able to give account of the different issues considered important by the ReMath community (e.g. theoretical and pedagogical assumptions; characteristics of the context in which a teaching experiment has to be carried out, etc.) and to adapt to different contexts and culture.

Meeting such requirements and accounting for the diverse perspectives and concerns that the project brings together, clearly, called for a design solution offering considerable expressiveness and a high degree of structural flexibility. To achieve such goal a model for pedagogical scenarios which is both flexible and expressive was developed (Bottino et al., 2008). The PPM is based on such model.

Pedagogical scenario is seen as a dynamic and modular vehicle for pedagogical planning able to foster consideration for, reflection on, and understanding of critical pedagogical and contextual aspects entailed in the design and enactment of learning activities. The model features a number of attributes for expressing (among other things) the reason why an intervention is proposed, the theoretical and didactical framework in which it is positioned, the innovation it is intended to introduce and the way it is to be implemented. The aim is to bring to light key (often implicit) issues involved in the designing process and to foster reflection on the adopted solutions.

The attributes of the pedagogical scenario are organized in a schema of descriptors which, when instantiated with data (open text and multimedia), form a pedagogical plan. Figure 2 shows one of the interfaces of the PPM devoted to the editing of pedagogical plans. For space reason, we cannot provide here a detailed description of the model adopted and of the prototype thus implemented (more details can be found in Bottino et al., 2008). Outputs of its use are currently under examination and will be further analysed at the end of the ReMath project (May 2009).

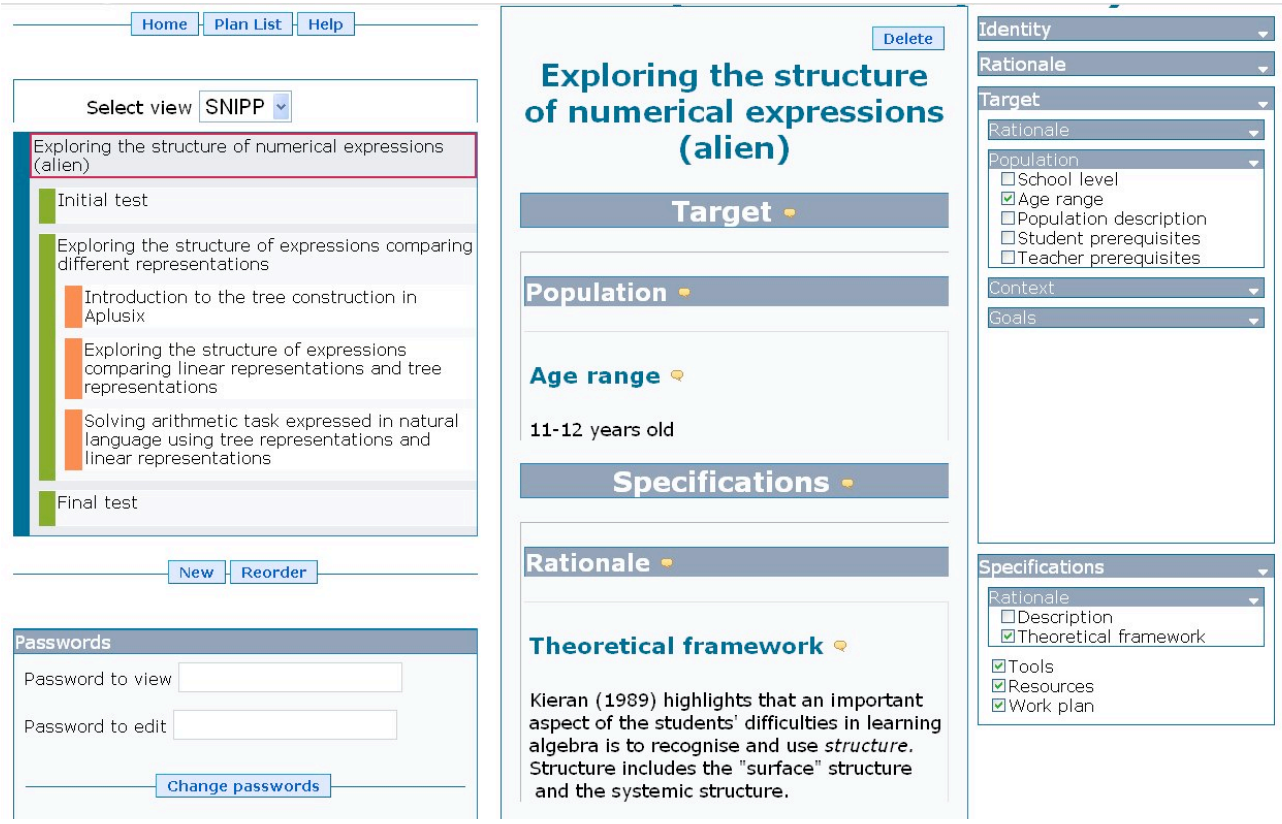


Figure 2: interface of the PPM supporting the editing of pedagogical plans.

CONCLUSIONS

Software designers, researchers and teachers may have different needs, different constrains, and different perspectives. This can be an obstacle for the effectiveness of technology enhanced learning in mathematics, also in terms of impact in school practice. The projects briefly presented tried to develop a coherent methodology for reducing the distance between the different stakeholders. In TELMA it was addressed the problem of networking research teams with different backgrounds and approaches by means of a specific collaborative methodology. In ReMath such methodology was extended, also through the development of a specific web-based tool, to involve all the stakeholders in the design, development and deployment of teaching and learning activities involving the use of technologies.

The outlined pathway includes researcher's explicitation of the actual role played by theoretical frameworks in the effective use of ICT tools and the individuation of the gap between theory and practice. This can help reducing the distance with teachers. The tool for pedagogical planning developed in the ReMath project is aimed at the same goal by involving teachers, from the beginning, also in the design of teaching activities with ICT-based tools. Such activities are seen as integral part in the design process of a technology. In this way we believe it can be possible to develop communities of practice that bring together teachers and researchers so that teaching practice and research could nurture one from each other favouring a better impact of technology enhanced learning in school practice.

NOTES

1. TELMA teams (whose acronyms are indicated in brackets) belong to the following Institutions: Consiglio Nazionale delle Ricerche, Istituto Tecnologie Didattiche, Italy (ITD); Università di Siena, Dipartimento di Scienze Matematiche ed Informatiche, Italy (UNISI); University of Paris 7 Denis Diderot, France (DIDIREM); Grenoble University and CNRS, Leibniz Laboratory, Metah, France (LIG); University of London, Institute of Education, United Kingdom (IOE); National Kapodistrian University of Athens, Educational Technology Laboratory, Greece (ETL-NKUA).

REFERENCES

Artigue, M. (2007). Digital technologies: A window on theoretical issues in mathematics education. In Pitta-Pantazi, D., Philippou, G. (eds.) *Proceedings of the V Congress of the European Society for Research in Mathematics Education CERME 5*, Department of Education, University of Cyprus. ISBN: 978-9963-671-25-0, pg. 68-82.

Artigue, M., Bottino, R.M., Cerulli, M., Georget, J.F., Mariotti, M.A., Maracci, M., Pedemonte, B., Robotti, E., Trgalova, J. (2007). Technology Enhanced Learning in Mathematics: the cross-experimentation approach adopted by the TELMA European Research Team". In *La matematica e la sua didattica,* 21 (1), Bologna: Pitagora Editrice, pg. 67-74.

Artigue, M., Haspékian, M., Cazes, C., Bottino, R.M., Cerulli, M., Kynigos, C., Lagrange, J., Mariotti, M. (2006). *Towards a methodological tool for comparing the use of learning theories in technology enhanced learning in mathematics*. TELMA Deliverable 20-4-1. Kaleidoscope Network of Excellence. http://telearn.noe-kaleidoscope.org/warehouse/Artigue-Kaleidoscope-2006.pdf (accessed September 2008).

Arzarello, F., Bosch, M., Gascón, J., Sabena, C. (2008). The ostensive dimension through the lenses of two didactical approaches. In S. Prediger, A. Arzarello, M. Bosch, Lenfant, A. (ed.) *Comparing, Combining, Coordinating – Networking Strategies for Connecting Theoretical Approaches*, ZDM Mathematics Education, 40 (2), Berlin / Heidelberg: Spriger, pg.179-188.

Bottino, R. M., Artigue, M., Noss, R. (in press). Building european collaboration in technology-enhanced learning in mathematics. In Balacheff, N., Ludvigsen, S., de Jong, T., Lazonder, A., Barnes, S. (eds.) *Technology-Enhanced Learning – Principles and Products*, Springer.

Bottino, R.M., Earp, J., Olimpo, G., Ott, M., Pozzi, F., Tavella, M. (2008). Supporting the design of pilot learning activities with the Pedagogical Plan Manager. In Kendall, M., Samwyas, B. Learning to Live in the Knowledge Society, Proceedings of IFIP World Computer Congress, ED-L2L Conference, New York: Springer, pg. 37-44.

Cerulli, M., Pedemonte, B., Robotti, E. (2006). An integrated perspective to approach technology in mathematics education. In Bosh, M. (ed.) *Proceedings of the IV*

Congress of the European Society for Research in Mathematics Education CERME 4, IQS, Fundemi Business Institute. ISBN: 84-611-3282-3, pg. 1389-1399.

Cerulli, M., Pedemonte, B., & Robotti, E. (Eds.) (2007). *TELMA Cross Experiment Guidelines*, Internal Report, ITD, Genova.

Cerulli, M., Trgalova, J., Maracci, M., Psycharis, G. & Georget, J.-P. (2008). Comparing theoretical frameworks enacted in experimental research: TELMA experience. In Prediger, S., Arzarello, F., Bosch, M., Lenfant, A. (eds.), *Comparing, Combining, Coordinating – Networking Strategies for Connecting Theoretical Approaches*, ZDM Mathematics Education, 40, 2, Berlin / Heidelberg: Springer, pg. 201-214.

Cuban, L., Kirkpatrick, H., Peck, C. (2001). High access and low use of technologies in high school classrooms: Explaining an apparent paradox. *American Educational Research Journal*, 38, 4, pg. 813-834.

Koper, R., Olivier, B. (2004). Representing the learning design of units of learning. *Educational Technology & Society,* 7, 3, pg. 97-11. Available at http://www.ifets.info/journals/7_3/10.pdf (accessed September 2008).

Lagrange, J. B., Artigue, M., Laborde, C., Trouche, L. (2003). Technology and mathematics education: Multidimensional overview of recent research and innovation. In Leung F. K. S. (ed.) *Second International Handbook of Mathematics Education*, 1, Dordrecht: Kluwer Academic Publishers, pg. 237-270.

Pratt, D., Winters, N., Cerulli, M. & Leemkuil, H. (in press). A patterns approach to connecting the design and deployment of mathematical games and simulations. In Balacheff, N., Ludvigsen, S., de Jong, T., Lazonder, A., Barnes, S. (eds.) *Technology-Enhanced Learning – Principles and Products*, Springer.

Prediger, S., Arzarello, F., Bosch, M., Lenfant, A. (2008). Comparing, combining, coordinating-networking strategies for connecting theoretical approaches, Editorial. In S. Prediger, A. Arzarello, M. Bosch, Lenfant, A. (eds.) *Comparing, Combining, Coordinating – Networking Strategies for Connecting Theoretical Approaches*, ZDM Mathematics Education, 40, 2, Berlin / Heidelberg: Springer, 40, 2, pg. 163-164.

Ruthven, K. (2007). Teachers, technologies and the structures of schooling. In Edt. Pitta-Pantazi, D., Philippou, G. (eds.) *Proceedings of the V Congress of the European Society for Research in Mathematics Education CERME 5*, Department of Education, University of Cyprus. ISBN: 978-9963-671-25-0, pg. 52-67.

Sutherland, R. (2004). Designs for learning: ICT and knowledge in the classroom. *Computers and Education, 43*, 5-16.

Vermesch, P., Maurel, M. (Eds.) (1997), *Pratiques de l'entretien d'explicitation*, Paris: ESF.

DYNAMIC GEOMETRY SOFTWARE: THE TEACHER'S ROLE IN FACILITATING INSTRUMENTAL GENESIS

Nicola Bretscher

King's College London

In the UK, use of dynamic geometry software (DGS) in classrooms has remained limited. Whilst the importance of the teacher's role is often stated in dynamic geometry research, it has been seldom elaborated. This study aims to address the apparent deficiency in research. By analysing teacher/pupil interactions in a DGS context, the intention is to identify situations and dialogue that are helpful in promoting mathematical thinking. The analysis draws on an instrumental approach to describe such interactions. Elements of instrumental genesis are distinguished in pupils' dialogue and written work which suggest strategies that teachers can employ to facilitate this process.

Keywords*: teacher's role, dynamic geometry, instrumental genesis*

INTRODUCTION

This study aims to elicit teaching strategies that teachers might employ in their classrooms to help pupils engage constructively with dynamic geometry software. Currently DGS has made little impact on classrooms in the UK. Research has tended to focus on elaborating situations of innovative use and student/machine interaction. This study hopes to re-focus on "the teacher dimension" (Lagrange et al, 2003). The author carried out this study in the role of a practitioner-researcher with a high ability year 8 class. Whilst the class cannot be deemed to be representative, nevertheless it is an 'ordinary classroom' and therefore this study can claim to respond to the need for research into how dynamic geometry software is integrated into 'the *regular* classroom' (Gawlick, 2002).

DGS – A CLASSROOM FAILURE?

Dynamic geometry software (DGS) appears to be following the cycle of oversell and high expectations, ending in limited classroom use identified by Cuban (2001) as a general pattern for technological innovation in education. Research in mathematics education generally presents DGS as a potentially important and effective tool in the teaching and learning of geometry (see for example Holzl, 1996; Marrades and Gutierrez, 2000; Mariotti, 2000). In their survey of geometry curricula, Hoyles et al (2001) state that although most countries seek to integrate ICT into teaching geometry, there is little explicit influence of ICT in classrooms. In the UK, despite recommendations in the Key Stage 3 Mathematics Framework (DfEE, 2001) for using DGS to develop geometrical reasoning, classroom use has remained limited (Ofsted, 2004). Syntheses of research findings generally conclude by favouring the strong potential of ICT but give few explanations for the contrasting poor reality of classroom use (Lagrange et al, 2003).

THE ABSENT TEACHER

A criticism of educational policy and discourse on ICT is that the predominant focus has been on technology rather than education (Selwyn, 1999). The picture painted by Lagrange et al (2003) of research on ICT within mathematics education is of a field dominated by "publications about innovative use or new tools and applications" where issues of the integration of technology into ordinary classrooms have been largely neglected. In particular, the voice and role of the teacher has been notably absent. Balacheff (1993) found that there was essentially no research focusing "on teachers and the explanation they might expect about a didactical interaction". Ten years later, Lagrange et al (2003) still found that very few papers considered what they termed "the teacher dimension", rather that there was a tendency for research to make "an implicit assumption that the transfer of innovative situations of use… would provide the teacher with sufficient material for integration". DGS is no exception: in his review of research on dynamic geometry software, Jones (2002) suggests that future research could usefully focus on teacher input and its impact, amongst other issues.

This study was designed with these issues in mind. The instrumental approach, described in the next section, was used to analyse teacher/pupil interactions in order to elicit teaching strategies which might facilitate pupils' instrumental genesis.

THEORETICAL BACKGROUND

Instrumental genesis is described as the process by which an artefact is transformed into an instrument by the subject or user (Guin and Trouche, 1999). An artefact is a material or abstract object, given to a subject. An instrument is a psychological construct built from the artefact by the subject internalising its constraints, resources and procedures (Guin and Trouche, 1999). Once the user has achieved instrumentalisation, he is able to reinterpret or reflect on the activity he is engaged in. Drijvers and Gravemeijer (2005) describe instrumental genesis as the "emergence and evolution of utilisation schemes". A utilisation scheme is a "stable mental organisation" including both technical skills and supporting concepts as a method of using the artefact for a given class of tasks (Drijvers and Gravemeijer, 2005). The interrelation between machine techniques and concepts seems important since Drijvers and Gravemeijer (2005) found that the apparent technical difficulties that students had often had a conceptual background.

The instrumental approach has been mainly developed and applied within the context of computer algebra software (Drijvers and Gravemeijer, 2005) and there remains a question over how general its applicability is. Drijvers and Gravemeijer (2005) cite two examples where the instrumental approach has been applied to DGS. Thus it seems instrumental genesis may be an appropriate tool to analyse observations of student behaviour within a dynamic geometry environment.

RESEARCH CONTEXT AND METHODOLOGY

This study was conducted as part of a Best Practice Research Scholarship-funded project on using DGS as a resource for teaching geometrical proof. Much of the previous research on DGS has focused on pupils in upper secondary school (Jones, 2000). It has been suggested that more research is needed on the impact of dynamic geometry software on students in lower secondary school (Marrades and Gutiérrez, 2000). The decision to conduct the research with the researcher's year 8 class was partly influenced by this consideration. Since the pupils were in year 8, there was an added advantage that they were not subject to public examinations, the curriculum is less pressurised and therefore ethical considerations about deviating from schemes of work were somewhat reduced. The school in which the research was conducted is a private day school for girls. The research was conducted with the highest attaining set in year 8, containing 23 pupils, with girls expected to achieve levels 7 or 8 at Key Stage 3 [1]. In common with several other research studies, this was seen as an advantage since students judged to be above average in mathematical ability are most likely to be able to engage with proving processes and therefore allow meaningful data collection to take place (Jones, 2000; Marrades and Gutiérrez, 2000).

In this paper, I consider data drawn from a sequence of 5 lessons in which pupils were engaged in investigating a series of construction problems in pairs using Cabri Geometre. The tasks were based upon the Phase 1 and 2 tasks developed by Jones (2000) and were intended to progress in difficulty. Each task consisted of a figure which the pupils were to construct in Cabri so that it remained constant under drag. The methods for constructing a figure were linked and developed from previous problems to encourage the pupils to examine how additional constraints might affect the resultant shape. They were prompted to say what the resultant shape was and, importantly, how did they know? The point of the teaching sequence was to encourage pupils to justify or prove these assertions.

The pupils were asked to choose a construction of their choice and produce a Power-point presentation on why their construction had worked which was presented to the class. Printouts of the pupils' Power-point presentations and audiotape recordings of their presentations to the class form one part of the data collected. During the lessons, the researcher carried an audiotape so that any teacher/pupil interactions would be recorded: these recordings form another part of the data collected. After the lessons, brief field-notes were made on the major events in the lesson.

The initial stage of data analysis concerned the transcription of tape-recordings made during lessons. Using field notes, the tapes were broken down into major events or "episodes" (Bliss et al, 1996). In the sense described by Bliss et al (1996) these episodes had "an internal coherence"; they were complete conversations which allowed the researcher to "interrupt momentarily, for the purpose of analysis, the 'relentless flow of the lesson'". A second stage of analysis involved going through the transcripts and pupils' work making notes, identifying critical incidents that build towards detailed accounts of practices. The final analysis was based on a grounded approach using narrative techniques (Kvale, 1996) which moved back and forth between the theoretical viewpoint developed in the review of literature and the pupils' work and transcribed episodes. Each step in this process eased the transition from emotionally involved participant towards objective observer. Using the concept of instrumental genesis to achieve a "rich and vivid description of events" (Hitchcock and Hughes, 1995), this study hopes to tease out the threads of a tapestry of complex social interactions to see if strategies for promoting mathematical thinking can be discerned in the weave.

ANALYSIS

From the analysis of data, three teaching strategies emerged for facilitating pupils' instrumental genesis in Cabri. Using excerpts from teacher/pupil dialogue, these strategies are described below, where T represents the teacher throughout.

Unravelling functional dependency in DGS

In common with other students, Pupils H and C experienced difficulty with specifying where they wanted objects to intersect when attempting to construct two circles sharing the same radius. They constructed the first circle successfully and correctly placed the centre of the second circle on its edge. The difficulty arose when they tried to adjust the size of the second circle so that its edge would pass through the centre of the first circle, thus ensuring that they would share a radius. The problem was that they made it *look* like the edge of the second circle passed through the centre of the first circle rather than specifying to Cabri that the circle should go "By this point" – as the Cabri pop-up phrase suggests if you hover over the required centre point. Although their Cabri drawing looked successful, when it was subjected to a drag-test, the circles changed size in relation to each other instead of maintaining their pattern:

T: Yeahhh. That's it because you see this computer program will only do exactly what you tell it so if you just make it look like it… sort of, yeah. I'm going to be able to change the shape of your circle so if you tell it, look….

crackle: teacher using the computer to show how the circle can still be messed up. Then creates a new one "by this point" method to show the difference

T: Ok now try and mess it up, you try and mess it up now mess up one of the other circles yeah… ok so…

There follows some unintelligible comments and crackling then…

H: You think a computer's smart but it's not, you can't just sit there and watch it do it for you, you have to know what to do and you have to <u>tell</u> it to do it so it's like a something…. like it's like a lightswitch.

The difficulties that students have in coming to terms with the concept of functional dependency in geometry exemplifies Drijvers and Gravemeijer's (2005) conception of utilisation schemes in which the technical and conceptual elements co-evolve. Pupil H articulates this point very clearly: “you have to know what to do and you have to tell it to do it”. Mathematical knowledge is knowing “what to do” and technical knowledge is required in order to tell the computer to do it. The *gap* in H and C’s knowledge was an appreciation of the functional dependencies inherent in Cabri: on the one hand, a conceptual gap of the *necessity* of specifying the required geometrical relationship and, on the other hand, a gap in the technical knowledge of *how* to specify the relationship using Cabri. The teacher explains the *need* to specify the geometrical relationship: the “computer program will only do exactly what you tell it”. The teacher goes on to illustrate the technical knowledge of *how* to specify the relationship by contrasting the construction ‘by eye’, which could still be messed-up, to the “by this point” version in which the geometrical relationships remained intact.

Pupil K had similar difficulties to H and C: although she seemed to be clear about how the circle should be positioned, she appeared unaware of the necessity to specify to Cabri that the circle should go “By this point”. Again the teacher makes the technical elements explicit:

> Ok. Keep your hand …[K: uhuh] yeah? So if you actually put it on the point and say I want it “by this point” that’s how the comp… that’s the only bit of IT you’re using. [K: But that’s…] That’s the only knowledge…IT knowledge you’ve used. And really then you’ve had to tell it to do that haven’t you?

In this case, the teacher is more direct in making the functional dependencies explicit, by guiding the pupil’s construction and referring to the software language “by this point”. The teacher even describes this technical knowledge of how to specify the relationship as “IT knowledge”, unravelling it from the mathematical knowledge of the geometrical relationship. The teacher again refers to the conceptual necessity of specifying the relationship: “you’ve had to tell it to do that”. Drijvers and Gravemeijer (2005) describe instrumental genesis as the “emergence and evolution of utilisation schemes, in which technical and conceptual elements co-evolve”. The role of the teacher in supporting instrumental genesis is partly in making the technical and conceptual elements explicit. In the case of dynamic geometry software such as Cabri, the role of the teacher is to unravel the notion of functional dependency by highlighting the *necessity* of specifying the required geometrical relationship and the technical knowledge of *how* to specify the relationship.

Exploiting dynamic variation to highlight geometric invariance

All the figures presented to the pupils for construction were based on the initial construction of a line which was apparently horizontal. Of course, there is no geometrical reason for the line to be horizontal, the figures had been presented in this way purely for neatness and it had not been given a second thought, until the teacher noticed that all students appeared to be constructing *intentionally* horizontal lines. The pupils had discovered that by pressing the “shift” key whilst constructing a line, the line would snap to the horizontal. According to the pupils, a similar feature of "snapping to a grid" occurs in a piece of completely unrelated software, which was how the discovery was made. Pupil K was insistent that the line should be horizontal:

T: Why do you always insist on that being horizontal? Does it matter if it….

The teacher draws attention to the pupil's misconception and, by dragging, attempts to convey that the horizontal constraint is artificial, that it can be broken without disturbing the figure under construction. As the pupils were presenting their work to the class, it became clear that all groups had produced figures with horizontal lines. The teacher again attempted to question this feature of their constructions but this time in a whole class context. Pupil MC was asked to reconstruct her solution to Problem 2 (a perpendicular bisector) without starting from a horizontal line. She did this successfully on an interactive whiteboard so that the whole class could see. She then dragged the figure, directed by the teacher, changing its orientation to show its invariance, including the situation with the initial line being horizontal. The teacher exploits dynamic variation to highlight the geometric invariance of the construction in order to help pupils differentiate between geometrical relationships which were or were not crucial.

A similar situation occurred when a pair of pupils, MC and ML, successfully completed the construction leading to a square (Problem 4). They both excitedly told the teacher that the shape they had produced was a diamond. The teacher dragged their construction so that the base of the shape was horizontal, at which point they both concurred that the shape was a square. Upon dragging it back to the original position, ML in particular returned to her previous statement that it was a diamond. Repeated dragging, more and more slowly to emphasise the continuous 'transformation' of the shape, convinced both students that the shape was, in fact, always a square. Again the teacher's strategy is to demonstrate the potential of the software, by exploiting dynamic variation to demonstrate the invariance of the constructed shape. Recognising the potential of the software and making its affordances explicit to pupils is a key element in supporting instrumental genesis.

Making connections between DGS and pencil-and-paper

Pupil N had constructed a rhombus but, as in the examples in the previous paragraph, had difficulty identifying the shape due to its unfamiliar orientation. The teacher employs dynamic variation to convince pupil N that the shape is indeed a rhombus but then continues the explanation on paper:

N: Is this a rhombus? But a rhombus supposed to be like tilted so…?

Teacher manipulating the diagram on screen

N: Oh so it can be, it can be any way up and it [T: Oh!] would still be a rhombus.

T: Well yeah… [N to another pupil: Well it is a rhombus.] it's like, look, this is a well no that's not. This a rectangle isn't it? Ok, it's still a rectangle. It's still a rectangle. However much I turn it, it's still a rectangle. Yeah, ok?

Diagram of rectangle drawn on paper and then the paper twisted and turned as a demonstration that orientation doesn't alter the shape.

Guin and Trouche (1999) suggest that teachers should highlight the constraints and limitations of the software to students in order to support their instrumental genesis. In these cases, the teacher is in fact using the dynamic nature of the Cabri software to highlight the constraints and limitations of the paper-and-pencil environment, exposing a misconception and thereby supporting the pupils' instrumental genesis in the more traditional medium. In the case of the tilted rhombus, the teacher sketched a rectangle on paper in order to further illustrate the concept that orientation does not affect the nature of the shape. This sketch was done on paper at the time mainly because it was quicker than constructing the shape on Cabri. The teacher's return to the paper-and-pencil environment is important because it makes a connection between the two environments: although dynamic variation makes it easier to appreciate that orientation does not affect the shape, the concept still holds in a paper-and-pencil environment. The return to paper-and-pencil is thus an attempt by the teacher to "build connections with the official mathematics outside the microworld", a responsibility which Guin and Trouche (1999) identify as being a crucial part of the teacher's role.

DISCUSSION

From the sequence of lessons, three teaching strategies have been distilled that serve to facilitate pupils' instrumental genesis in a DGS context. These strategies are clearly not exhaustive: exploiting anomalies of measurement in Cabri such as rounding errors might be another way to promote mathematical thinking, for example. These strategies are specific to DGS in general and Cabri Geometre. They are also analogous to teaching strategies used in other contexts. Guin and Trouche (1999) suggest that teachers should highlight the constraints and limitations of the software to students: in the case of Derive, the discrete and finite nature of the software. Similarly, a dynamic geometry environment such as Cabri is only a discrete model of Euclidean geometry, despite its continuous appearance. All tools and resources have constraints and limitations. In the case of paper and pencil, a limitation is the static nature of the environment. Thus strategies such as those identified in this paper may apply to any teaching resource. In a sense, the teaching strategies mentioned here essentially highlight general principles of mathematics teaching applied to a specific context, in this case DGS. The resource provides a context for learning but cannot teach. The focus of research needs to shift away from the context, towards teachers and the teaching strategies they may employ in order to aid pupils' instrumental genesis. In this way research on ICT may avoid the criticism that the predominant focus has been on technology rather than education.

NOTES

1. Key Stage 3 covers the first three years of secondary schooling in England: Year 7 (age 11-12), Year 8 (age 12-13) and Year 9 (age 13-14). Average attainment at the end of KS3 is at level 5/6. Level 8 is the highest level possible in maths at KS3.

REFERENCES

Balacheff, N. (1993). Artificial Intelligence and Real teaching. In Keitel, C. and Ruthven, K. (eds.), *Learning from Computers: Mathematics education and technology*, New York: Springer Verlag, 131-158

Bliss, J., Askew, M. and Macrae, S. (1996). Effective Teaching and Learning: scaffolding revisited. *Oxford Review of Education,* **22** (1), 37-61

Cuban, L. (2001). *Oversold and Underused: Computers in the classroom*. Harvard University Press

DfEE (2001). *Key Stage 3 National Strategy: Framework for teaching mathematics: Years 7,8 and 9*. DfEE Publications

Drijvers, P. and Gravemeijer, K. (2005). Computer algebra as an instrument: examples of algebraic schemes. In D. Guin, K. Ruthven and L. Trouche (eds.), *The Didactical challenge of Symbolic Calculators: turning a computational device into a mathematical instrument,* New York: Springer Verlag, 163-196

Gawlick, T. (2002). On Dynamic Geometry Software in the Regular Classroom. *Zentrallblatt fur Didaktik der Mathematik,* **34** (3), 85-92

Guin, D. and Trouche, L. (1999). The Complex Process of Converting Tools into Mathematical Instruments: the case of calculators. *International Journal of Computers for Mathematics Learning*, **3**(3), 195-227

Hitchcock, G. and Hughes, D. (1995). *Research and the Teacher: A Qualitative Introduction to School-Based Research*. 2nd edition, London: Routledge

Hölzl, R. (1996). How does 'dragging' affect the learning of geometry? *International Journal of Computers for Mathematics Learning*, **1** (2), 169-187

Hoyles, C., Foxman, D. and Kuchemann, D. (2001). *A Comparative Study of Geometry Curricula*. Institute of Education, University of London for QCA

Jones, K. (2000). Providing a Foundation for Deductive Reasoning: students' interpretations when using dynamic geometry software. *Educational Studies in Mathematics*, **44** (1&2), 55-85

Jones, K. (2002). Research on the use of Dynamic Geomatery Software: implications for the classroom. *Micromath*, **18**(3), 18-20

Kvale, S. (1996). *Interviews: an Introduction to Qualitative Research Interviewing.* Sage Publications

Lagrange, J-B., Artigue, M., Laborde, C. and Trouche, L. (2003). Technology and Mathematics Education: A Multidimensional Study of the Evolution of research and Innovation. In A.J. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick, F.K.S. Leung (eds.), *Second International Handbook of Mathematics Education*, Dordrecht: Kluwer Academic Publishers, 237-269

Mariotti, M.A. (2000). Introduction to Proof: the mediation of a dynamic software environment. *Educational Studies in Mathematics*, **44** (1&2), 25-53

Marrades, R. and Gutiérrez, A. (2000). Proofs produced by secondary school students learning geometry in a dynamic computer environment. *Educational Studies in Mathematics*, **44** (1&2), 87-125

Ofsted (2004). *2004 report: ICT in schools – the impact of government initiatives, Secondary Mathematics* London, A report from the Office of Her Majesty's Chief Inspector of Schools http://www.ofsted.gov.uk/assets/3646.pdf

Selwyn, N. (1999). Why the Computer is not Dominating Schools: a failure of policy or a failure of practice? *Cambridge Journal of Education,* **29** (1), 77-91.

STUDENT DEVELOPMENT PROCESS OF DESIGNING AND IMPLEMENTING EXPLORATORY AND LEARNING OBJECTS

Chantal Buteau & **Eric Muller**

Department of Mathematics, Brock University, St. Catharines (CANADA)

In 2001 a core undergraduate program, called Mathematics Integrated with Computers and Applications (MICA) *was introduced in the Department of Mathematics at Brock University, Canada. In this program that integrates evolving technologies, students complete major projects that require the design and implementation of 'Exploratory and Learning Objects' (ELO). In this paper, we propose schematic representations and descriptions of the student development process as s/he completes an ELO project. We highlight the important role that ELO interfaces play in this development process.*

Keywords*: Exploratory and Learning Objects (ELO); student development process; students designing and implementing ELO; university mathematics education.*

INTRODUCTION

There have been a number of publications (Muller, 1991, 2001; Muller & Buteau, 2006; Buteau & Muller, 2006; Pead et al, 2007; Muller et al., forthcoming) about the long-term implementation of evolving technology use in undergraduate mathematics education at Brock University (Canada) that started in the early 80s. The most recent development is the 2001 implementation of the core undergraduate mathematics program called *Mathematics Integrated with Computers and Applications* (MICA). Two of the program aims are to (1) develop mathematical concepts hand in hand with computers and applications; and (2) encourage student creativity and intellectual independence (Brock Teaching, 2001). Three innovative core courses, called *MICA I, II, III*, were implemented in addition to a review of all traditional courses to incorporate the MICA program aims. Results of a 2006 MICA student survey and an enrolment analysis covering the years 2001 to 2006 are reported in Ben-El-Mechaiekh et al. (2007). Highlights include

> Students overall rated the use of technology in their mathematics courses as positively beneficial (77.74% of responses; 79.36% when restricted to mathematics majors). (p.10)

and, furthermore,

> ... students overwhelmingly rated the use of technology in [MICA] courses as [positively] beneficial (91.13% of responses) (p.9)

In this paper proposal, we focus on one of the major student activities in the MICA courses, namely their designing, implementing (VB.net, Maple, C++), and using of interactive and dynamic computer-based environments, called Exploratory and Learning Objects (ELO). By Exploratory Object (EO) and Learning Object (LO), we mean the following.

> *An Exploratory Object is an interactive and dynamic computer-based model or tool that capitalizes on visualization and is developed to explore a mathematical concept or conjecture, or a real-world situation*
>
> and,
>
> *A Learning Object is an interactive and dynamic computer-based environment that engages a learner through a game or activity and that guides him/her in a stepwise development towards an understanding of a mathematical concept.* (Muller et al., forthcoming, p.5)

To illustrate these objects, we provide without comment three examples of original student ELO projects that can be accessed at (MICA Student Projects website, n.d.): (1) *Structure of the Hailstone Sequences* EO by first-year student Colin Phipps for the investigation of a mathematical conjecture; (2) *Running in the Rain* EO by second-year students Matthew Lillie and Kylie Maheu for the investigation of a real-world situation; and (3) *Exploring the Pythagorean Theorem* LO by first-year student Lindsay Claes for the learning of a school mathematical concept.

In previous publications, we have elaborated how the MICA I course is designed to progressively bring the students to acquire the skills and understanding required for the development of ELOs (Muller & Buteau, forthcoming). In brief, as the course progresses, our students are guided through each step in the development process of ELOs that we describe in the next section of this paper. We have also explained that this requires a significant change in the teaching paradigm of faculty involved in these courses, and motivates a change in attitude in the students about learning and doing mathematics with technology at the university level (Muller et al., forthcoming). And also, we have argued that learning activities in the MICA program accelerates students' growth towards independence in doing mathematics (Buteau & Muller, 2006).

In this paper we propose a first attempt at defining a structure for the student development process in their activity of designing, implementing, and using an ELO. These final MICA projects are completed individually or in pairs selecting a topic of their own choice. We also briefly discuss the role of interfaces in the student development of an ELO. As in the past, we, as mathematicians in a mathematics department, look forward to receiving constructive feedback from mathematics educators. We hope that the presentation of our innovative student learning activities, as part of the systemic integration of technology in our university mathematics curriculum, will instigate educational research questions on learning mathematics with use of technology in tertiary education.

STUDENT DEVELOPMENT PROCESS OF EXPLORATORY AND LEARNING OBJECTS

In what follows, we suggest schematic representations of the development process for ELOs. Even though the schematic representations are worded generally, in their descriptions we focus on students in MICA courses.

Development Process of an Exploratory Object to Investigate a Conjecture

We propose the following diagram (Figure 1) to illustrate this development process.

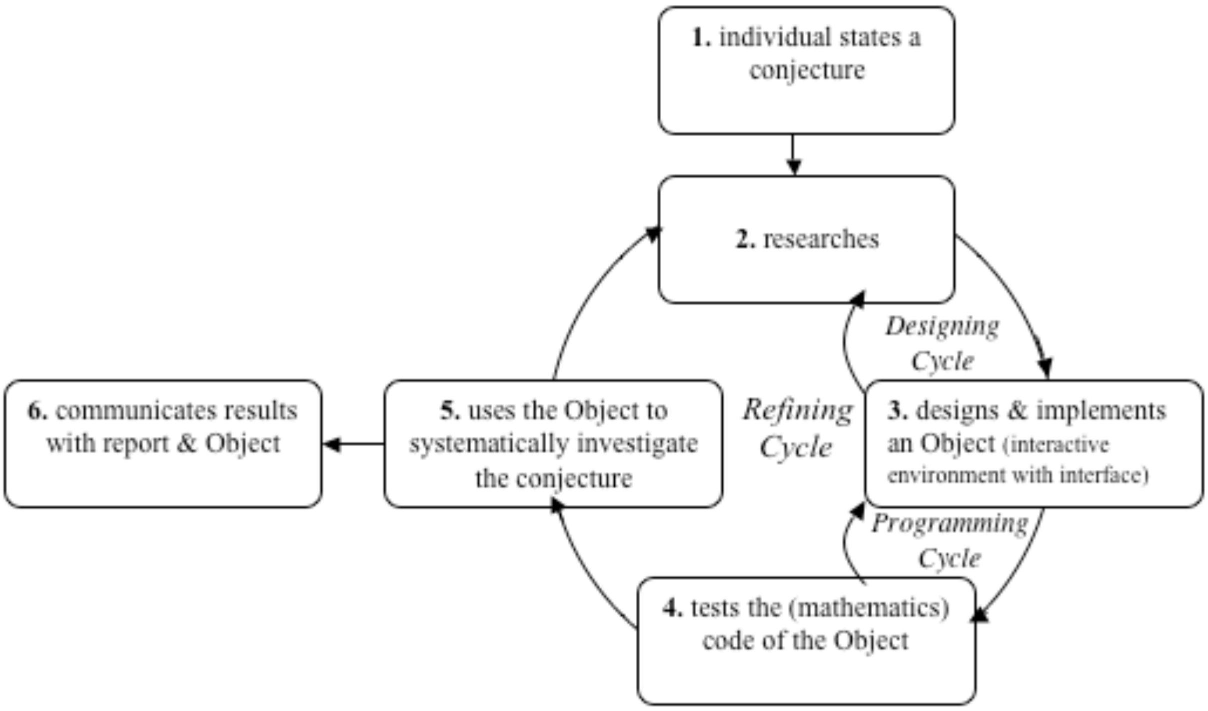


Figure 1. Development process of an Exploratory Object for the purpose of investigating a conjecture.

Here is a description of each step in the diagram.

1. Student states a conjecture, and may discuss it with the instructor; some of the more independent students wait until step 3 to discuss their project.

2. Student researches the conjecture using library and Internet resources, and may refine his/her conjecture. In conjunction with step 3, student identifies the mathematics, such as variables, parameters, etc., and is involved in a *Designing Cycle*.

3. With his/her understanding of the conjecture, student starts designing and implementing (i.e., coding) an interactive environment (i.e., program with interface) with a view to testing the conjecture. Student organizes the interface to make parameters accessible and to display diverse representations of results. As the interface plays such an important role in EO, we discuss it further in the next section.

4. Student selects, in a step-wise fashion, simple and more complex cases to test that the mathematics is correctly encoded and that the interface is fully functional. Together with step 3, the code testing and revising involve the student in a *Programming Cycle*.

5. At this step, student now returns to focus on his/her conjecture and uses the Object to systematically investigate it. Following the results of the investigation, the student may decide to refine the Object, e.g., introducing new parameters, etc., and be involved in *a Refining Cycle* (with steps 2, 3, and 4).

6. Student produces a report of his/her results and submits it with the Object. The report includes a statement of the conjecture, the mathematical background (from step 2), results of the exploration including an interpretation of the data and graphs (from step 5), a discussion, and a conclusion. This is somewhat similar to a science laboratory report. Building on this analogy, the Object is the laboratory itself. In other words, student submits his/her self-designed 'virtual laboratory' for the investigation of a self-stated conjecture together with his/her laboratory report.

Development Process of an Exploratory Object to Investigate a Real-World Situation

We propose the following diagram (Figure 2) to illustrate this development process.

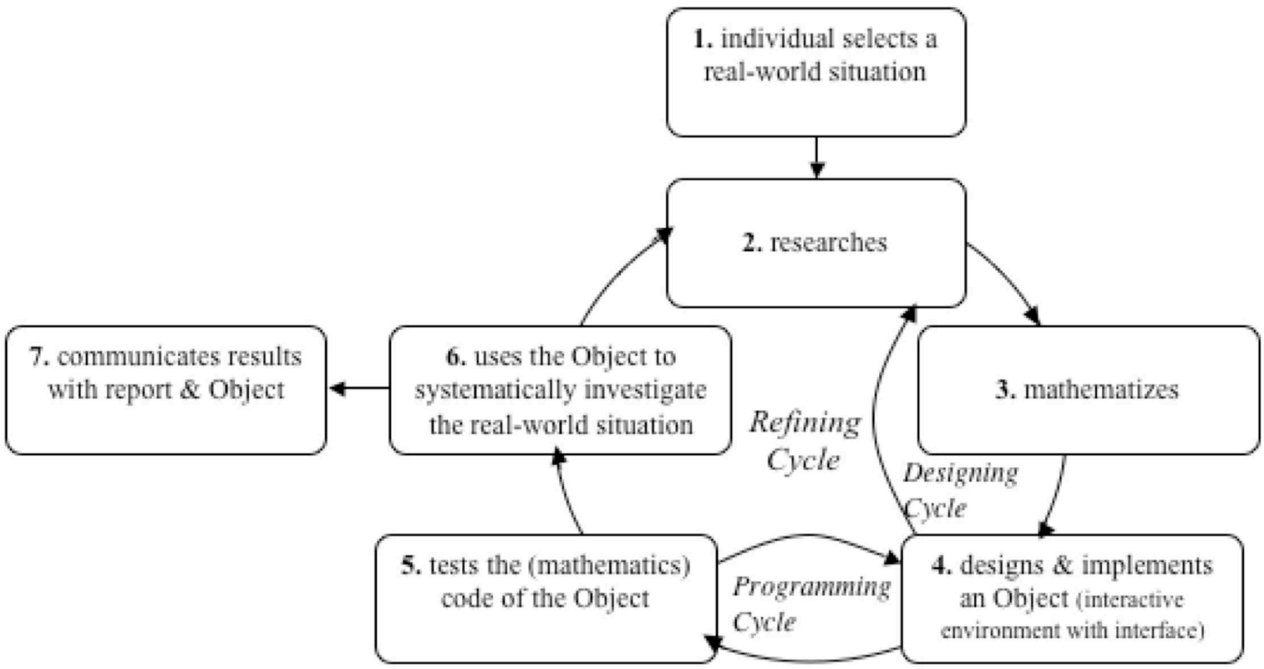


Figure 2: Development process of an Exploratory Object for the purpose of investigating a real-world situation.

Here is a description of each step in the diagram.

1. Student selects a real-world situation of particular interest, and may discuss it with the instructor; some of the more independent students wait to discuss their project until step 3 or 4.

2. Student researches the real-world situation using library and Internet resources, and may restrict or modify the scope of the real-world situation. In conjunction with steps 3 and 4, student identifies the mathematics, such as variables, parameters, etc., and is involved in a *Designing Cycle*.

3. Student develops a mathematical model of the real-world situation using the variables and parameters selected in step 2 and in the majority of cases, consults the instructor.

4. With his/her understanding of the model, student starts designing and implementing (i.e., coding) an interactive environment (i.e., program with interface) with a view to investigating the real-world situation. Student organizes the interface to make the model parameters accessible and to display diverse representations of solutions. As the interface plays such an important role in EO, we discuss it further in the next section.

5. Student selects, in a step-wise fashion, simple and more complex cases to test that the mathematical model is correctly encoded and that the interface is fully functional. Together with step 4, the code testing and revising involve the student in a *Programming Cycle*.

6. At this step, student now returns to focus on his/her real-world situation and uses the Object to systematically investigate it. Following the results of the investigation, the student may decide to refine the model and the Object, e.g., introducing or deleting, new parameters and variables, new conditions, etc., and may be involved in a *Refining Cycle* (with steps 2, 3, 4 and 5).

7. Student produces a report of his/her results and submits it with the Object. The report includes a description of the real-world situation, a development of the mathematical model (from step 3), results of the exploration (from step 6) including an interpretation of the data and graphs, a discussion, and a conclusion. This is somewhat similar to a science laboratory report. Building on this analogy, the Object is the laboratory itself. In other words, student submits his/her self-designed 'virtual laboratory' for the investigation of a self-selected real-world situation together with his/her laboratory report.

Development Process of a Learning Object of a Mathematical Concept

We propose the following diagram (Figure 3) to illustrate this development process.

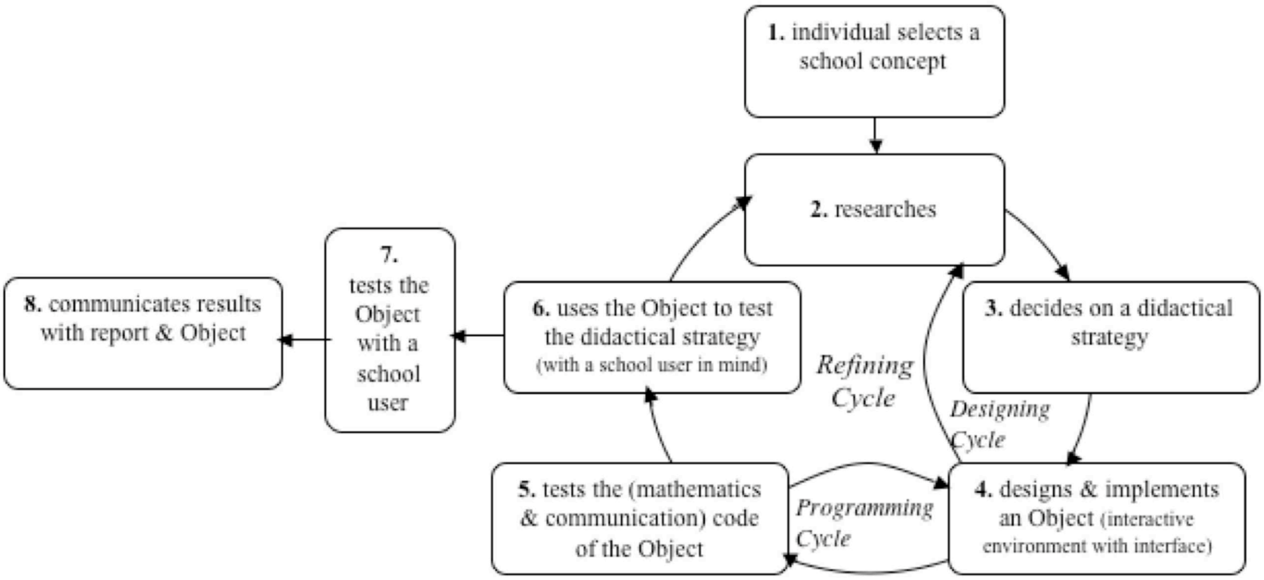


Figure 3: Development process of a Learning Object of a mathematical concept.

Here is a description of each step in the diagram.

1. Student selects a school concept.

2. Using library and Internet, student looks at resources about the concept and its teaching. In particular, student identifies when in the school curriculum the concept is taught, reviewed and expanded, what previous mathematical understanding, general knowledge and reading capabilities can be assumed, etc. In conjunction with steps 3 and 4, student identifies and develops the mathematics didactical features that could be used for his/her Object, and is involved in a *Designing Cycle*.

3. Based on the information gathered in step 2, student selects a didactical strategy for a fictive school pupil learning of the concept that may include developing a game or activity to engage the learner, breaking down the concept, setting up a testing procedure, etc. Student may discuss the strategy with the instructor or wait until the next step.

4. Student starts designing and implementing (i.e., coding) an interactive environment (i.e., program with interface) with a view to implement the didactical strategy. Student structures a self-contained interface realizing that the fictive school pupil will be using the LO independently. As the interface plays such an important role in LO, we discuss it further in the next section.

5. Student tests that the interface (communication, navigation, etc.) is fully functional and tests with simple and more complex cases that the mathematics is correctly encoded. Together with step 4, the code testing and revising involve the student in a *Programming Cycle*.

6. At this step, student now returns to focus on his/her didactical strategy and works through the Object with a school pupil in mind. Following the results of this investigation, the student may decide to refine the Object, e.g., changing the sequence of activities, improving the clarity of communication, etc., and may be involved in a *Refining Cycle* (with steps 3, 4, and 5).

7. Student tests his/her Object by observing a school pupil, at appropriate grade level, working with the Object. In some cases, student returns to the refining cycle and revises the Object.

8. Student produces a report of his/her results and submits it with the Object. The report includes the didactical purpose, the target audience, the mathematical background of the target audience, a brief account of the school pupil experience (step 7), and a discussion. This report is somewhat similar to a lesson plan, including a post-lesson reflection, though without a description of the lesson. Building on this analogy, the Object is the lesson itself. Thus, student submits his/her lesson plan of a self-selected mathematical concept in which the written description of the lesson is replaced by an 'interactive self-directed lesson (with a virtual learner)', i.e., by the Object.

ROLE OF THE INTERFACE IN THE DEVELOPMENT PROCESS OF EXPLORATORY AND LEARNING OBJECTS

The interface provides interactivity and (dynamic) visualization. In the Development Process of ELOs (Figures 1, 2, and 3), the student creates an interface in the Designing Cycle with the aim of using it for his/her mathematical or didactical investigation (step 5 in Figure 1 and step 6 in Figures 2 and 3).

During the Designing Cycle of an Exploratory Object, the potentiality of interactivity encourages the student to make explicit the parameters that could play a role in the investigation of his/her conjecture or real-world situation in such a way that they are accessible from the interface. The potentiality of visualization urges the student to decide on the representations to be displayed in his/her interface so as to best support his/her investigation.

At the step in the Development Process when the student uses the Object for his/her investigation (step 5 in Figure 1 and step 6 in Figure 2), both interactivity and visualization aspects of the interface play a role in the student's systematic investigation. The latter can be seen as a dialogue between the student and the computer, though the discussion is fully controlled by the student. During the systematic investigation, the student sets a question by fixing values to parameters (interactivity), the computer answers the question (visualization), and the dialogue continues in that way unless the student concludes that the answers are not satisfactory to meet his/her goal and decides to refine the Object (Refining Cycle). In other words, the student is in an *intelligent partnership* (Jones, 1996) with technology.

The interface plays a central role in Learning Objects but which is different than in the Exploratory Objects. A Learning Object is designed for other users to use by themselves, i.e., without the Object designer who is the student in our case. Thus the navigation in the interface should be very clear and easy. The interface should also provide, at any time, motivation for the intended users to go to a next step in the Object. As such, the visual presentation and the wording should be adapted to the intended users:

> For Learning Objects students [are] reminded constantly that they are designing interfaces for people who are not experts and that they need to take into account such issues as the user's age, educational level, gender, cultural background, experience with computers, motivation, disabilities, etc. (Muller et al., forthcoming, p.12)

Also, students should

> ... break away from the linearity of the written tradition in order to take full advantage of the technological paradigm. (Muller et al., forthcoming, p.12)

In step 8 of the Development Process of the LO (Figure 3), we introduced an analogy where the Object is a 'lesson with a virtual learner'. Using this analogy, the interface's potentiality of interactivity encourages the student during the Designing Cycle to develop an active 'lesson', i.e., a lesson that is interactive, with the intended fictive pupil. The interface's potentiality of visualization facilitates the development of transparent communication of the 'lesson' flow and makes it possible for the student to test his/her 'lesson' (steps 6 and 7 in Figure 3). In other words, we suggest that these two potentialities allow the student to develop a 'guided intelligent partnership' between a fictive pupil and technology.

REFLECTIONS

Diagrams shown in Figures 1, 2, and 3 clearly indicate our view that the student mathematics learning experience through the designing and implementing of an ELO is richer than what is experienced through activities of only programming mathematics. The interface plays a major role through its interactivity and visualization potentialities as it provides students with an opportunity to be involved in an 'intelligent or guided intelligent partnership' with the technology.

In a recent collaborative project between a local elementary school, *École Nouvel Horizon*, and our Department of Mathematics, MICA student Sarah Camilleri was involved as part of her Honour's project in the development of *Fractions Fantastiques/Fantasy Fractions* Learning Objects (Camilleri, 2007; Buteau et al., 2008a and b; MICA Student Project website, n.d.). In this development, she worked with a Grade 5 class, the teacher, and the school principal. It is worthwhile to explore the ways in which individuals took different roles and responsibilities in the Development Process (Figure 3).

Sarah and the teacher selected the fraction concept (step 1), and Sarah researched it (step 2). The teacher taught fractions to the class and presented the collaborative project. In the *Designing Cycle*, guided by the teacher and the principal, the Grade 5 pupils developed the dynamic mathematics lessons, interactive mathematics games, story line of the Object, its characters, etc., and provided drawings and written materials to communicate their ideas to Sarah who had to select and adapt some of them for programming purposes. The pupil design work was achieved in class discussions and in smaller groups of two or three. Within the *Programming Cycle* Sarah took the responsibility of faithfully implementing the pupils' design which also involved the digitizing of the pupils' drawings. The *Refining Cycle* involved Sarah and the teacher for testing the functionality of the Learning Object and checking the faithful integration of the pupils' ideas. *Fractions Fantastiques* Learning Object was presented by Sarah to the Grade 5 class and each pupil received a CD-ROM copy of *their* Learning Object (step 8).

REFERENCES

Ben-El-Mechaiekh, H., Buteau, C., & Ralph, W. (2007). MICA: A Novel Direction in Undergraduate Mathematics Teaching. *Canadian Mathematics Society Notes*, *29* (6), 9-11.

Brock Teaching (2001). Retrieved August 28, 2008, from: http://www.brocku.ca/ctl/pdf/Brock_Teaching_1.pdf

Buteau, C., Camilleri, S., Fodil, K., Mgombelo, J., & Lacroix, M.-E. (2008a). When a Grade 5 Class Design Computer Mathematics Games. *Ontario Mathematics Gazette, 46* (3), 26-30.

Buteau, C., Camilleri, S., Fodil, K., Mgombelo, J., & Lacroix, M.-E. (2008b). Fractions Fantastiques: Lorsqu'une classe de 5e année crée un jeu informatique de mathématiques. *Revue Envol*, *143*, 19-23.

Buteau, C., & Muller, E., R. (2006). Evolving technologies integrated into undergraduate mathematics education. In L. H. Son, N. Sinclair, J. B. Lagrange, & C. Hoyles (Eds.), *Proceedings of the ICMI Study 17 Conference: Digital technologies in mathematics education—Rethinking the terrain*. Hanoi University of Technology, Hanoi, Vietnam.

Camilleri, S. (2007). *Fantasy Fractions Learning Object: A Collaborative Grade 5 ClassProject*. Unpublished Honour's Thesis, Brock University, Canada.

Jones, P. L. (1996). Handheld technology and mathematics: Towards the partnership. In P. Gomez & B. Waits (Eds), *Roles of calculators in the classroom* (pp. 87-96).

MICA Student Project website, (n.d.). Retrieved August 28, 2008, from Brock Department of Mathematics: http://www.brocku.ca/mathematics/studentprojects

Muller, E., R. (1991). Symbolic mathematics and statistics software use in calculus and statistics education. *ZDM, 91* (5), 192-198.

Muller, E., R. (2001). Reflections on the sustained use of technology in undergraduate mathematics education. In D. Holton (Ed.), *The Teaching and Learning of Mathematics at University Level: An ICMI Study* (pp.381-394). Dordrecht: Kluwer Academic Publishers.

Muller, E., R., & Buteau, C. (2006). Un nouveau rôle de l'informatique dans la formation initiale des enseignants. In N. Bednarz, & C. Mary (Eds.), *L'enseignement des mathématiques face aux défis de l'école et des communautés*, Actes du colloque EMF 2006 (cédérom). Sherbrooke: Éditions du CRP.

Muller, E., R., & Buteau, C. (forthcoming). An innovative integration of evolving technologies in undergraduate mathematics education. *ATINER publication*.

Muller, E., Buteau, C., Ralph, B., & Mgombelo, J. (forthcoming). Learning mathematics through the design and implementation of Exploratory and Learning Objects. In *International Journal for Technology in Mathematics Education*.

Pead, D. & Ralph, B. with Muller, E. (2007). Uses of Technologies in Learning Mathematics through Modelling. In Blum et al. (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp. 308-318). New York: Springer.

ISSUES IN INTEGRATING CAS IN POST-SECONDARY EDUCATION: A LITERATURE REVIEW

Chantal Buteau[1], **Zsolt Lavicza**[2], **Daniel Jarvis**[3] & **Neil Marshall**[1]

[1]*Brock University (Canada),* [2]*University of Cambridge (UK),*

[3]*Nipissing University (Canada)*

We propose a discussion on preliminary results of a literature review pilot study regarding the use of CAS in post-secondary education. Several issues surrounding technology integration emerged from our review, and these are described in detail within the paper. The brief report on the type of analysis and the integration scope in curriculum suggest that the multi-dimensional theoretical framework proposed by Lagrange et al. (2003) needs to be adapted for our focus on systemic technology integration in post-secondary education.

Keywords: *CAS, university education, literature review, technology implementation issues*

Introduction

A growing number of international studies have shown that Computer Algebra System (CAS-based) instruction has the potential to positively affect the teaching and learning of mathematics at various levels of the education system, even though this has not been widely realized in schools and institutions (Artigue, 2002; Lavicza, 2006; Pierce & Stacey, 2004). In contrast to the large body of research focusing on technology usage that exists at the secondary school level, there is a definite lack of parallel research at the tertiary level. However, Lavicza (2008) highlights that university mathematicians use technology at least as much as school teachers, and that the innovative teaching practices involving technology that are already being implemented by mathematicians in their courses should be researched and documented. Further, Lavicza (2008) found that within the research literature there existed only a small number of papers dealing with mathematicians and university-level, technology-assisted teaching. In addition, most of these papers are concerned with innovative teaching practices, whereas few deal with educational research on teaching with technology. These findings coincide with school-focused technology studies conducted by Lagrange et al. (2003) and Laborde (2008).

We aim to point out that it is particularly important to pay more attention to university-level teaching, because universities face new challenges such as increased student enrollment in higher education, decline in students' mathematical preparedness, decreased interest toward STEM subjects, and the emergence of new technologies (Lavicza, 2008). Mathematicians must cope with these challenges on a daily basis and only a few studies have offered systematic review and developed recommendations in this area. Our project aims at both documenting university teaching practices involving technology, and formulating recommendations for individual and departmental change. Our research program also aims at raising the amount of attention paid to tertiary mathematics teaching from a research point of view and, from a more practical side, elaborating on specific issues and strategies for the systemic integration of technology in university mathematics courses.

Method Design and Implementation

Based on the above-mentioned Lavicza (2008) findings and recommendations, we designed a mixed methods research study which involves a systematic review of existing literature regarding CAS use at the tertiary level. The theoretical framework developed by Lagrange et al. (2003) involved several stages. They first reviewed a large number of papers in relevant journals and then categorized these papers into five "types." Based on these types, they then selected a sub-corpus of papers dealing specifically with educational research papers focusing on technology use mainly in the secondary school. Through the careful analysis of this sub-corpus of papers, they further developed seven dimensions, each with key indicators, and then proceeded to identify and further analyze papers that best described each of these dimensions.

The theoretical framework of Lagrange et al. (2003) provided our research team with a helpful foundation from which to prepare for our own literature review which will involve approximately 1500 papers/theses. It was decided to implement a pilot study for this large literature review in order to begin to work with the Lagrange et al. framework and to determine if it would be sufficient for our purposes, or may be in need of certain modifications. In the summer of 2008, we therefore began our pilot study focusing on 326 contributions dealing with CAS use in secondary/tertiary education. These papers were drawn from two well-regarded journals, namely the *International Journal for Computers in Mathematical Learning* (issues since its beginning in 1996) and the *Educational Studies in Mathematics* (since 1990). We also selected proceedings from two technology-focused conferences, namely the *Computer Algebra in Mathematics Education* (since its first meeting in 1999) and the *International Conference on Technology in Collegiate Mathematics* (since 1994 with first electronic proceedings). A sub-corpus of 204 papers dealing specifically with CAS use at the post-secondary level was also identified to further focus the analysis.

While the descriptive categories found within the Lagrange et al. template were helpful, we began to notice that several other category/theme columns would be helpful at this stage of the instrument/template development (e.g., we added fields such as "computer/calculator," "implementation scope," and "implementation issues"). An important point to note here is that in contrast to the Lagrange study where the majority of papers were those describing educational research results, our selection of papers revealed a majority that focused on practitioner innovations with very few involving educational research. Thus, we realized that in order to develop our template for reviewing the large number (1500) of papers in the research study proper, we would have to separate the practitioner report type papers from the educational research papers, and further modify the template in both of these areas. In this paper we outline preliminary results of our ongoing pilot study, with a specific focus on a series of "issues of implementation" at the tertiary level of education.

Results

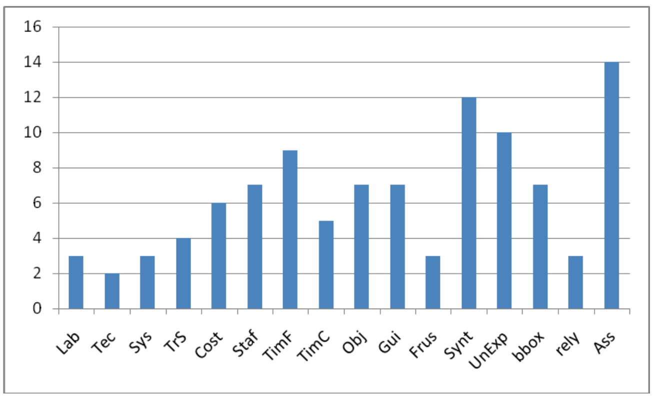
The majority of the papers in the corpus are practice reports by practitioners (88%), whereas the remaining contributions are education research papers (10%) or letters to journal editors (1%) (see Table 1). Among the practice reports, different types of contributions become apparent. Some (94) are merely examples of CAS usage. Other papers (41) are mostly examples of CAS but feature reflections by the practitioner. A few (13) have the practitioners go further and include classroom data and perform some basic analysis. There are also papers (5) that focus on classroom surveys and a small set (7) that examines a specific issue in detail. The remaining contributions (23) are conference abstracts only. The analysis of the education research papers according to Lagrange et al.'s multi-dimensional framework (2003) is still in progress. In this paper, we focus our analysis mainly on practitioner reports.

Table 1: Type of Contribution	
Presentation of Examples	46%
Examples with practitioner reflections	20%
Classroom Study	6%
Classroom Survey	3%
Examinations of a specific issue	3%
Abstract only	11%
--	-------
Education research papers	10%
Letters	1%

In addition, nearly all papers are American (87%). The computer use is more evident (59%) than the use of graphical calculators (29%) or than the combined use of both computer and graphing calculators (10%). Furthermore, the most widely used CAS in the corpus is the graphing calculator (83 papers), followed by *Maple* (53) and *Mathematica* (43). *Derive* (21) and *Matlab* (11) are also common, as well as 27 papers dealing with other CAS. In what follows, we elaborate on one particular significant aspect of the study, namely "integration issues" that emerged from our review, and also briefly report on "integration scope."

Issues of CAS Integration

Education researchers and practitioners widely wrote about issues surrounding the use and implementation of CAS at post-secondary education (72 papers). With regard to practitioner reports, 56 papers identify some issues; of these there are 20 that go into considerable detail. These papers could be further divided into two categories: Seven of them deal with a specific problem relating to CAS (e.g., rounding error) and thirteen discuss various implementations of CAS while underlining the hurdles the authors encountered. Of the sixteen issues identified in the corpus and



summarized in Figure 1, we divide them into three categories: Technical (first four columns), cost-related (fifth column), and pedagogical (last 11).

There are four issues discussed in the literature dealing specifically with technological aspects: Lab availability (Lab), reliability of technical support (Tec), system requirements (Sys) and troubleshooting (TrS). These issues may not be independent from each other. For example, May (1999, p. 4) urges instructors to test out their *Maple* worksheets on the lab computers rather than their own workstations due to such machines having less memory installed in them. Weida (1996, p. 3) notes that in troubleshooting, various hardware problems arise and his “experience and lots of calls to the Computer center” helps. An unexpected issue for him was the class interruption of students not enrolled in his class. While they would never think to disrupt a lecture, they would see nothing wrong with walking into his lab session to complete homework for other courses.

Many reports mention the issue of costs (Cost) incurred by integrating CAS into instructors’ courses, providing few further details beyond the existence of the financial obstacle. An exception occurs in one paper where the authors argue for a particular choice of open-source (free) technology, namely *GeoGebra* (Hohenwarter et. al, 2007, p. 5).

Wu (1995) notes that besides the cost aspect, enacting calculus reform “requires more talent and training” (p. 1). This need for trained staff (staf) is mentioned in seven papers, often in conjunction with other issues. For example, to deal with technical difficulties during labs, Weida relies on his own experience to assist in troubleshooting (1996, p. 3). At the beginning of an attempt at CAS integration, Schurrer and Mitchell (1994, p. 1) wondered, “how they could go about motivating [sceptical mature faculty] to consider introducing the available technology and making the curricular changes this would require?”

Schurrer and Mitchel (pp. 1-2) further discuss the need for time for the faculty (TimF) to design courses and meaningful activities with technology. Their department required decisions on types of technology used and on what technology curriculum package had a “right mix.” They note that program-wide integration takes time. In their case at University of Iowa, it took seven years to implement (p. 3). Even after a curriculum change, additional time demands on faculty are reported by practitioners. Wrangler (1995, p. 8) notes that near constant improvement is needed in lab experiments and stresses that for faculty there is “no resting on laurels.” A closely related issue is the problem of time management in courses (TimC). Wrangler (p. 8) remarks that besides the time he spent outside of class, he had to take his students into the lab and walk them through basic commands. Many other practitioners, such as May (1999, p. 4), expresses similar sentiments. While this issue is discussed less frequently than time spent outside the classroom, practitioners report about both issues in conjunction (e.g., Wrangler p. 8).

CAS integration also affects classroom time management with respect to course content. Dogan-Dunlop (2003, p. 4) remarks that, “since class time was allocated for in-class demonstrations and discussions, detailed coverage of all the topics that were included in the syllabus was not possible.”

Another source of pressure on time management is the failure of students to achieve learning objectives (Obj). Krishanamani and Kimmons (1994, p. 4) note that students failed to learn material assigned in labs and they had to include it in later lectures.

One particular type of student error that clashes with learning objectives is the assumption on the part of students that their methodology is correct if their paper-and-pencil calculations match up with results obtained from the computer. As Cazes et. al. (2006 p. 342) write, “a correct answer does not mean the method is correct or is the best one. Teachers and students must be aware of such… pitfalls.” Often students engaged in trial and error strategies, with students guessing the answer from feedback without making a proper mathematical argument (p. 347). Instructors sometimes failed to ensure that students found an “optimal” solution to a particular problem rather than just having a “correct” answer (pp. 342-343).

Pedagogical difficulties with learning objectives can place demands on faculty time not only inside but also outside of the lecture hall. Dogan-Dunlap (2003 p. 4) had to redesign his course and the use of CAS within it three different times because of such concerns. As previously discussed, there is an ongoing time commitment by faculty to improve their lecture and laboratory instruction and Dogan-Dunlap’s experiences show that student difficulties may greatly influence the nature of those changes.

Related to the learning objectives issue, that of guidance (Gui) also emerges from the review. Often practitioners show concerns as to how much help they should give their students without compromising learning objectives. Westhoff (1997) designed a student project for Multivariate Calculus on the lighting and shading of a 3-dimensional surface. He found that the difficulty in the project, due to its complexity, lays in determining how much he could tell his students (p. 6). Another area in which guidance becomes an issue is mentioned by Weida (1996). Noting that there is a “fine line between helping students… and ‘giving away’ the answers,” he remarks that such a problem is “particularly exacerbated at the end of a lab when the slower workers are running out of time” (pp. 3-4). Weida further presents the idea that careful scheduling could help alleviate this by ensuring that there isn’t a need to leave immediately after the lab.

Student frustration (Frus) is another issue related to learning objectives. Cazes et. al. (2006, p. 344) note that students would often seek help either online or via the instructor “after having encountered the first difficulty” rather than attempting to solve the problem on their own. Krishahamani and Kimmons (1994) took steps to reduce anxiety both in course design and in providing additional help for students. Several measures, including reduced expectations, more time for tests, increased extra credit problems and a homework hotline were implemented (p. 2). Clark and

Hammer (2003, p. 3) had a project for first year calculus modeling a rollercoaster. They found that "students who were not as "good" at *Maple* struggled, found the project (and *Maple* syntax) frustrating and were just happy to produce one mathematical model." This suggests possible relationship between student frustration and failure regarding activity learning objectives, and the CAS syntax issue.

Syntax (Synt) is the second most frequent concern for both practitioners and students. Cherkas (2003) found this to be a source of student dissatisfaction. He quotes a student complaining, "*Mathematica* would cause a lot of problems. If I make a mistake in the syntax, I couldn't do my work" (p. 31).

Tiffany and Farley (2004) exclusively focus on common mistakes in *Maple*, emphasizing the hurdle for practitioners caused by syntax. Practitioners employ various schemes attempting to minimize this difficulty. Some such as May (1999) design interactive workbooks that eliminate the need for teaching syntax entirely. Others like Herwaarden and Gielen (2001, p. 2) provide *Maple* handouts with expected output to their students. Some emphasize a pallet-based CAS such as *Derive* (Weida, 1996, p. 1) because it is easier to learn and has, according to them, a more straightforward notation.

Another source of student frustration is the unexpected behaviour of CAS (UnExp) even when their reasoning is syntactically and mathematically correct. Sometimes this is merely the case of paper-and-pencil calculations not easily matching up with CAS output. CAS may employ an algorithm efficient for computation and not necessarily one that matches a hand technique. For example, Holm (2003, p. 2) found that an online integral calculator would (rather than using the substitution method for $\int x(3x^2-1)^7\,dx$) simply expand the product and use the power rule. He notes that such cases provide an opportunity for learning, and that, referring to another classroom assignment, the more "savvy student would… expand $\frac{1}{48}(3x^2-1)^8$." Unexpected behaviour of CAS also takes the form of errors by the computers themselves. Due to the nature of floating point arithmetic and in spite of correct input by the user, roundoff error can cause the output to be wrong (Leclerc, 1994, p. 1). To encourage her students to adapt, Wu (1995, p. 2) purposely designed a lab with roundoff error. LeClerc urges students to be instructed in the nature of floating point arithmetic so that they "will be able to detect when roundoff has corrupted a result and hopefully find better ways to formulate or evaluate the computation" (1994, p. 4).

The concept of the "black box" (bbox) is examined in seven papers. Though this issue tends to be explored in more detail in education research papers, practitioners comment on it as well. O' Callaghan (1997, p. 3) writes that faculty at Southeastern Louisiana University expressed concern that "students would become button pushers rather than problem solvers." The managed used of the black box as an opportunity for students to explore complex mathematics beyond their level is discussed in great detail in education research papers (e.g., Winsløw, 2003, p. 283). Practitioners do not emphasize this potential as much. However, Cherkas (2003, p. 234) notes that CAS allows practitioners “to teach at a higher level of mathematical sophistication than is possible without such technology.”

Closely related to the “black box” issue, is the fear that students become too reliant on the technology (rely). This, along with student frustration, is the least mentioned pedagogical issue. Cherkas reports on a student complaint that s/he could not do questions on tests because “Mathematica usually did them for me” (pp. 231-232). An over-reliance on technology may interfere with learning objectives. Considering this, Shelton (1995, p. 1) emphasizes her “top-down” approach and writes that “students can avoid the technology crutch and approach the goal of developing determination and mathematical maturity to perform mathematics without the technology.”

The last and most commonly examined issue encountered in the literature is that of assessment (Ass). Practitioners encounter problems in evaluation. Schlatter (1999) allowed for CAS use during his exam for his multivariate calculus course. Unfortunately, in a question designed to test student understanding of the divergence theorem, several students simply used the CAS capabilities to solve the integral in a “brute force” approach (pp. 8-9). A poorly designed assessment thus leads to a failure in learning objectives. Schlatter further writes that he expected “to spend more time during this semester… more carefully designing exam questions” (p. 8), pointing again to the issue of faculty time.

Interpreting CAS output is discussed frequently. Quesada and Maxwell (1994, p. 207) never accept a decimal answer (even if correct) if there is a proper algebraic expression. Many papers that discuss mathematical projects stress the use of written reports (e.g. Westhoff, 1997, p. 1). Lehmann (2006, p. 3) writes in his assignment “the important part of this assignment is the thought you put into it, the analysis you do and the presentation of your solution, not the answers themselves.” Xu (1995, p. 1) found that students were finding derivatives of easy functions by hand on assignments, but using graphing calculators to solve the more difficult questions. To show students “that the calculator could not do everything for them” he found functions in the textbook that “were easy to handle by hand but could not be done easily on the calculator.”

CAS Integration Scope

Policy making regarding the curriculum in tertiary education is rather different than in school education. Hodgson and Muller (1992) mention that school mathematics curricula are in general developed by Ministries or Boards and implemented in the classroom by teachers, whereas tertiary mathematics curricula are developed and implemented by the same actors, i.e., faculty in departments of mathematics. However, change involving technology in tertiary curriculum, like in its secondary school counterpart, seems to remain very slow (Ruthven & Hennesssy, 2002). Lavicza (2006) argues that due to academic freedom, "Mathematicians have better opportunities than school teachers to experiment with technology integration in their teaching". This ad hoc basis is strongly reflected in our literature review. A large majority (67%) of the corpus restricted to practice reports discusses CAS usage with regards to one course, or in other words, CAS integration by one practitioner. While 16% has a scope that reaches across a series of courses (e.g. calculus courses), 11% discusses a CAS implementation with a grouping of courses (e.g. all first year courses). Only 6% discusses a program-wide implementation within a department.

Conclusions

There is a need to develop a framework for the review of literature on the use of CAS at tertiary education that will integrate specificities of university-level education and technology integration. A significantly stronger majority of papers in our study stemmed from practitioner use (88%) than in Lagrange et al.'s (2003) study (60%) which stated, "Most of the [practitioner] papers lack sufficient data and analysis and we could not integrate them into the [detailed (statistical) analysis]" (p.242). Our selection of journals and conferences for our pilot study may have influenced the above percentage. Nevertheless, this reality will clearly influence the development of our analytical framework henceforth. Lagrange et al. (2003) further state,

> [Practitioner] papers offer a wealth of ideas and propositions that are stimulating, but diffusion is problematic because they give little consideration to possible difficulties. Didactical research has to deal with more established uses of technology in order to gain insights that are better supported by experimentation and reflection. We have then to think of these two trends as complementary rather than in opposition. (p.256)

We aim at elaborating upon these complementary trends at the post-secondary level by both analyzing existing instructional practices and scrutinizing problematic issues within implementation. Lagrange et al. (2003) further state that the "integration into school institutions progresses very slowly compared with what could be expected from the literature" (pp. 237-8). This might be the case for school education, but apparently less so for tertiary education (Lavicza, 2008). The research literature about school mathematics and technology seems to pay less than adequate attention to the actual classroom implementation piece. The literature about tertiary mathematics and technology tends to inform us more about (individual) implementation than its didactical issues and benefits. This suggests that there may be a need for more education research focusing on the integration of technology in tertiary education. It also points, as suggested by Table 2, to the need of resources for departments of mathematics for systemic integration of technology in curriculum. At the recent ICME 11 conference, the results of a special survey highlighted concerns about the international trend of disinterest in university mathematics (ICME 11, n.d.). Departments of mathematics have a responsibility to question the current curriculum. We contend that part of this responsibility includes the careful consideration of the role and relevance of technology within that 21st-century curriculum and classroom.

References[8]

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work, *IJCML, 7*(3), 245-274.

Cazes, C., Gueudet, G., Hersant, M. & Vandebrouck, F.: 2006, 'Using E Exercise Bases in Mathematics: Case Studies at University', *IJCML*, 11(3), pp. 327-350.

Cherkas, B.: 2003, 'Finding Polynomial and Rational Function Turning Points in Precalculus', *IJCML*, 8(2), pp. 215-234.

Clark, J. & Hammer, T.: 'Interactive Web-Based Calculus Projects at Hollins University: Area of U.S. States and Roller Coasters', *Electronic Proceedings of the 16th ICTCM, available at* */VOL16/C019/paper.pdf.

Dogan-Dunlop, H.: 2003, 'Technology-Supported Inquiry-Based Learning In Collegiate Mathematics', *Electronic Proceedings of the 16th ICTCM*, available at */VOL16/C047/paper.pdf.

Herwaarden O. & Gielen J.: 'Linking computer algebra systems and paper-and-pencil techniques to support the teaching of mathematics', Proceedings of the CAME 2001 Symposium, available at http://www.lkl.ac.uk/research/came/events/freudenthal/1-vHerwaarden.doc

Hodgson, B., H., & Muller, E., R. (1992). The impact of symbolic mathematical Systems on mathematics education. In B. Cornu & A. Ralston (Eds.), *The influence of computers and informatics on mathematics and its teaching* (pp. 93-107). UNESCO Science and Technology Education, #44.

Hohenwarter, M., Preiner J., & Yi, T.:2007, 'Incorporating GeoGebra Into Teaching Mathematics At The College Level', *Electronic Proceedings of the 19th ICTCM,* available at */VOL19/S100/paper.pdf.

Holm, J.: 2003, 'Potential Pitfalls for Students Using Online Math Utilities', *Electronic Proceedings of the 16th ICTCM*, available at */VOL16/C009/paper.pdf.

ICME 11. (n.d). *Recruitment, entrance and retention of students to university mathematics studies in different countries.* Retrieved August 31, 2008, from http://www.icme11.org/node/1606.

Krishanamani, V. & Kimmons, D.: 1994, 'Using Technology as a Tool in Abstract Algebra and Calculus: The MTSU Experience', *Electronic Proceedings of the 7th ICTCM*, available at */VOL07/C007/paper.pdf.

Lagrange, J. B., Artigue, M., Laborde, C., & Trouche, L. (2003). Technology and mathematics education: multidimensional overview of recent research and innovation. In

[8] ***http://archives.math.utk.edu/ICTCM***

F. K. S. Leung (Ed.), *Second International Handbook of Mathematics Education* (Vol. 1, pp. 237-270). Dordrecht: Kluwer Academic Publishers.

Lavicza, Z. (2006). The examination of Computer Algebra Systems (CAS) integration into university-level mathematics teaching. In C. Hoyles, J. Lagrange, L. H. Son & N. Sinclair (Eds). *Proceedings for the Seventeenth ICMI Study Conference: Technology Revisited, Hanoi University of Technology, 3rd-8th December, 2006* (c33)[CD-ROM].

Lavicza, Z. (2008). Factors influencing the integration of Computer Algebra Systems into university-level mathematics education. *IJTME.*

Leclerc, A.: 1994, 'Should We be Concerned about Roundoff Error?', *Electronic Proceedings of the 7th ICTCM*, available at */VOL07/C024/paper.pdf.

Lehmann, J.:2006, 'Calculus Computer Laboratory: Experience Guiding Current Practice', *Electronic Proceedings of the 18th ICTCM,* available at */VOL18/C006/paper.pdf.

May, M.: 1999, 'Designing Courseware with Maple', *Electronic Proceedings of the 12th ICTCM,* available at **/VOL12/P013/paper.pdf*.

O'Callaghan, B.: 1997, 'Calculators, Attitudes, and Success', *Electronic Proceedings of the 10th ICTCM*, available at */VOL10/C019/paper.pdf.

Pierce, R. L., & Stacey, K. (2004). A framework for monitoring progress and planning teaching towards the effective use of Computer Algebra Systems. *IJCML, 9*, 59-93.

Quesada, A. & Maxwell, :1994, 'The effects of using graphing calculators to enhance college students' performance in precalculus', *ESM*, 27(2), pp 205-215.

Ruthven, K., & Hennessy, S. (2002). A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics, 49*, 47-88.

Schlatter, M.:1999, 'Using Matlab in a Multivariable Calculus Course', *Electronic Proceedings of the 12th ICTCM*, available at */VOL12/C021/paper.pdf.

Schurrer, A. and Mitchell D.: 1994, 'Technology and the Mature Department', *Electronic Proceedings of the 7th ICTCM,* available at */VOL07/C002/paper.txt.

Shelton, T.: 1995, 'Reducing Tedium in Teaching and Learning', *Electronic Proceedings of the 8th ICTCM*, available at */VOL12/C021/paper.pdf.

Tiffany, P. & Farley, R.: 2004, 'Jumpstarting Technology in the Calculus Curriculum', *Electronic Proceedings of the 17th ICTCM*, available at */VOL10/C011/paper.pdf.

Weida, R.: 1996, 'Computer Laboratory Implementation Issues at a Small Liberal Arts College', *Electronic Proceedings of the 9th ICTCM*, available at */VOL09/C051/paper.pdf.

Westhoff, R.: 1997, 'Developing Shading Functions for Surfaces using Mathematica', *Electronic Proceedings of the 10th ICTCM,* available at */VOL10/C011/paper.pdf.

Winsløw, C.: 2003, 'Semiotic and discursive variables in cas-based didactical engineering', *ESM*, 52(3), pp. 271-288.

Wrangler, T.: 1995 'Paradigm Lost: A Modern Approach to Teaching Ordinary Differential Equations', *Electronic Proceedings of the 8th ICTCM,* available at */VOL08/C059/paper.pdf.

Wu, D.: 1995, 'Some Examples on Using Maple to Increase Students' Understanding of Calculus', *Electronic Proceedings of the 8th ICTCM*, available at */VOL08/C052/paper.pdf.

Xu, M.: 1995, 'Using Graphing Calculators to Motivate Students in a Calculus Course', *Electronic Proceedings of the 8th ICTCM*, available at */VOL08/C037/paper.pdf

GEOMETERS' SKETCHPAD SOFTWARE FOR NON-THESIS GRADUATE STUDENTS: A CASE STUDY IN TURKEY

Berna CANTÜRK-GÜNHAN , Deniz ÖZEN

Dokuz Eylul University, Izmir, Turkey.

The purpose of this paper is to determine mathematics teachers' views about Geometers' Sketchpads Software (GSP) and to analyze the effects of training sessions on prospective teachers' ability to integrate instructional technology into geometry concepts. For that purpose, we selected two graduate pre-service teachers and investigated GSP activities. Training sessions about using GSP were proposed to them. The data come from interviews with them and GSP activities improved by them. The results of this study indicate their awareness level was increased about GSP.

Keywords: *Teacher Education, Secondary Mathematics Education, Non-thesis Graduate Program, Integrating Technology, Geometers' Sketchpad Software.*

INTRODUCTION

Today's use of technology as a learning tool supplies the students with gaining the mathematics skills in their lessons. According to Newman (2000), the use of technology in learning arouses curiosity and thinking, and challenges students' intellectual abilities. Kerrigan (2002) state that using mathematics software promote students' higher order thinking skills, develop and maintain their computational skills. For this reason, teacher training is crucial in order to use technology in mathematics education.

Computers could be used in school for teaching geometry, and since then a lot of work has been done that discusses many aspects of using Dynamic Geometry Software (DGS) in education (Kortenkamp, 1999). In this study, it was concerned with DGS activities developed by non-thesis graduate student teachers. Non thesis graduate program is in Turkey was opened for the purpose of educating future teachers. The secondary school (grade 9-11) mathematics teacher training program made up of two different programs. The Five-Year Integrated Programs (3.5+1.5) in Faculty of Education and Non Thesis Graduate Program (4+1.5) in Faculty of Science. Last 1.5 year part is the same for both 3.5+1.5 and 4+1.5 programs. Of these programs 3.5 and 4 year are spent on taking the mathematics courses and remainder years on pedagogical courses. After graduation, they can be secondary school mathematics teacher. This program is described in more detail in YOK (1998). The aim of this study was to investigate whether their views changed after the education process and to determine the outcomes about student teachers' proficiency.

THEORETICAL FRAMEWORK

In geometry, teachers are expected to provide "well-designed activities, appropriate tools, and teachers' support, students can make and explore conjectures about geometry and can learn to reason carefully about geometric ideas from the earliest years of schooling" (NCTM, 2000). Mathematics teachers can help students compose their learning by using geometry sketching software. Geometer's Sketchpad allows younger students to develop the concrete foundation to progress into more advanced levels of study (Key Curriculum Press, 2001).

Reys et al. (2006) point out young learners of mathematics need to

- experience hands-on (concrete) use of manipulative for geometry such as geoboards, pattern blocks and tangrams,
- connect the hands-on to visuals or semi concrete models such as drawings or use the sketching software on a computer,
- comprehend the abstract understanding of the concepts by seeing and operating with the picture or symbol of the mathematical concept (cited in Furner and Marinas, 2007).

GSP is an excellent tool for students to understand the properties of geometric shapes and to model for them mentally manipulating objects. GSP can also provide students to visualize the solid in their mind. In literature, McClintock, Jiang and July (2002) found GSP provides opportunities to have a distinct positive effect on students' learning of three dimensional geometry. In another study, Yu (2004) stated that the students' concurrent construction of figurative, operative and relational prototypes was facilitated by dynamic geometric environment. That's why, the knowledge about which DGS and DGS activities how prepared should be given the student teachers.

METHOD

Participants

Case study was used in this paper. This research was conducted during the spring term of 2007–2008 academic years in spring term. The study was conducted with two secondary school preservice teachers attending the 4+1.5 Integrated Secondary Mathematics Teacher Education Program at Dokuz Eylul University in Turkey. Of the ten students in this program there were two volunteers. In this process, they took the courses about mathematics content knowledge, pedagogical content knowledge and general pedagogical knowledge. All participants had basic computational skills but none of them knew how to use DGS.

Data Collection

The data were collected from interviews and the activities which are prepared by the student teachers. The interviews were semi-structured in nature. In the beginning of the research, the opinions of the participants towards GSP software are taken with semi-constructed interview form. Each interview took approximately 15-20 minutes and recorded with a tape. Then the participants attended a six-hour GSP training sessions which is given by the researchers. After the program, it was demanded that the participants developed the GSP activities. Finally, the participants` opinions towards GSP software are taken again.

The Geometer's Sketchpad Training Sessions

The training sessions allowed the instructor to prepare the non-thesis graduate student teachers to enter their future mathematics classrooms not only knowledgeable about the capabilities of instructional technology, but also experienced enough to appropriately integrate their selected software. The GSP training sessions' content is given Table 1.

	Training Sessions	Topics	Duration
DAY 1	Introductory *(Guided & Discussed)*	• major concepts of mathematics education • the aim of the involved Software • introduction to dynamic geometry environment with GSP • introduction to tools and menus of the Software	1 hour
	Constructing Geometrical Concepts *(Guided & Discussed)*	• to construct basic concepts of geometry • to transform the rotation, reflection, and dilation of the figures • to construct regular and non-regular polygons, and its interiors • to measure in geometry (length, distance, perimeter, area, circumference, arc angle, arc length, radius, etc.) • to graph various functions and its derivative	1 hour
DAY 2	Animation and Presentation (*Guided & Discussed)*	• to use action and hide/show buttons • to tabulate the data • to prepare presentations	2 hours
	Activity Planning *(Guided & Individual)*	• to plan activities and practice it	2 hours

Table 1: Training Sessions

DAY 1 included two sessions. Each session is formed of an hour.

<u>*Introductory Session:*</u> The introductory session contained the major concepts of mathematics education, introduction to dynamic geometry environment with GSP and the aim of the involved Software.

In the beginning of the session, the participants discussed the major concepts - conceptual development, problem solving, modelling verbal problems, creative thinking, analytical thinking etc.- in order to determine their readiness with researcher. Then, they argued the aim of the involved Software. Afterwards, the participants introduced Dynamic Geometry Environment, the menus, sub-menus and tools of the GSP Software. When the participants get information about tool box, text palette, file menu, edit menu, display menu, construct menu etc., the researcher advanced next session.

Constructing Geometrical Concepts: In this session, the participants find out how to construct the basic concepts of geometry; such as ray, line, segment, paralel line, perpendicular line, angle bisector, median of triangle, altitude of triangle, arc etc.

When the participants learned how to use the menus, sub-menus and tools, the researcher showed them some operations. The participants learned about constructing regular and non-regular polygons, and its interiors. After that, they learned to change the color and width of the lines and figures.

Then, they transformed the rotation, reflection, and dilation of the figures. Subsequently, they measured length, distance, perimeter, area, circumference, arc angle, arc length, radius, etc. with using GSP.

When they reached the graph menu, they defined coordinate system, chose grid form and they draw some graphs with GSP, such as sinus, cosinus, tangent, etc. Afterwards, they graphed various functions and its derivatives. During this session, the participants discussed the functions of GSP each other if it was necessary or it was forgotten.

DAY 2 comprised two sessions. Each session are made up of two hours.

Animation and Presentation: In this session, the participants found out text palette on advanced level. Next they learned motion controller, how to paste picture and then passed animation and hide/show buttons. They learned how to utilize animations and change it's' speed. Then they learned to trace points, segments, rays and lines. Afterwards they focused on tabulate the data on tables in order to show them regularly.

After they learned animation and presentation clues, they started to organize page setup and document options in order to prepare excellent presentations.

Activity Planning: This session includes all of the applications learned. The researchers wanted the participants to prepare activities. And they also wanted to apply all the operations learned in their activity. In the preparation period, if the participants needed to be supported, the researchers could be guiding them.

Data Analyses

In the interview, four open-ended questions were asked to the participants and the interview guide was used in this stage. During the interview, the questions like "What are the GSP aims in mathematics learning environment?" "Which students' skills are able to improve by GSP activities?", "What do you take into account while the GSP activities are composed?" and "How can you assess the students with the GSP activities?" were answered by the students.

The evaluating criteria were determined in order to assess the activities improved by the student teachers. These criteria were adapted from Roblyer (2003).

1.	Connection to mathematics standards.
2.	Appropriate approach to mathematics topics with respect to grade, ability.
3.	Presence of conceptual development, problem solving/higher order thinking skills.
4.	Use of practical applications and interdisciplinary connections.
5.	Suitability of activities (interesting, motivating, clear, etc.)

Table 2: Evaluation Criteria adapted from Roblyer (2003)

RESULT

In this section, the analysis of data obtained from two preservice teachers' view transcripts and activities which they prepared are presented.

Handan's Case

Handan is working as an assistant teacher in private teaching institution for a year. During the pre-interview, four questions were asked her. She made explanations as follows:

Researcher : What are the GSP aims in mathematics learning environment?
Handan : It supplies the students with learning and visualizing in math lessons and preparing animations.
Researcher : Which students' skills are able to improve by GSP activities?
Handan : The students' spatial thinking skills are improved.
Researcher : What do you take into account while the GSP activities are composed?
Handan : It should be appropriate the students' cognitive level.
Researcher : How can you assess the students with the GSP activities?
Handan : I don't know because of lacking knowledge about GSP.

As can be seen in her statements, although she mentioned that she did not know GSP, she could be able to estimate its aims, skills to be improved and rules taken into account when the activities had done.

After training sessions, the researcher wanted her to prepare GSP activities whatever topics she wished. She chose the congruence as a subject of geometry instruction. Her activity is given Figure 1.

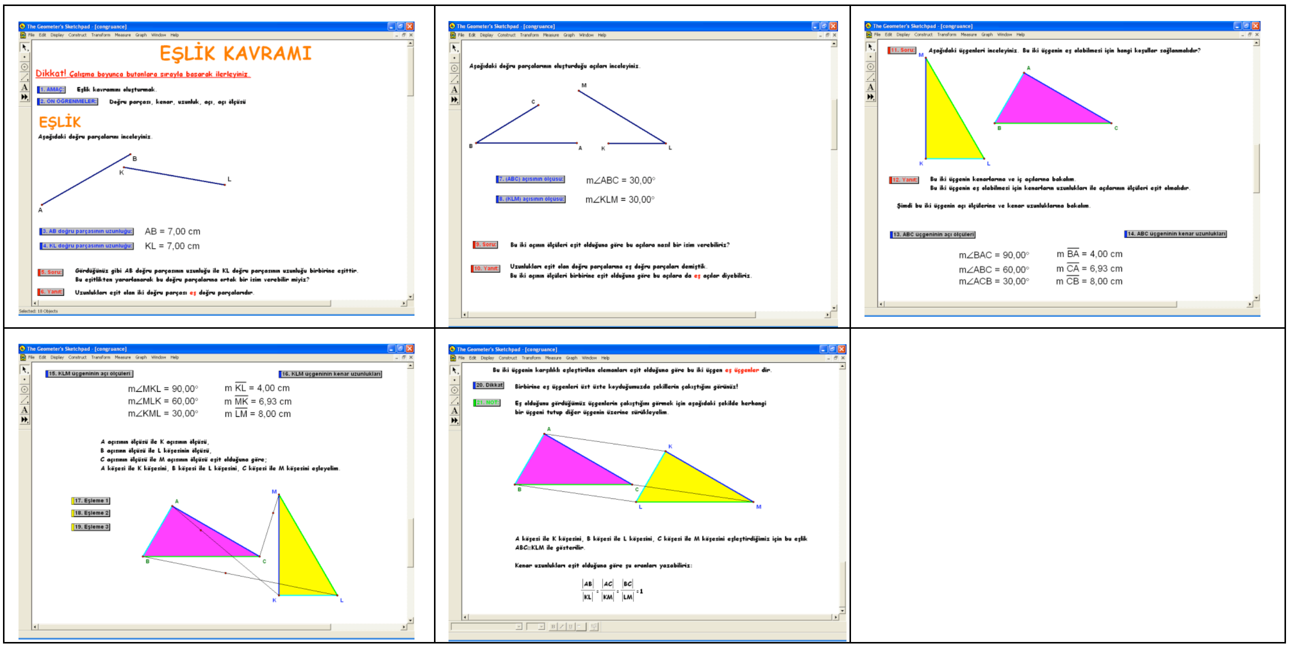


Figure 1: Handan's Activity

The content of her activity was about congruence. She decided to plan her activity for constructing the concept of congruence. As regards to the activity, the student knows the aim of the subject (step 1) and the concepts related to the subject (step 2). Handan gave directions to the students in her activity, in general. Therefore the student follows the instructions and carries on step by step. Afterwards, she gave two segments as AB and KL. She demonstrated the length of AB and KL segments (step 3-4). In the next step of the activity, she wanted students to compare the length of AB segment with KL segment. She asked whether the students call a common name to these segments (step 5) and explained it simply (step 6). Subsequently, she gave two angles and its measurements (step 7-8). She told the angles have the same measurement (step 9) and asked what the common name of the angles is (step 10). Later she constructed two triangles (ABC and KLM) and asked the students in what conditions they are congruent (step 11). Later on she showed the conditions of the congruence (step 12) and measurements of the triangles (step 13-14-15-16). In following steps, she paired each corners of the triangles and animated them (step 17-18-19). Finally, she drew the students' attention for the coincidence of triangles and demonstrated this (step 20-21).

When her activity arranged was assessed via the so-called evaluation criteria in Table 2, it was seen that the activity was connected to mathematics standards organized by Ministry of National Education (MNE) in Turkey, suited approach to mathematics topics -to explain congruence of triangle- with respect to 10^{th} grade but it was too simple and like 8^{th} grade level. It was provided conceptual development, also clear but not engaged the students in real life situations and interdisciplinary connections. It is useful for constructing the concept of congruence but not provide satisfactory knowledge. It wasn't prepared for improving the students' problem solving skills also. Handan utilized the mathematical language adequately. In respect of technicality, the activity is good. Each step's button is made as hide/show button. The 17th and 19th steps' button have the same function, so one of them is needless. The activity hasn't got any other technical problem.

Afterwards she had done activity; the post-interview was carried out with her and it was given her comments as follows:

Researcher	: What are the GSP aims in mathematics learning environment?
Handan	: It provides the students learn geometrical concepts…their problem solving skills are improved and the concepts are visualized.
Researcher	: Which students' skills are able to improve by GSP activities?
Handan	: The students' spatial thinking…. and problem solving skills are improved.
Researcher	: What do you take into account while the GSP activities are composed?
Handan	: It should be interesting…. appropriate for the students' cognitive level and the students' opinions can be taken while the activities are prepared.
Researcher	: How can you assess the students with the GSP activities?
Handan	: The students can be able to do the applications involved in GSP and these are evaluated.

Considering her statements, it is seen that her views changed after training sessions and her activity. She has primarily information about GSP and she awakes of what taking into account while the GSP activities are composed.

Mualla's Case

Mualla is also working as an assistant teacher in private teaching institution for a year. In time of the pre-interview, she gave responses as follows:

Researcher	: What are the GSP aims in mathematics learning environment?
Mualla	: …It constitutes long lasting learning in math lessons and provides the teachers and the student drawing figures, preparing animations.
Researcher	: Which students' skills are able to improve by GSP activities?
Mualla	: GSP improves the students' spatial thinking skills.
Researcher	: What do you take into account while the GSP activities are composed?
Mualla	: It should be interesting…
Researcher	: How can you assess the students with the GSP activities?
Mualla	: I don't know…

In the analysis of this interview, she determined which skills improved and what she pays attention during the GSP activities are composed. Besides it is seen that Mualla's responses are similar to the Handan's statements.

After training sessions, the researcher wanted her to prepare GSP activities whatever topics she wished. She chose the similarity as a subject of geometry instruction. The activity involved is given Figure 2.

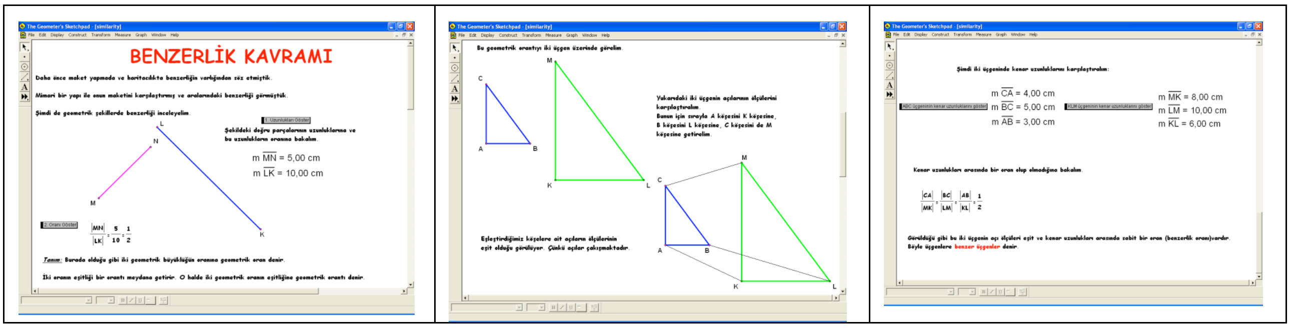


Figure 2: Mualla's Activity

Mualla's activity deals with similarity of triangles. She tried to carry out her activity for constructing the concept of similarity. According to her activity, she acknowledged that the students have little knowledge about the subject. Mualla generally gave directions to the students in her activity, as Handan did. However, her activity didn't similar to in terms of following the instructions step by step. In the beginning of the activity, she mentioned few real-life examples to the students about similarity and then she passed the similarity between geometrical concepts. She gave two segments, like Handan, and she compared the length of them under the first button. The second button shows the students the ratio of the lengths of the segments. After that, the definition -geometrical ratio and geometrical proportion- was given, and demonstrated. Then, she compared the measures of each angle of the triangles and mentioned the coincidence of each angle. Afterwards, she showed and compared the length of sides of the triangle and stated whether the sides of both triangles have a ratio or not. Lastly, she defined a stable ratio, as the ratio of similarity.

When her activity organized was assessed by means of the evaluation criteria in Table 2, it was seen that the activity was overlapped mathematics standards organized by MNE in Turkey, partly suited approach to mathematics topics -to explain similarity of triangle- with respect to 10th grade. It was provided conceptual development, but not connected to the students in real life situations and interdisciplinary connections. Her activity was clear and understandable but it was also towards 8th grade and too simple. It wasn't also provides sufficient knowledge. It wasn't prepared for improving the students' problem solving skills also. Mualla used the mathematical language few adequately. In respect of technicality, the activity is not bad. Each step's button was made as hide/show button, as Handan did. It didn't include enough animation and demonstration. Finally it was said that, the activity hasn't got any technical problem.

After she had done activity; her comments during the post-interview was given as follows:

Researcher : What are the GSP aims in mathematics learning environment?

Mualla : It provides the students learn geometrical concepts and problem solving, proof geometrical theorems. In addition to, it can be long lasting learning.

Researcher : Which students' skills are able to improve by GSP activities?

Mualla : The students' spatial thinking was improved.

Researcher : What do you take into account while the GSP activities are composed?

Mualla : It should be appropriate the students' cognitive level and the mathematics standards

Researcher : How can you assess the students with the GSP activities?

Mualla : It can be ask some question in GSP aiming at determining whether they learned the geometric concepts. We expect that the students reveal the relationships between geometric concepts.

As her statements, she increases information about GSP. It follows from her responses that her point of view enlarged after training sessions. She encouraged and determined carefully what she does with GSP in mathematics learning environment after she prepared activities herself.

DISCUSSION AND CONCLUSION

In this study, the data indicated that Dynamic Geometry Software (DGS) is importance for in geometry education. Generally speaking, Handan and Mualla learned some properties of the GSP and their views look like. At the end of the study, they realized how they can use GSP prepare the activities on GSP. It is pleasing a finding. As the case of Handan, she gave directives detailed in her activity. Although she expected that the students said the concept of congruence, this concept was given by her at the beginning of the study. As the other case, Mualla set out the similarity proportion when she prepared her activity. Both of them did not mention the kinds of congruence and similarity. They may be encouraged that the students found the kinds of its. However, these activities were regular with regard to visual. Their activities did not include assessment. As Key Curriculum Press (2001) mentioned, teachers can use GSP to create worksheets, exams, and reports by exporting GSP figures and measurements to spreadsheets, word processors, other drawing programs, and the Web. These results indicate that DGS is important in teacher education and DGS training must be in non-thesis graduate education in a way of detailed.

REFERENCES

Furner, J.M. & Marinas, C. A. (2007). Geometry Sketching Software for Elementary Children: Easy as 1,2,3 . *Eurasia Journal of Mathematics, Science & Technology Education*, 3(1), 83-91.

Kerrigan, J. (2002*).* Powerful software to enhance the elementary school mathematics program. *Teaching Children Mathematics*, *8*(6), 364–377.

Key Curriculum Press. (2001). The geometer's sketchpad reference manual. Emeryville CA: Author. Also found at: http://www.keypress.com/sketchpad/

Kortenkamp, U. (1999). Foundations of Dynamic Geometry. Unpublished Doctoral Thesis. Swiss Federal Institute of Technology, Zurich.

McClintock, E., Jiang, Z., & July, R. (2002). Students' development of three-dimensional visualization in the geometer's sketchpad environment. Research paper completed as part of a grant from the National Aeronautics and Space Administration. (ERIC Document Reproduction Service No. ED 471759).

National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. NCTM: Reston, VA.

Newman, C. (2000) Online testing rated, *Advertising-Age,* 71(20), p. 64.

Roblyer, M.D. (2003). *Integrating educational technology intoteaching*. Upper Saddle River, NJ: Pearson Education, Inc.

YOK, (1998). Eğitim Fakülteleri Öğretmen Yetiştirme Programlarının Yeniden Düzenlenmesi

Yu, P. W. (2004). Prototype development and discourse among middle school students in a dynamic geometric environment. Unpublished Doctoral Thesis. Illinois State University, USA.

THE DESIGN OF NEW DIGITAL ARTEFACTS AS KEY FACTOR TO INNOVATE THE TEACHING AND LEARNING OF ALGEBRA: THE CASE OF ALNUSET

G. Chiappini, B. Pedemonte

Istituto per le Tecnologie Didattiche – CNR Genova

The integration of CAS systems into school practices of algebra is marginal. To integrate effectively digital technology in the teaching and learning of algebra, it is necessary to go beyond the experience of CAS and of their instrumented techniques and to face the design of new artefacts. In this paper we discuss design problems faced in the development of a new digital artefact for teaching and learning of algebra, the Alnuset system. We present the key ideas that have oriented its design and the choices we have worked out to instrument its incorporated algebraic techniques. We compare the quantitative, symbolic and functional instrumented techniques of Alnuset with those of CAS highlighting crucial differences in the teaching and learning of algebra.

Keywords: Alnuset, Instrumented technique, CAS, Algebraic learning

INTRODUCTION

In the last 15 years a scientific debate on the role of technology in supporting teaching and learning processes in the domain of algebra has been going on. This debate originates from research studies carried out in different countries with the purpose of studying the use of Computer Algebra Systems (CAS) in school contexts. In particular, near benefits (Heid, 1988, Kaput, 1996, Thomas, Monaghan and Pierce, 2004) obstacles and difficulties have been identified in using this technology by students and teachers (Mayes, 1997, Drijvers, 2000, Drijvers, 2002, Guin & Trouche, 1999). Results of these research works (Artigue, 2005) highlight that the integration of CAS systems into the school practice of algebra remains marginal due to different reasons. CAS expands the range of possible task-solving actions. As a matter of fact, techniques involved in a CAS (instrumented techniques) are in general different from those of the paper and pencil environment. Managing the complexity of CAS instrumented techniques and highlighting the potential offered by the machine to the student is hard work. As shown by some experiments (Artigue, 2005), CAS use may cause an explosion of techniques which remain in a relatively simply-crafted state. Moreover, any technique that goes beyond a simple, mechanically learnt gesture, should be accompanied by a theoretical discourse. For the paper and pencil techniques this discourse is known and can be found in textbooks. For instrumented techniques it has to be built and its elaboration raises new, specific difficulties. Even if the use of CAS seems fully legitimate in the class, in general, instrumented techniques cannot be institutionalised in the same way as paper and pencil ones (Artigue, 2005).

THE RATIONALE

To frame the results carried out by these research studies and the complexity of the processes involved in the educational use of CAS, some French researchers (Lagrange 2000, Artigue, 2002, Artigue, Lagrange, Guin and Trouche, 2003) have elaborated a theoretical framework, named 'instrumental approach', integrating both the ergonomic theory (Rabardel, 1995) and the anthropological theory (Chevallard, 1992). The 'instrumental approach' provides a frame for analyzing the processes of instrumental genesis both in their personal and institutional dimensions, and the effect of instrumentation issues on the integration of CAS in the educational practice. Using this framework, Artigue observes that CAS are extremely effective from a pragmatic standpoint and for this reason professionals (mathematicians, engineers..) are willing to spend time to master them (Artigue, 2002). At pragmatic level the effectiveness often comes with the difficulty to justify, at a theoretical level, the instrumented techniques used. In particular, this is true for users who do not fully master mathematical knowledge and techniques involved in the solution of the task. As a consequence, the epistemic value of the instrumented technique can remain hidden. This can constitute a problem for the educational context where technology should help not only to yield results but also to support and promote mathematical learning and understanding. In educational practice, techniques should have an epistemic value contributing to the understanding of objects involved. "*Making technology legitimate and mathematically useful from an educational point of view, whatever be the technology at stake, requires modes of integration that provide a reasonable balance between the pragmatic and the epistemic values of instrumented techniques*" (Artigue, 2007, p. 73). These results might account for the marginalization of CAS integration into the school algebraic practices. For some researchers, to integrate digital technology effectively in the domain of algebra, it is necessary to go beyond the experience of CAS and of their instrumented techniques and to face the design of new artefacts. As underlined by Monaghan (2007) up to now CAS-in-education workers have paid little attention to design issues, preferring, in general, to work with the design supplied by CAS designers (Monaghan, 2007). Moreover, it should be noted that no comparison between the design of CAS and of technological tools for education has been developed so far. This article aims at pointing out design issues that can effectively support teaching and learning processes in algebra. This goal will be pursued considering the design of ALNUSET (ALgebra on the NUmerical SETs), a system developed to improve teaching and learning of crucial topics involved in the mathematical curricula such as algebra, functions and properties of numerical sets. In particular, in this article we compare design aspects of Alnuset and of CAS and we highlight the relevance of differences in their instrumented techniques for the teaching and learning of algebra.

PROBLEMS OF DESIGN IN DEVELOPING NEW DIGITAL ARTEFACTS

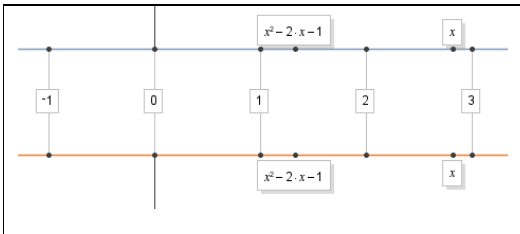
Going beyond the design of CAS requires new creative ideas to instrument techniques for mathematical activity different from those of CAS. The advent of both the dynamic geometrical artefacts and of spreadsheets has evidenced that even a single creative idea can determine a new typology of innovative artefacts. This can occur when new creative ideas allow to instrument mathematical techniques characterizing them with new operative and representative dimensions such as the drag of the variable point of a geometrical construction, as in the case of dynamic geometrical software, or the automatic re-computation of formulas of the table, as in the case of spreadsheet. Moreover, when a technique must be instrumented on the basis of an idea, various types of design problems emerge. They regard the way tasks and responsibilities have to be distributed between user and computer and the management of the interactivity, namely the operative modalities of the input by the user, the representation of the result by the computer (output), the visualisation of specific feedback to support the user action or to accompany the presentation of the result. Moreover, problems of design regard also the way in which the instrumented techniques have to be connected between each other. The way these problems are solved affects the accessibility of techniques, their usefulness for the task to be solved, the meaning that the instrumented technique evidences in the interaction, the discourse that can be developed about it. Hence, the way these problems are solved affects the balance between pragmatic and epistemic values of instrumented techniques within the didactical practice and this can affect mathematics teaching and learning. The anthropological framework is the theoretical tool used to analyse the way in which techniques are implemented and their effectiveness on the educational level. Ideas are evaluated on the base of this framework. We discuss these general assumptions in the domain of algebra referring to Alnuset System.

ALNUSET: IDEAS AND CHOICE OF DESIGN

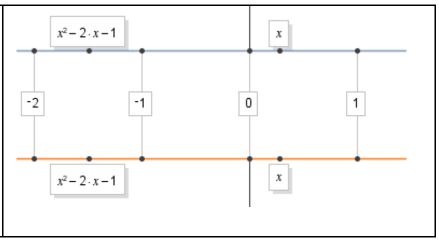
ALNUSET is a system designed, implemented and experimented within the ReMath (IST - 4 - 26751) EC project that can be used to improve the teaching and learning of algebra at lower and upper secondary school level. The design of ALNUSET is based on some ideas that have oriented the realisation of the three, strictly integrated components: the Algebraic Line component, the Algebraic Manipulator component, and the Function component. These three components make available respectively techniques of quantitative, symbolic and functional nature to support teachers and students in developing algebraic objects, processes and relations involved in the algebraic activity. In the following we present the main ideas that have oriented the realisation of the three components of Alnuset and illustrate the choices and decisions taken to instrument algebraic techniques so that an appropriate balance between their epistemic and pragmatic values can emerge when used in the educational practice.

Algebraic line component

The main idea in the design of the Algebraic line component is the representation of algebraic variables on the number line through mobile points associated to letters, namely points that can be dragged on the line with the mouse. In this component the user can edit expressions to operate with. The computer automatically computes the value of the expression on the basis of the value of the variable on the line and it places a point associated to the expression on the algebraic line. When the user drags the mobile point of a variable, the computer refreshes the positions of the points corresponding to the expressions containing such a variable in an automatic and dynamic manner. This is possible only thanks to the digital technology that allows to transform the traditional number line into an algebraic line. The following two figures report the representation of a variable and of an algebraic expression on the lines of this component. Note that the presence of two lines is motivated by operative necessities regarding the use of the algebraic editor based on geometrical models that is available in this component. This editor is not considered in this report.



The drag of the variable x determines the dynamic movement of the expression containing it

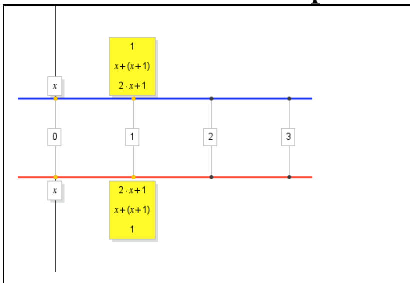


Through its visual feedback, this technique can be used either to explore what an expression indicates in an indeterminate way or to compare expressions. The design of this component is associated to every point represented on the line by a post-it. The computer automatically manages the relation among expressions, their associated points and post-it. The post-it of a point contains all the expressions constructed by the user that denote that point. By dragging a variable on the line, dynamic representative events can occur in a post-it. They might be very important for the development of a discourse concerning the notions of equality and equivalence between expressions. As a matter of the fact, the presence of two expressions in a post-it may mean:

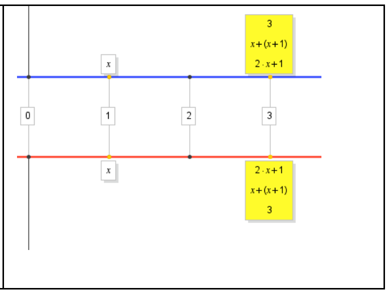
A relationship of equality, if taking place at least for one value of the variable during its drag along the line

A relationship of equivalence, if taking place for all the values assumed by the variable when it is dragged along the line.

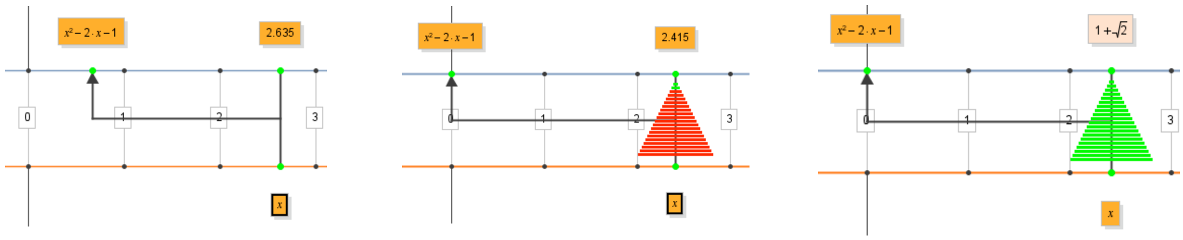
A relationship of equivalence with restrictions, if taking place for every value of the variable when it is dragged along the line, but for one or more values, for which one of the two expressions disappears from the post-it and from the line.



The expressions x+(x+1) and 2*x+1 are equivalent, because they refer to the same point on the Algebraic line and they are contained in the same post-it whatever the value of the x variable is during the drag.



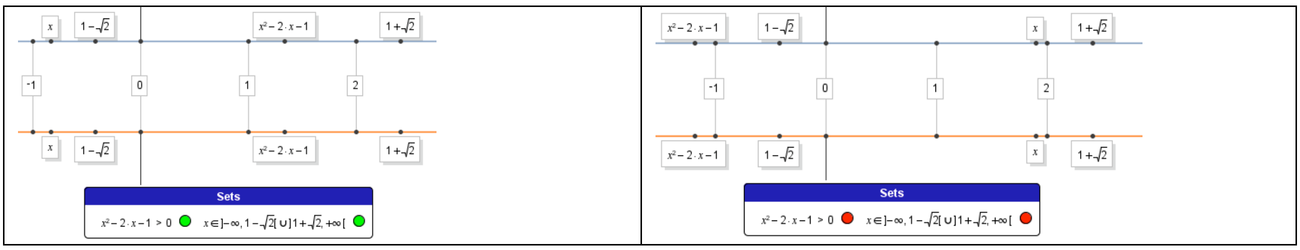
Moreover, the algebraic line component has been designed to provide two very important instrumented techniques for the algebraic activity, i.e. for finding the roots of polynomial with integer coefficients and for identifying and validating the truth set of algebraic propositions. The root of a polynomial can be found dragging the variable on the algebraic line in order to approximate the value of the polynomial to 0. When this happens, the exact root of the polynomial is determined by a specific algorithm of the program and it is represented as a point on the line.



This technique, that can be controlled by the user through his visual and spatial experience, is effective not only at a pragmatic level but also at an epistemic level, because it can concretely support the development of a discourse on the notion of root of a polynomial, as value of the variable that makes the polynomial equal to 0. The truth set of a proposition can be found through the use of a specific graphical editor. Let us consider the inequation x^2-2x-1>0, that once edited, is visualised in a specific window of this component named “Sets”. Once the root of the polynomial associated to the inequation has been represented on the line, a graphic editor can be used to construct its truth set (see the figure).

Two open intervals on the line, respectively on the right and on the left side of the roots of the polynomial x^2-2x-1, have been selected with the mouse. The system has translated the performed selection into the formal language.

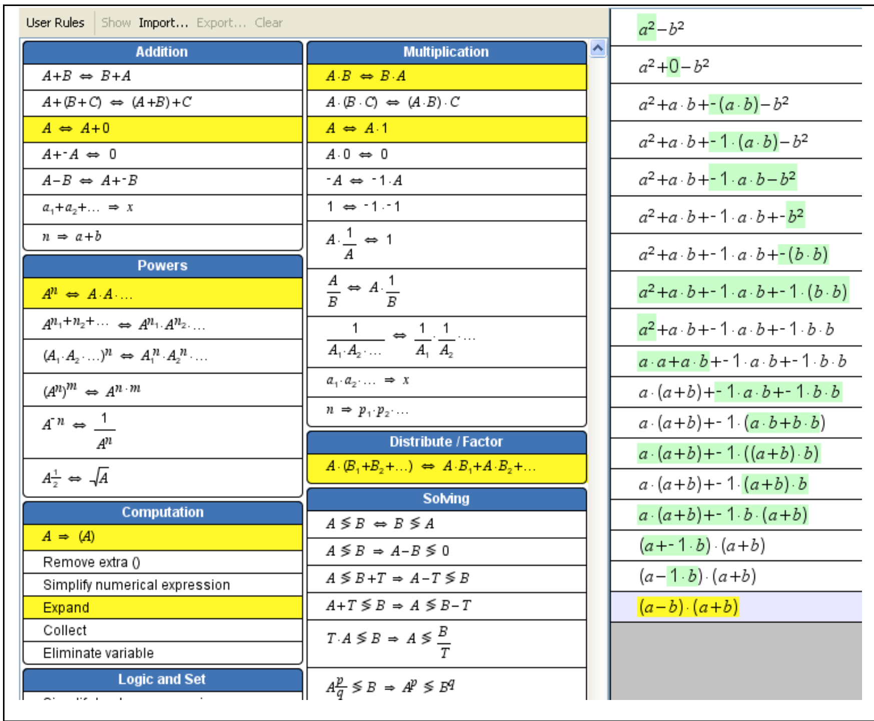
Once the truth set of a proposition has been edited, it can be validated using a specific feedback of the system. In the set window propositions and numerical sets are associated to coloured (green/red) markers that are under the control of the system. The green/(red) colour for the proposition means that it is true/(false) while the green/(red) colour for the numerical set means that the actual variable value on the line is/(is not) an element of the set. Through the drag of the variable on the line, colour accordance between proposition marker and set marker allows the user to validate the defined numerical set as truth set of the proposition (see figure below). The validation process is supported by the accordance of colour between the two markers and by the quantitative feedback provided by the position of variable and of the polynomial on the algebraic line during the drag.



This feedback offered by the system during the drag of the variable is important to introduce the notions of truth value and of truth set of an algebraic proposition and to develop a discourse on their relationships. All the described instrumented techniques that are specific of the Algebraic line component make a quantitative and dynamic algebra possible.

Algebraic manipulator component

The interface of this component has been divided into two distinct spaces: a space where symbolic manipulation rules are reported and a space where symbolic transformation is realised.



This figure shows a part of the commands available with the interface and an example of algebraic transformation.

The figure shows a characteristic of the interactivity of this manipulator: the selection of a part of an expression determines the activation of the commands of the interface that can be applied on it. This characteristic can help students to explore the systems of rule for the algebraic transformation and the effects they produce

The main idea characterizing the design of the Algebraic Manipulator component is the possibility to exploit pattern matching procedures of computer science to transform algebraic expressions and propositions through a structured set of basic rules that are deeply different from those of the CAS. In CAS pattern matching procedures are exploited according to a pragmatic perspective oriented to produce a result of symbolic transformation that could be also very complex, as in the case of command like factor or solve. As a consequence, the techniques of transformation can be obscure for a not expert user. In the Algebraic Manipulator component of Alnuset pattern matching procedures have been exploited according to three specific pedagogical necessities. The first necessity is to highlight the epistemic value of algebraic transformation as formal proof of the equivalence among algebraic forms. To this aim we have designed this manipulator with a set of basic rules that correspond to the basic properties of addition, multiplication and power operations, to the equality and inequality properties between algebraic expressions, to basic logic operations among propositions and among sets. Every rule produces the simple result of transformation that is reported on the icon of its corresponding command on the interface, and this makes the control of the rule and the result easy to control. Moreover a fundamental function of this component allows the student to create a new transformation rule (user rule) once this rule has been proved using the rules of transformation available on the interface. For example, once the rule of the remarkable product a^2-b^2= (a+b)*(a-b) has been proved, it can be added as new user rule in the interface a^2-b^2←→(a+b)*(a-b) and it can successively be used to transform other expressions or part of them whose form match with it. Moreover, a specific command allows to represent every transformed expression on the algebraic line automatically. Through this command it is possible to verify quantitatively the preservation of the equivalence through the transformation, observing that all the transformed expressions belong to the same post-it when their variables are dragged along the line. These characteristics of the algebraic manipulator of Alnuset can have a great epistemic importance because they can be effectively exploited to support the comprehension of the algebraic manipulation in terms of formal proof of the equivalence between two algebraic forms. The second necessity is to support the integration of practice of quantitative and manipulative nature. In this manipulator three rules allow the user to import the root of a polynomial, the truth set of a proposition and the value assumed by a variable on the algebraic line from the Algebraic line component to be used in the algebraic transformation. For example the rule "Factorize" uses the root of polynomial found in the Algebraic Line to factorize it. The way in which this rule works, makes the factorization technique of Alnuset different from that of CAS. In CAS this technique is totally under the control of the system, and the result can appear rather obscure for not expert users. In Alnuset, the factorization can be applied on the polynomial at hand only if its roots have been previously determined on the algebraic line. In Alnuset the distribution of tasks between user and computer and the way they interact, can contribute to understand the link between the factorization of a polynomial and its roots. The third necessity is to offer more powerful rules of transformation when needed for the activity and when specific meaning of algebraic manipulation have been already constructed. Two specific rules, also present in the CAS are available in this manipulator. They determine the result of a numerical expression and the result of a computation with polynomials respectively. These rules of transformation contribute to increase the pragmatic value of the instrumented technique of algebraic transformation in Alnuset and they can be used to introduce to the use of CAS

Moreover, the technique of algebraic transformation has been instrumented in this manipulator to provide not expert users with cognitive supports in the development of specific manipulative skills. A first support is the possibility to explore, through the mouse, the hierarchical structure that characterises the expression or the proposition to be manipulated. By dragging the mouse pointer over the elements of the expression or proposition at hand (operators, number, letters, brackets…), as feedback the system dynamically displays the meaningful part of the expression or proposition determined by such pointer. In this way it is possible to explore all meaningful parts of an expression in the different levels of its hierarchical structure.

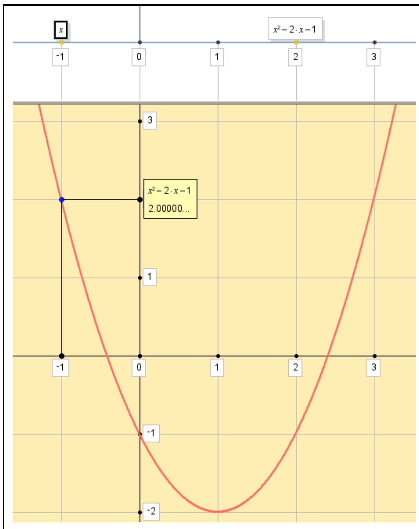
Another feedback occurs when a part of expression has been selected. Through a pattern matching technique, the system, as feedback, activates only the rule of the interface that can be applied on the selected part of expression. This is a cognitive support that can be used to explore the connection among the transformational rules of the interface, the form on which it can be applied, and the effects provided by their applications.

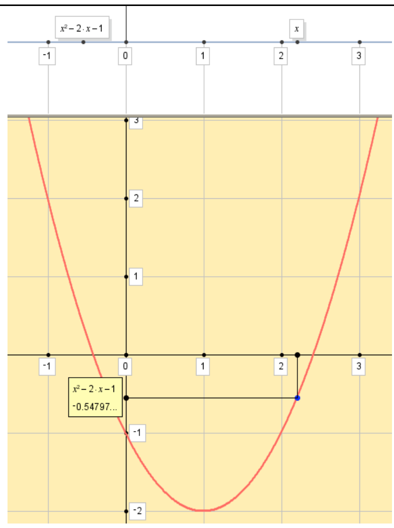
Functions component

The main idea characterizing the design of the Functions component is the possibility to connect a dynamic functional relationship between variable and expression on the algebraic line with the graphical representation of the function in the Cartesian plane. As a consequence, the interface of this component has been equipped with the Algebraic line and a Cartesian plane. This idea makes this component deeply different from other environment for the representation of function in the Cartesian plane. Through a specific command and the successive selection of the independent variable of the function, an expression represented on the Algebraic line is automatically represented as graphic in the Cartesian plane. Dragging the point corresponding to the variable on the algebraic line, two representative events occur:

- on the algebraic line, the expression containing the variable moves accordingly
- on the Cartesian plane, the point defined by the pair of values of the variable and of the expression moves on the graphic as shown in the following figure.

This instrumented technique supports the integrated development of a dynamic idea of function with a static idea of such a notion (Sfard 1991). The functional relationship between variable and expression is visualized dynamically on the algebraic line through drag of the variable point, and statically in the Cartesian plane through the curve. The movement of the point along the curve during the drag of the variable on the algebraic line supports the integration of these two ideas, showing that the curve reifies the infinite couples of values corresponding to the variable and to the expression on the line. This instrumented technique can be very useful to orient the interpretation of the graphics on the Cartesian plane and to develop important concepts of algebraic nature.





For example, it contributes to assign an algebraic meaning to the intersection of two curves (for the value of the variable that determines the intersection, the two expressions are contained in the same post-it on the algebraic line) or to the intersection of a curve with the x-axis (in this case the expression is contained in the post-it of 0).

Other examples are related to the construction of meaning for the sign of a function (in terms of the position of the corresponding expression on the line with respect to 0), or to relationships of order among functions (in term of the respective position of the expressions on the algebraic line)

CONCLUSIONS

In this paper we have presented the main ideas that oriented the realisation of Alnuset and the choices we made to instrument specific functions of algebraic activity that can be useful for the teaching and learning of algebra. We have shown that the quantitative, symbolic and functional techniques available in the three environments of Alnuset to operate with algebraic expressions and propositions have characteristics that are deeply different from the instrumented technique of CAS. The technique of Alnuset was designed having in mind two types of user that are different from the target user considered by CAS designers. The former type of user is the student that is not an expert of the knowledge domain of algebra and that uses the instrumented techniques of Alnuset to learn it carrying out the algebraic activity proposed by the teacher. The latter type of user is the teacher that has difficulties to develop algebraic competencies and knowledge in students and that uses the instrumented technique of Alnuset to acquaint them with objects, procedures, relations and phenomena of school algebra. The technique of Alnuset was designed to be easily controlled during the solution of algebraic tasks, to produce results that can be easily interpreted and to mediate the interaction and the discussion on the algebraic meaning involved in the activity. The techniques of Alnuset structure a new phenomenological space where algebraic objects, relations and phenomena are reified by means of representative events that fall under the visual, spatial and motor perception of students and teachers. This contributes to provide an appropriate balance between the pragmatic and epistemic values of the techniques made available by Alnuset. In the phenomenological space determined by the use of the instrumented technique of Alnuset algebra can become a matter of investigation as evidenced by two papers presented in group 4 by Trgalova J., Chiappini G., Robotti E: and in group 2 by Pedemonte B. of this edition of CERME.

REFERENCES

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning,* 7(3), 245-274

Artigue, M. (2005) The integration of symbolic calculators into secondary education : some lessons from didactical engineering. *The didactical challenge of symbolic calculators. Turning a Computational Device into a Mathematical Instrument.* 231-294 D. Guin, K. Ruthven, L. Trouche (eds) Springer

Artigue, M. (2007) Digital technologies: a window on theoretical issues in mathematics education Plenary 4. Pitta-Pantazi D. & Philippou G. (eds.) *Proceedings of the fifth congress of the European Society for Research in Mathematics Education CERME,* 68-82

Chevallard Y. (1992). Concepts fondamentaux de la didactique: perspectives apportées par une approche anthropologique. *Recherches en Didactique des Mathématiques*, 12/1, 77-111

Drijvers, P.H.M. (2000). Students encountering obstacles using a CAS. *International Journal of Computers for Mathematical Learning,* 5(3), pp. 189-209.

Guin, D. & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: the case of calculators. *International Journal of Computers for Mathematical Learning* 3: 195–227.

Heid, M.K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for Research in Mathematics Education* 19(1): 3–25.

Kaput, J. (1996). Algebra and technology: New semiotic continuities and referential connectivity. In F. Hitt, T. Rojano, & M. Santos (Eds.), *Proceedings of the PME-NA XXI Annual Meeting*, Cuernavaca, Mexico.

Lagrange, J.-B. (2001) L'intégration d'instruments informatiques dans l'enseignement: une approche par les techniques, *Educational Studies in Mathematics* 43, 1–30.

Lagrange, J.-B., Artigue, M., Laborde, C., Trouche, L. (2003) Technology and Mathematics Education : A Multidimensional Study of the Evolution of Research and Innovation *Second International Handbook of Mathematics Education,* 239-271 A.J. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick and F.K.S. Leung (eds.) Kluwer

Mayes, R. (1997). Current state of research into CAS in mathematics education. In: J. Berry, J. Monaghan, M. Kronfellner and B. Kutzler (Eds), *The State of Computer Algebra in Mathematics Education,* Bromley: Chartwell-Bratt, pp. 171–189.

Monaghan, J. (2007). "Computer algebra, instrumentation and the anthropological approach". *International Journal for Technology in Mathematics Education*, 14(2), 63-72.

Rabardel (1995): Les hommes & les technologies. Approche cognitive des instruments contemporaines Ed.: A. Colin Paris.

Sfard (1991) On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics 22,* 1-36.

Thomas, M. O. J., Monaghan, J., Pierce, R. (2004). Computer algebra systems and algebra: Curriculum, assessment, teaching, and learning. In K. Stacey, H. Chick, & M. Kendal (Eds.), *The teaching and learning of algebra: The 12th ICMI study* (pp. 155-186). Norwood, MA: Kluwer Academic Publishers.