ESTABLISHING A LONGITUDINAL EFFICACY STUDY USING SIMCALC MATHWORLDS®

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We describe the construct of a 4-year longitudinal efficacy study implementing dynamic mathematics software and wireless networks in Algebra 1 and 2 classrooms. We focus on student learning and motivation over time, and issues of effective implementation in establishing a longitudinal study.

INTRODUCTION: BACKROUND TO DYNAMIC MATHEMATICS

New forms of mathematics technology (e.g., dynamic geometry) can provide *executable representations*—representations that transform the mathematics made by students into a more tangible and exciting phenomenon (Moreno-Armella, Hegedus & Kaput, 2008). In particular, we have designed and used SimCalc MathWorlds® to transform students' mathematical constructs into fascinating motion phenomena. Second, networks can intimately and rapidly *link private cognitive efforts to public social displays*. Consequently, students can each be assigned a specific mathematical goal (e.g., playing the part of a single moving character by making a graph with certain mathematical characteristics), which instantly links to public social display (e.g., the parade constituted by all characters moving simultaneously). This approach shifts the types of critical thinking that are possible in mathematics classrooms and transforms the role of instructional technology by integrating it into the social and cognitive dimensions of the classroom.

Our connected approach to classroom learning highlights the potential of classroom response systems to achieve a transformation of the classroom-learning environment. Similarly other investigators have expanded their approaches to include devices that allow aggregation of mathematical objects submitted by students. (Stroup, Ares & Humford, 2005).

SITUATED NEED

Our proposed work addresses three essential needs: (i) the Algebra Problem (RAND, 2002), (ii) the related problem of student motivation and alienation in the nation's schools, especially urban secondary schools (National Research Council, 2003), and (iii) the widely acknowledged unfulfilled promise of technology in education, especially mathematics education (e.g., Cuban, 2001).

An important analysis by the National Academies Institute of Medicine (National Research Council, 2003) of student motivation at the high school level reveals in painful detail what most high school teachers (and parents) know only too well: that student motivation in high schools, and even more acutely in urban high schools, is an urgent and complex national problem. The report also recommends that high school courses and instructional methods need to be redesigned in ways that will increase adolescent engagement and learning.

Ethnographical studies of high school students (Davidson & Phelan, 1999; Phelan, Davidson, & Yu, 1998) reveal a world of alienation with strongly negative responses to standard practices (Meece, 1991) and strong sensitivity to interactions with teachers and their strategies (Davidson, 1999; Johnson, Crosnoe & Elder, 2001; Skinner & Belmont, 1993; Turner, Thorpe, & Meyer, 1998). Negative responses, particularly as they are intimately connected with self image and sense of personal efficacy, can be deeply debilitating, both in terms of performance variables (Abu-Hilal, 2000) as well as in the ability to use help when it is available (Harter, 1992; Newman & Goldin, 1990; Ryan & Pintrich, 1997). See the comprehensive reviews by Brophy (1998), Newmann (1992), Pintrich & Schunk (1996), and Stipek (2002). On the other hand, students exhibit consistently positive responses to alternative modes of instruction and content (Ames, 1992; Boaler, 2002; Mitchell, 1993), particularly those that build upon intrinsic instead of external motivation (Linnenbrink & Pintrich, 2000).

The literature on motivation in education and social situations in general has focused on intrinsic and extrinsic motivation with a great deal of debate (Sansone & Harackiewicz, 2000). Intrinsic motivation reflects the propensity for humans to engage in activities that interest them. Extrinsic motivation, such as rewards, can have an undermining effect and decrease intrinsic motivation, i.e., the reason why the person chose to want to do the activity in the first place (Deci, 1971). Yet both intrinsic and extrinsic motivation, as a key feature of participation in mathematics classrooms, have appeared to be an orthogonal field of inquiry to the development and instruction of content, with motivation hesitantly intersecting with education in the form of "motivational strategies," incentivizing students to learn mathematics because it is "fun" or "applicable" to *their* life, through relevant contexts, e.g., sports or vocations.

Relevance, unfortunately, is a somewhat indirect means to link motivation and mathematics—the link between immediate cognitive effort and later applications that may seem improbable to students. There is a more direct alternative. Students can become motivated because they want to participate more fully in what their classroom is doing now. The alternative, thus, is to link motivation and mathematics through *participation*.

We advocate two radically new forms of participatory activity in technology-enhanced environments:

1. Mathematical Performances. These activities emphasize individual student creations, small group constructions, or constructions that involve coordinated

interactions across groups that are then uploaded and displayed, with some narration by the originator(s).

2. Participatory Aggregation to a Common Public Display. These activities involve systematic variation, either within small groups, across groups, or both, where students produce functions that are uploaded and then systematically displayed and discussed to reveal patterns, elicit generalizations, expose or contextualize special cases, and help raise student attention from individual objects to families of objects.

These activities aim at enhancing mathematical literacy, debate and coherent argumentation—all fundamental mathematical skills. The central point is that each requires and rewards students for cognitive engagement in producing tangible phenomena that are simultaneously phenomenologically exciting and mathematically enlightening. This happens not at some future time when mathematics can be applied to a career or personal goal; instead these activities draw students in and sustain their interest because they are exciting and enlightening in the moment, in the classroom. These activities create an intrinsic motivation context with a socio-cultural view to "motivation in context" (Hickey, 2003) that is also intrinsically mathematical, accomplishing a much more intimate intertwining of motivation and mathematics that can be typically accomplished in existing classrooms.

PRIOR WORK

SimCalc MathWorlds® creates an environment where students can be part of a *family of functions,* and their work contributes to the mathematical variation across this mathematical object. Consider this simple activity, which exemplifies a wider set of activity structures. Students are in numbered groups. Students must create a motion (algebraically or graphically) that goes at a speed equal to their group number for 6 seconds. So, Group 1 creates the same function, Y=(1)X, Group 2, Y=(2)X, etc. When the functions are aggregated across the network via our software, students' work becomes contextualized into a family of functions described algebraically by Y=MX (see Figure 1 below). Students are creating a variation of slope and in doing so this can help each student focus on their own personal contribution within a set of functions.

At the heart of SimCalc is a pedagogical tool to manage classroom flow. This tool allows teachers to control who is connected to the teacher computer using a simple user interface, and choose when to "freeze" the network and aggregate students' work or allow students to send a number of tries via the TI-Navigator™. In addition, teachers have control over which set of contributions (e.g., Group 1's functions) and which representational perspectives (e.g., tables, graphs, motions) to show or hide. Thus, the management tool encapsulates a significant set of pedagogical strategies supported by question types in existing curriculum materials to satisfy a variety of pedagogical needs, focus students' attention depending on their progress, and promote discussion, reasoning and generalization in a progressive way at the public level.

In our prior research, students build meaning about the overall shape of the graphs and have demonstrated gestures and metaphorical responses in front of the class when working on this activity. For example, in two entirely different schools, students have raised their hand with fingers stretched out (see Figure 1 below), and said it would look like a "fan." In this socially-rich context, students appear to develop meaning through verbal and physical expressions, which we observe as a highly powerful way of students engaging and developing mathematical understanding at a whole group level. Various forms of formative assessment can said to be evident as each student's work emerges in a public display, and representations can be "executed" (Moreno-Armella & Block, 2002) to test, confirm or refute ideas. These forms of reflection, enabled through particular question-types and classroom dialogue focused on the dynamic representations, can be attributed to students learning and resonate with established research on formative assessment (Black & William, 1998; Boston, 2002).

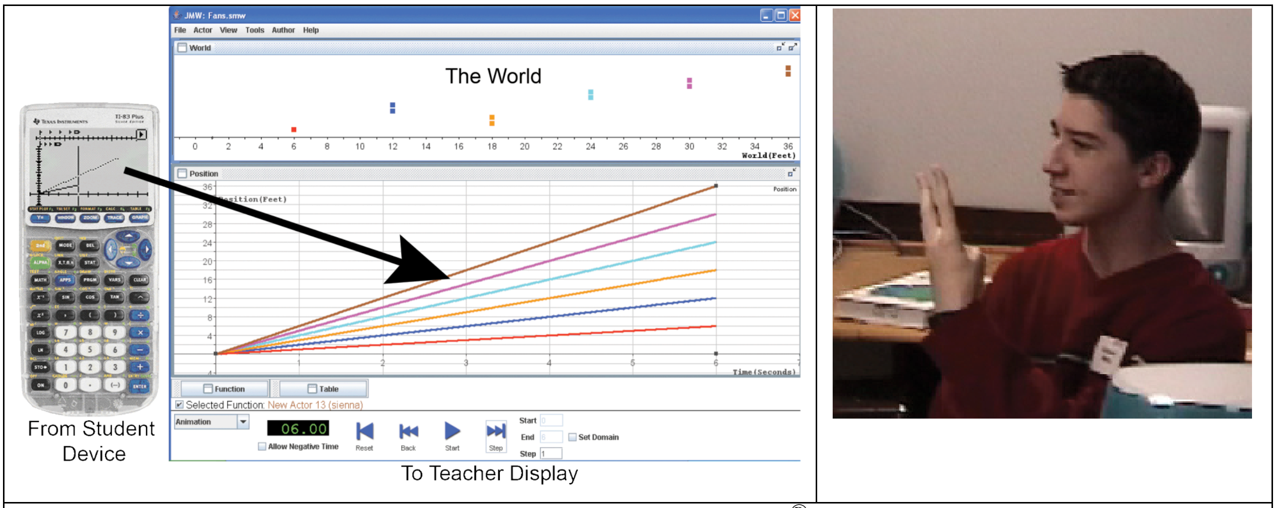

Figure 1. Sample Function in SimCalc MathWorlds®

Over the past ten years, over the course of three consecutive research and development projects (NSF ROLE: REC-0087771; REC-0337710; REC-9619102) and related projects at TERC (NSF REC-9353507), the SimCalc project has examined the integration of the Mathematics of Change and Variation (MCV) as a core approach to algebra-intensive learning. This work has led to a Goal 3 IERI-funded study (NSF REC-0437861), led by SRI International, focusing directly on large-scale implementability and teacher professional development in TX, and a recently funded IES Goal 2 project in the high school grades (IES Goal 2 # R305B070430) focusing on longitudinal impact of our curriculum and software products distributed by Texas Instruments on their popular graphing calculators in combination with a commercially available wireless network (TI-Navigator™ Learning system).

The Scale-Up pilot work employed a set of SimCalc resources in a delayed-treatment design. Teachers were initially randomly assigned to one of two groups. An ANOVA of difference scores (again teacher nested within condition) was significant [$F(1,282)=178.0$, $p<0.0001$]. The effect size for the gain in the group that used SimCalc is 1.08. In our main study, which is a randomized controlled trial in which 95 7th-grade mathematics teachers were randomly assigned to implement a 3-week SimCalc curriculum unit following training, our analyses show an effect size of 0.84 (Roschelle, Tatar, Shectman et al., 2007).

Prior work has documented statistically significant evidence for impact of SimCalc materials in connected "networked" environments with computers and calculators (Hegedus & Kaput, 2004) under multiple quasi-experimental interventions across grades 8-10 and college students demonstrating statistically significant increases ($p<0.001$) in student mean scores (effect=1.6) but with an even higher effect on the at-risk 9th grade population (effect=1.9). A major finding of our work was that critically important skills such as graphical interpretation were improved, i.e., cognitive transfer was evident. Recent studies show similar statistically significant results in terms of student learning and shifting attitudes towards learning mathematics in connected environments (Hegedus, Kaput, Dalton et al., 2007). We have also analyzed the changing participation structures using frameworks from linguistic anthropology (Duranti, 1997; Goffman, 1981). Our work has described new categories of participation in terms of gesture and language (Hegedus, Dalton, Cambridge et al., 2006) new forms of identity (Hegedus & Penuel, 2008), and theoretical advances in dynamic media and wireless networks (Hegedus & Moreno-Armella, 2008; Moreno-Armella et al., 2008).

DESIGN ASPECTS OF EFFICACY WORK

In this context, our research program (funded by the US Department of Education, IES Goal 2 # R305B070430) builds on prior work to examine this problem. It is focused on outcomes in terms of both grade-level learning gains and longitudinal measures that relate to students' progress and motivation in mathematics across the grades in Algebra 1 and 2 classrooms.

SimCalc combines two innovative technological ingredients to address core mathematical ideas: Software that addresses content issues through dynamic representations and, wireless networks that enhance student participation in the classroom. We have begun to develop materials that fuse these two important ingredients in mathematically meaningful ways and developed new curriculum materials to replace core mathematical units in Algebra 1 (8-12 weeks) and Algebra 2 (4-8 weeks) at high school. We are measuring the impact of implementing these materials on student learning (high-stakes State examinations in Massachusetts (MA), USA) and investigating whether one or multiple involvements in this type of learning environment over the course of their high school years affects their motivation to continue studying mathematics effectively and enter STEM-career trajectories.

Our work is conducted in eight school districts in MA offering a wide variety of settings in terms of performance on State exams and Socio-Economic Status (SES). Our treatment interventions are in 9^{th} and 11^{th} grade classrooms (Algebra 1 then 2) but we will also track some students when they are in 10^{th} and 12^{th} grade collecting simple questionnaire data. Our study is a small-scale cluster randomized experiment where we cluster at the classroom level, randomly assigning two classrooms in each school to treatment in our main studies (total of 28 classrooms and a. 500 students in each main study).

We are using two instruments comprised of standardized test items to measure student's mathematical ability and problem-solving skills before and after each intervention. We are also collecting survey data on student's attitude before during and after the intervention. We are administering these tests and surveys at similar times (with respect to curriculum topics covered) in treatment and control classrooms. Video data from periodic classroom visits are being analyzed using participation frameworks from prior work and triangulated with variations in student survey data on attitude.

We are using suitable statistical methods to assess gain relative to the control groups, and between-cluster variation using mixed-Hierarchical Linear Modeling. We are also collecting survey and classroom observation data to assess changes in attitudes and participation, and daily logs by teachers to monitor fidelity of implementation.

We have completed our first year of 4 years work with our first cohort of students that we will track for the duration of their high school career and will present initial findings from our pilot study and challenges we have addressed in sampling and establishing a longitudinal program of research. We focus on results from factor analyses of our survey instruments on student and teacher attitude and correlations with student learning. Following a minimal effect size in our pilot study, we aim to present findings for improving effective implementation from analyses of teacher daily logs and classroom video.

Such methodologies build a comprehensive program for evaluating how prior findings (briefly highlighted above) can scale to larger implementations whilst being cognizant of issues of fidelity. Our ongoing work and preliminary analyses report of the potential effect on outcome measures such as student learning and motivation.

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ENHANCING FUNCTIONAL THINKING USING THE COMPUTER FOR REPRESENTATIONAL TRANSFER

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The area of functional thinking is complex and has many facets. There are several studies that describe the specific difficulties of functional thinking. They show that the main difficulties are the transfer between the various representations of functions, e.g. graph, words, table, real situation or formula, and the dynamic view of functional dependencies (process concept of a function). Interactive Geometry Software allows the visualization of the dynamic aspect of functional dependencies simultaneously in different representations and offers the opportunity to experiment with them. The author presents and discusses the potential of two interactive learning units that focus on the dynamic aspect of functional thinking in a special way. Some preliminary results from a first adoption of one of the activities in class are presented. Resulting research questions and plans for further research are stated.

Keywords: Functional thinking, representational transfer, Interactive Geometry Software, Interactive learning unit, empirical study.

THEORETICAL BACKGROUND

Functional Thinking – Concept and Relevance

In Germany the term 'functional thinking' was first used in the 'Meraner Reform' of 1905. The 'education to functional thinking' was a special task of the reform. Functional thinking was meant in a broad sense: As a common way to think which affects the whole mathematics education (Krüger 2000). In the 60s and 70s the impact of functional thinking in the above sense on the mathematics curriculum in Germany was very low. Since the 80s it regains importance although not in the broad sense of the Meraner Reform. A common definition of functional thinking derives from Vollrath (1989): 'Functional thinking is the typical way to think when working with functions'. Functional thinking in this sense is strongly connected to the concept of function. In the german mathematics curriculum the 'idea of functional dependency' is one of five central competencies, which form the mathematics education (Kultusministerkonferenz 2003).

The concept of function and functional thinking includes many aspects and competencies: On one hand functional dependencies can be described and detected in several representational systems like graphs, words, real situations, tables or formulas. On the other hand the nature of functional dependencies has different characteristics (Vollrath 1989 or Dubinsky, Harel 1992):

- Functional dependency as a pointwise relation (horizontal, static aspect)

- Functional dependency as a dynamic process (aspect of covariation and change, vertical aspect)
- Functions viewed as objects

Many studies (e.g. Janvier 1978, Müller-Philipp 1994, Swan 1985, Kerslake 1981) show the following main difficulties and misconceptions concerning functional thinking:

The interpretation of functional dependencies in different representations and the representational transfer is a main difficulty. Especially the interpretation of functional dependencies in situations and the transfer to e.g. the graphical representation and vice versa causes problems. For example: graphs are often interpreted as photographical images of real situations. Another main difficulty is the aspect of covariation or the dynamic view of a functional dependency. This is evident in problems with the interpretation of slopes or distance-time graphs for instance.

An illustrative example

The above difficulties were affirmed by written tests the author gave to either 10th class students and to university students who just started their study on mathematics. Based on the problems in the test the interactive learning units, which we describe below, were built. Figure 1 shows one of the problems (see: Schlöglhofer 2000) from the tests.

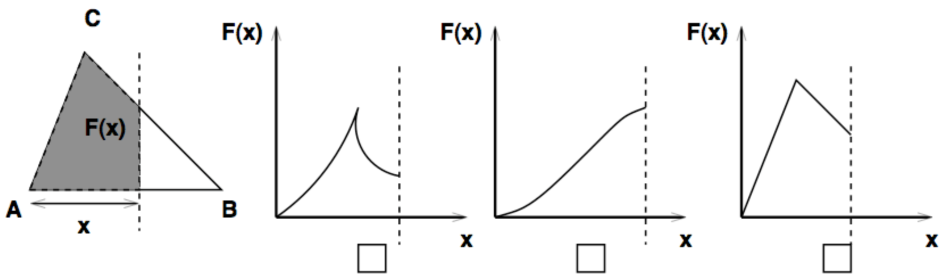

Fig. 1: The dashed line moves rightwards. F(x) is the area of the grey part of the triangle dependent on the distance x. Which graph fits and why?

Only 66% of about 100 university students made their cross at the graph in the middle. Giving the problem to sixteen 9th and 10th grade high school students, resulted in only 37% correct answers. The main mistake was to put a cross at the graph on the right side. The reason for this choice was usually given by a statement like: The area [of the graph on the right side] is just like the area F(x).

The graph is interpreted as a photographical image of the situation. The specific difficulty is the transfer between situation and graphical representation, which is caused by the inability to interpret the functional dependency dynamically (covariation of x and F(x)).

The chances of Interactive Geometry Software

When using the computer in classrooms on the topic functions one might think immediately of Computer Algebra Systems (CAS). Most studies about the use of the computer when working with functions are about using CAS, e.g. Müller-Philipp (1994), Weigand (1999), Mayes (1994). While CAS is input/output based and gives back information and changes asynchronously, the use of Interactive Geometry Software (IGS) allows interactivity and gives immediate response. This difference will be used to emphasize the dynamic view of functional dependencies.

IGS offers the possibility to create a platform for experimentation with functional dependencies. Mathematical objects can be dynamically visualized by using the dragging mode and the covariation aspect of functional dependencies becomes visual. Moreover it is possible to interactively connect different representations of functions. Mental operations like the representational transfer can be externally presented, which is a chance to enhance relational understanding.

Especially the software Cinderella includes a functional programming language called *CindyScript*. This enables the teacher to create learning units and own teaching material like the ones described below by using standard tools (Kortenkamp 2007).

DESIGN OF THE ACTIVITIES AND CONCEPTUAL BASIS

Main research question

The learning activities are designed with regard to the following research question:

Is it possible to enhance the dynamic aspect of functional thinking by dynamically visualizing functional dependencies simultaneously in different representations and by giving the opportunity to experiment with them?

General design ideas and concept

We developed two interactive learning units (joint work with Andreas Fest). The learning units consist of single Java applets embedded into a webpage and can be used without prior installation with a standard Internet browser. The applets are built with the IGS Cinderella and are accessible by using the links on the webpage http://www.math.tu-berlin.de/~hoffkamp.

Figure 2 shows the typical design of a learning unit. Next to the applet there is a short instruction on how to use the applet and some work orders. The students are asked to investigate and describe the functional dependency between the distance A-D and the dark (if coloured: blue) area within the triangle. The applet in Figure 2 allows the following actions:

- Moving point D and watching the corresponding point in the graph.
- Moving points B and C, which changes the triangle, and watching the effects on the graph.

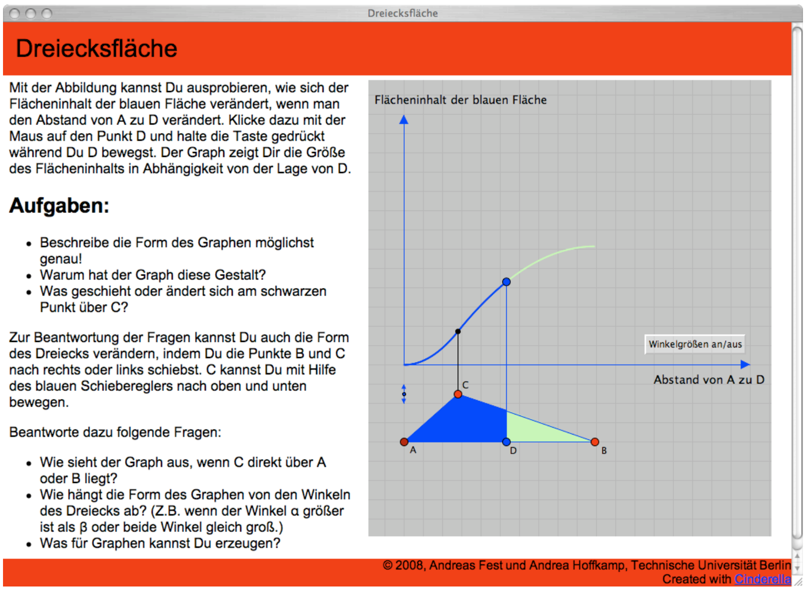

Fig. 2: Interactive learning unit 'Dreiecksfläche' ('Area of a triangle').

The learning activities have the following conceptual and theoretical ideas in common:

Connection situation-graph: The starting point is a figurative description of a functional dependency, which is simultaneously connected to a graphical representation. The graphical representation was chosen, because it relates to the covariation aspect in a very eminent way. As analysed by von Hofe (1995) students are able to establish 'Grundvorstellungen' (GV) more easily when an imaginable situation is given. GV's are mental models connecting mathematical concepts, reality and mental concepts of students. Rich GV's of the functional dependencies are necessary to succeed in problem solving processes.

Language as mediator: The students are asked to verbalise their observations in their own words. Janvier (1978) emphasises the role of the language as a mediator between the representations of the functional dependency and the mental conceptions of the students.

Active processing assumption: According to the cognitive theory of multimedia learning of Mayer (2005) humans are actively engaged in cognitive processing in order to construct a coherent mental representation. The activities are conceptualized as attempt to assist students in their model-building efforts. Therefore the activities allow to experiment with different representations of the functional dependencies. At the same time the actions of the user are limited to focus on the dynamic view of the functional dependencies.

Two levels of variation: The activities allow two levels of variation. First, one can vary within the given situation. This visualizes the covariation aspect. Secondly, one can change the situation itself and watch the effects on the graph. We will call this **meta-variation**. Meta-variation allows the user to investigate the covariation in several scenarios. It changes the functional dependency itself and allows a more global view of the dependency. Therefore it refers to the object view of the function. To understand the covariation aspect one needs to find correlations between different points of the graph in order to describe changes. This requires a global view of the graph. For example the property 'strict monotony' of a graph is a global property and therefore refers to the object view of a functional dependency. But to describe it in terms of 'if x>y then f(x)>f(y)' one has to understand the covariation of different points of the graph.

Low-overhead technology: To work with the interactive units there is no special knowledge of the technology necessary. The activities make use of the students' experience with Internet browsing (actions like dragging, using links, using buttons etc.). The low-overhead technology allows the students (and the teachers) to work directly on the problems without special knowledge of the software and the software's mathematical background. This is important especially with regard to time economy.

Practicability: The activities are designed with respect to their practicability. Besides the activities learning material in form of a worksheet is provided. The activities can be employed in class without great effort in a block period of 2x45 minutes.

Learning unit 'Die Reise' ('The journey')

On the basis of the conceptional ideas above the learning activity 'Die Reise' was developed. Figure 3 shows the first part of the activity. Like the learning unit 'Dreiecksfläche' it is adapted from a problem (see: Swan 1985) the author gave to university students and 10th grade students within a written test. The unit consists of three parts. Part one (Fig. 3) is about the transfer situation-graph. A car advances from Neubrandenburg (top of the map) to Cottbus (bottom of the map). The graph shows the corresponding distance-time graph for the journey. The cars are movable in the map and the graph. The students are asked to mark the positions A-F on the map with the flags.

Part two of the learning unit (without Figure) refers to the first level of variation (visualization of the covariation aspect in the given situation). It shows the distance-time graph of part one again together with the corresponding velocity-time graph. Again there are movable cars in both graphs. The applet is about the transfer between two graphs with emphasis on the dynamic view of functional dependencies (here: slopes). The work orders aim at interpreting the slopes in the distance-time graph and see how the velocity-time graph corresponds to the slopes.

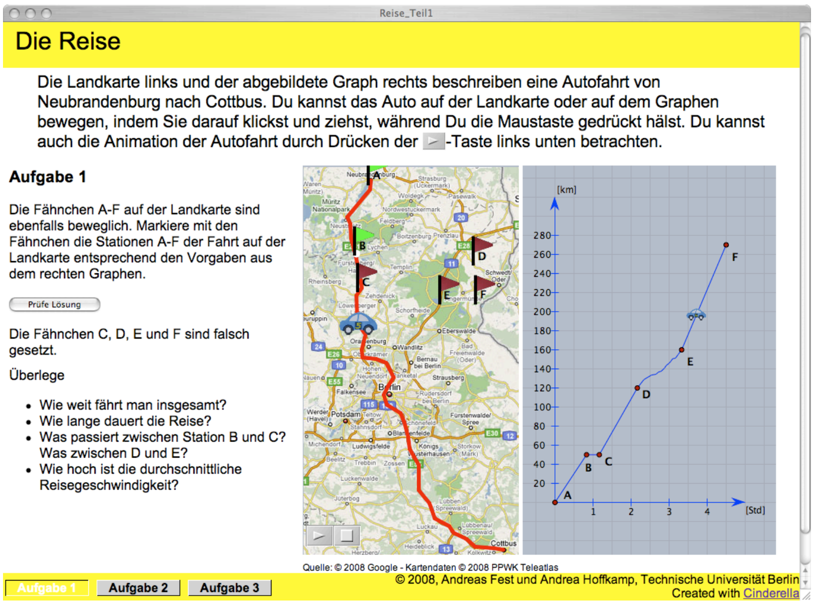

Fig. 3: Part one of the learning unit 'Die Reise' ('The journey'). Using the 'Prüfe Lösung'-button ('Check your solution'-button) reveals the text 'Flags C, D, E and F are in the wrong position'.

Figure 4 shows the applet within part three of the learning activity. It refers to the second level of variation (meta-variation). Besides moving the cars in both graphs one can move the bars in the velocity-time graph vertically and change the width of the bars while watching the changes of the corresponding distance-time graph.

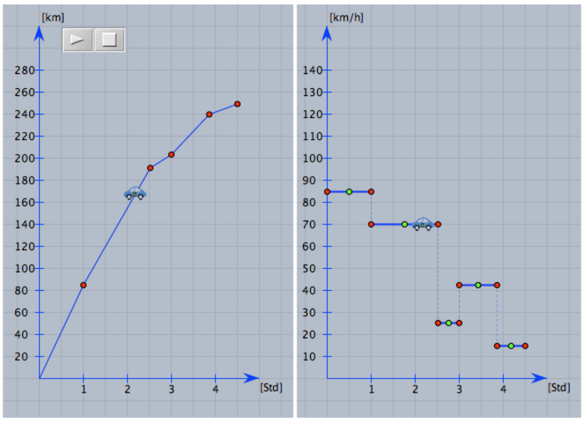

Fig. 4: Level of meta-variation in part three of the learning unit 'Die Reise'.

A FIRST STUDY

Setting and methods

The learning unit 'Dreiecksfläche' was tested with 19 secondary school students of age 14-15 (10^{th} class) in a block period of 2x45 minutes. The teacher characterized the learning group as being rather slow. The students were not prepared to either the topic (they were not currently working with functions in class) or the special use of technology. A worksheet was prepared which contained the Internet address of the learning unit and the questions to work on. The students had to start on their own using the instructions of the worksheet. They wrote their answers and solutions on the worksheet. To provoke discussion and first reflection about the problems two or three students worked together. Afterwards the solutions were discussed in class. The results of the study are based on student observations during their work with the computer, general impression of the discussion in the class, a short written test and a questionnaire. All material can be found on www.math.tu-berlin.de/~hoffkamp.

The study was conceived as a preliminary study with the following aims:

- Test the learning unit and work it over for further studies.
- Specify further research questions.
- Create a study design for a larger study based on the experiences made.

Results and discussion

The results mainly have qualitative character and will be presented by commenting on the observations made, by picking confirming statements of the students' answers on the questionnaire and by presenting some results from the written test.

Computer-aided work and work with the activities in general:

The observations clearly showed that the concept of low-overhead technology was successful in the sense that it was no problem to handle the activities without further instructions by the teacher although the students never used the computer in this specific way before. The design of the units made it possible to introduce a 'new' topic in a very short period of time.

The use of the computer had a very positive effect on the students' motivation. This is not only caused by the fact that the use of the computer in math classes was new for most of the students but also from the fact that the students appreciated to work autonomously. The students pointed out that the autonomy allowed them to find their own tempo and follow their own train of thoughts. This is confirmed by the following statements on the questionnaire:

> Question: Is there something special you like when working with the computer?
>
> Answer 1: It is less monotonous and the lesson is organized differently. You learn by means of a different learning aid, which allows a better imagination. The studious atmosphere is more comfortable. You do not have to follow the group's train of thoughts.
>
> Answer 2: That I can work independently (without teacher). One can use his own mistakes to come to the right result.

Statements like answer 2 were made several times. The students had the impression that they were able to use their mistakes in a productive way, a statement which is worth to be studied in further research. Students also appreciate that the computer takes over actions like drawing, calculating etc., which increases the time to work on the problems themselves.

It was a strong observation that computer-aided work allowed for a better internal differentiation of the learner group. Slowly learning students asked the teacher for help more often than more advanced students, but they still worked independently for longer periods. But of course there is still a high need for reflection of the train of thoughts within the class to avoid establishment of wrong mental representations.

Effects on functional thinking

It was obvious during the discussion and while observing the students, that the applets – by limiting the actions allowed – forced the students to focus on the dynamic view of the functional dependency.

The discussion of the results – which mainly consist of verbalisations of the properties of the functional dependency – ran pretty smooth. The students seemed to have created a mental image by using the applets. As an effect it was easy to communicate about the topic in the sense that the students were highly engaged in making contributions to the discussion. But it is still not clear what sort of mental image the students had created the more so as the verbalisations were mostly superficial. It only shows that working with the applets set a profound basis on which it seemed to be easy to continue to work on the topic.

Figure 5 shows the results from a problem on the written test. The students had to sketch graphs describing the dependency between x and the grey area F(x). An answer was 'meaningful' when the graph was strictly increasing, but e.g. left and right turn were mixed up.

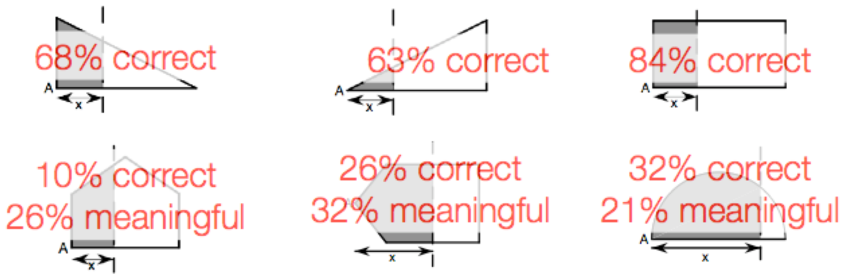

Fig. 5: A problem from the written test with percentages of correct and meaningful graph sketches.

As seen from the results in figure 5 the students by majority seemed to have created an appropriate mental concept concerning the dynamics of the functional dependency as far as the solution of problems like the one above is concerned. But it was still difficult to adapt the concept to other situations (here: other forms in line two of figure 5). Moreover the mistake to interpret graphs as photographical images nearly disappeared, but was still present in two or three student solutions. To find out more about the students' mental concept other approaches are necessary.

The potential of the level of **meta-variation** in order to enhance the understanding of the dynamic aspect of the functional dependencies seems to be high. This is an impression from student observations, class discussion and could be assumed when considering the following student statements:

> Question: Can you say what exactly you understood better by using the computer?
>
> Answer 1: How the graph changes when changing the triangle.
>
> Answer 2: I liked this form of figurative illustration that was given directly when changes were made because it is easier to understand something by watching it.

Of course the above question implies that the students are able to reflect their own thinking, but it is only used to find out which parts of the activity were considered by the students as showing them something new. To prove the effectivity of meta-variation further studies are needed.

OUTLOOK

The results of the first study give valuable hints for the direction of further research. In order to get deeper insights the learning unit 'Die Reise' was developed based on the experiences with the activity 'Dreiecksfläche'. It shall be used for more extensive tests. It is planned to record the computer actions and the student interactions with a camera integrated in the computer. The results will be available at CERME.

The following research questions are of interest and will guide our future research:

Main question: Is it possible to enhance the dynamic aspect of functional thinking by dynamically visualizing functional dependencies simultaneously in different representations and by giving the opportunity to experiment with them?

Further questions:

- Is it possible to enhance the dynamic view of functional dependencies faster or more sustainable by using the learning units?
- Which elements of the applets have a positive effect on the dynamic view of functional dependencies?
- Is it possible to distinguish types of students who get along better or worse with the learning units? How do slow learners deal with the units compared to more advanced students?
- How can we use computer-based activities like these as diagnostic tools?

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THE SYNERGY OF STUDENTS' USE OF PAPER-AND-PENCIL TECHNIQUES AND DYNAMIC GEOMETRY SOFTWARE

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This study is part of an ongoing research[1] centered on the interpretation of students' behaviors when solving plane geometry problems in Dynamic Geometry Software and paper-and-pencil media. Our theoretical framework is based on Rabardel's (2001) instrumental approach to tool use. We seek for synergy relationships between students' thinking and their use of techniques by exploring the influence of techniques on the resolution strategies. Our findings point to the existence of different acquisition degrees of geometrical abilities concerning the students' processes of instrumentation and instrumentalization when they work together in a computational and paper-and-pencil media. In this report we focus on the case of a student.

INTRODUCTION

We report research on the integration of computational technologies in mathematics teaching, in particular on the use of Dynamic Geometry Software (DGS) in the context of students' understanding of plane geometry through problem solving. We focus on the interpretation of students' behaviors when solving plane geometry problems by analyzing connections and synergy among techniques used in both media, DGS and paper-and-pencil, and geometrical thinking (Kieran & Drijvers, 2006). Many pedagogical environments have been created such as Cinderella, Geometer's Sketchpad, and Cabri Géomètre II. We focus on the use of GeoGebra (GGB) because it is a free DGS that also provides basic features of Computer Algebra Software. As said by Hohenwarter and Preiner (2007), the software links synthetic geometric constructions (geometric window) to analytic equations, coordinate representations and graphs (algebraic window). Our aim is to analyze the relationships between secondary students' problem solving strategies in two environments: paper-and-pencil (P&P) and GGB. Laborde (1992) claimed that a task solved using DGS may require different strategies to those required by

[1] The research has been funded by Ministerio de Educación y Ciencia MEC-SEJ2005-02535, 'Development of an e-learning tutorial system to enhance student's solving competence'.

the same task solved with P&P; this fact has an influence on the feedback provided to the student.

Our broadest research question aims at how the use of GGB in the resolution of plane geometry problems interacts with the students' P&P skills and their conceptual understanding. We analyze and compare resolution processes in both environments, taking into account the interactions (student-content, student-teacher and student-GGB). In this report we focus on two research goals as being interpreted in the case of one student, Santi. We analyze this student's instrumentation and instrumentalization processes, and we compare his resolution strategies when using P&P and GGB within each problem. In the whole research we work with a total of fourteen individual cases from the same class group and establish some commonalities and differences among them.

THEORETICAL FRAMEWORK

We first draw on the instrumental approach (Rabardel, 2001). According to Kieran and Drijvers (2006), a theoretical framework that is fruitful for understanding the difficulties of effective use of technology, GGB in our case, is the perspective of instrumentation. The instrumental approach to tool use has been applied to the study of Computer Algebra Software into learning of mathematics and also to Dynamic Geometry Software. The instrumental approach distinguishes between and artifact and an instrument. Rabardel and Vérillon (1995) claim the importance of stressing the difference between the artefact and the instrument. A machine or a technical system does not immediately constitute a tool for the subject; it becomes an instrument when the subject has been able to appropriate it for her/himself. This process of transformation of a tool into a meaningful instrument is called instrumental genesis. During the instrumental genesis, mental schemes are built up by the student. In these mental schemes, technical and conceptual components are interwoven (Rabardel, 2001). This process is complex and depends on the characteristics of the artifact, its constraints and affordances, and also on the knowledge of the user. The process of instrumental genesis has two dimensions, the instrumentation and the instrumentalization:

- Instrumentation is a process through which "the affordances and the constraints of the tool influence the students' problem solving strategies and the corresponding emergent conceptions" (Kieran & Drijvers, 2006, p. 207). "This process goes on through the emergence and evolution of schemes while performing tasks" (Trouche, 2005, op. cit., p. 148).

- Instrumentalization is a process through which “the student’s knowledge guides the way the tool is used and in a sense shapes the tool” (Kieran & Drijvers, op. cit., p. 207). “This process can lead to enrichment of an artifact, or to its impoverishment” (Trouche, 2005, op. cit., p. 148).

In our research, we need to select different problems for being solved first with P&P and then with the help of GGB. In order to analyze the connectivity and synergy between the students’ resolution strategies in both environments, the problems are to be somehow similar. The basic space of a problem is formed by the different paths for solving the problem. We transfer the similarity of the problems to the similarity of their basic spaces. For example, the problems considered in this article, share common strategies for reaching the solution such as equivalence of areas due to complementary dissection rules, application of formulas (area of a triangle), particularization, etc.

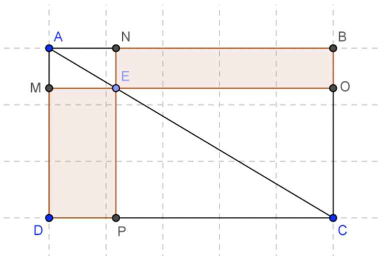
We plan to design an instructional sequence, focusing on a systematization of the interactions produced between artifacts (P&P, GGB), the mathematical actions and the didactical interactions. The theoretical framework is based on both instrumental approach and activity theory (Kieran &Drijvers, 2006). We connect the activity theory as part of the “orchestration” (Trouche, 2005). The actions consist in different problem sequences to be proposed by the teacher to the students, to be solved in both media. The teacher proposes different indications or new problems. For each problem, we prepare a document with pedagogical messages understood as Cobo, Fortuny, Puertas and Richard (2007), that provide differing levels of information, and we group them according to the phases of the solving processes which are being carried out- familiarization, planning, execution, etc. We classify the pedagogical messages, for each phase, in three levels. Level 0 contains suggestions that do not imply mathematical contents or procedures in the solving process. The messages of level 1 only convey the name of the implied mathematical contents or procedures. Level 2 provides more specific information on these contents or procedures. For the problems to be solved in a technological environment we also prepare contextual messages. These messages are related to the use of GGB. The teacher can help the students in case they have technical difficulties with GGB.

We also specify some terms that will be used in this study of students’GGB resolutions such as figure and drawing. We use these terms with their usual meaning in the context of the Dynamic Geometry Software (Laborde & Capponi 1994). Hollebrands (2007) uses this distinction between figure and drawing in order to describe the way in which students interpret the representations generated on the computer.

CONTEXT AND METHOD

The study is conducted with a group of fourteen 16-year-old students from a regular class in a public high school in Spain. These students are used to working on Euclidean geometry in problem solving contexts. They have been previously taught GGB. The main source of data for this paper comes from the experimentation with two problems:

1. Rectangle problem: *Let E be any point on the diagonal of a rectangle ABCD such as AB =8 units and AC=6 units. What relation is there between the areas of the shaded rectangles in the figure below?*



2. Triangle problem: *Let P be any point on the median [AM] of a triangle ABC. What relation is there between the areas of the triangles APB and APC?*

These problems have to do with comparing areas and distances in situations of plane geometry. They admit different solving strategies; they can be solved by mixing graphical and deductive issues, they are easily adaptable to the specific needs of each student, and they can be considered suitable for the use of GGB. For all the problems, we start by exploring the basic space of the problems in the P&P and DGS environments. After having identified the different resolution strategies and conceptual contents of the problems, the focus is on analyzing the necessary knowledge to solve them. Finally, we prepare a document with the pedagogical and technical messages that provide differing levels of information.

All the activities with students are planned to take four sessions of one hour each with an average of two problems per session. The two problems above were developed in the first two lessons in which the students worked on their own. The inquiry-based approach to the lessons leads the students to assume the responsibility for the development of the task. The teacher fosters the students' autonomy by only intervening in certain moments and giving some messages, established a priori, concerning the resolution.

For the experimentation with each problem, the whole set of data is: a) the solving strategies in the written protocols (P&P and GGB); b) the audio and video-taped interactions within the classroom (student-teacher, student-content and student-GGB); and c) the GGB files. All these data were examined in order to inform about our research goals. The integration of data concerning these goals led us to the description of the students' processes of instrumentation and instrumentalization processes. For the description, different variables were considered, among them: the students' heuristic strategies (related to geometric properties, to the use of algebraic and measure tools or to the use of both…); the use of GGB (visualization, geometrical concepts, overcoming difficulties…); the obstacles encountered in each environment (conceptual, algebraic, visualization, technical obstacles…); etc.

For each case, we first analyze the P&P resolution with data coming from the tapes and the protocols. We consider the student's solving strategies and the use of mathematical contents. Then we analyze the GGB resolutions with data coming from the tapes and especially from those tapes that show the screen. We consider again the student's solving strategies, the use of mathematical contents and now we also pay attention to instrumented techniques and technical difficulties. After having developed these two types of analysis, we compare GGB and P&P resolutions by looking at the use of the two environments within each problem, when it is possible. To analyze the problem solving process, we also consider the phases of the problem solving process (Schoenfeld, 1985) as a whole in each group of problems (GGB and P&P).

THE CASE OF SANTI: An episode of exploration/analysis

The mathematical content of the problem was dealt with in courses prior to the one Santi is currently taking. Santi has procedural knowledge relating to the application of formulas for calculating the area of the figure, and sufficient knowledge of the concepts associated with geometric constructions. He is a high-achieving student. Santi is asked to solve the first problem with P&P and the second problem with the help of GGB. In this section we summarize his problem solving process for both problems.

- Resolution of the rectangle problem (P&P):

In the resolution of the first problem, after reading the explanation of the problem, Santi observes the figure and then he states that he does not know enough numerical data. The teacher suggests the student to consider a particular case (heuristic cognitive message of level 1 in the planification/execution phase). Santi reacts to this message, considering the particular case in which E is the midpoint of the diagonal and he conjectures that both areas should be equal. Then he tries to prove the conjecture for the particular case in which the length AE is 2 units. The student reaches a solution to the particular case by using trigonometry. He obtains the angles in the triangle EAN (Figure 1) and he calculates the measures of the sides AN and AM. Finally he obtains the numerical value of both areas and he observes that he gets different values. Santi requests a message about the solution because he expected to obtain equal values. The teacher observes that there is an algebraic mistake in his resolution and suggest Santi to review the process he has followed because there are algebraic mistakes (metacognitive message of level 1 in the verification phase). The student finds the mistake and obtains the equal values of both areas (Figure 1). He then tries to use the same strategy for the general case using the relation: $\tan(\sphericalangle MAE) = \frac{8}{6} = \frac{AN}{AM}$.

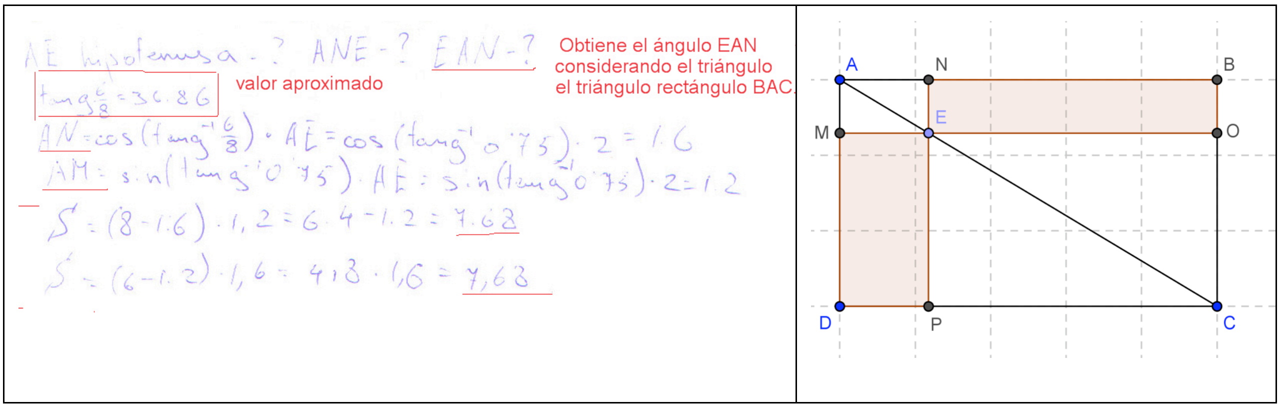


Figure 1: Resolution with paper and pencil of the first problem (Santi)

Santi bases his resolution strategies on applying trigonometry and he does not tries to use the strategy based in comparing areas of congruent triangles (strategy based on equivalence of areas due to complementary dissection rules). The teacher proposes other problems to be solved with P&P and with GGB. In the following paragraph we consider one of these problems.

- Resolution of the triangle problem (GGB):

After reading the explanation of the problem, Santi draws a graphic representation without coordinate axes before constructing the figure with GGB. The teacher observes that Santi has considered the point P in the side AC of the triangle instead of the median. The teacher says to Santi "Try to understand the conditions of the problem" (metacognitive message of level 0 in the familiarization phase). Santi constructs a new figure with GGB (Figure 2) and he observes the figure trying to find a solving path. Then he proposes a conjecture and ask the teacher for verification: " *the triangles APC and APB have a common side and the same area (he verifies this with the tool area of a polygon). How could I prove that these two triangles are equal"*? *I have tried to prove that they have the same angles but I don't see it..."*

We observe that Santi does not validate his conjecture with the help of GGB. The teacher gives him a validation message of level 1 " *Are you sure that these triangles are equal?*. Santi reacts to this message changing the triangle ABC. He moves the vertex A (Figure 3) and he observes without measure tools that the triangles are different.

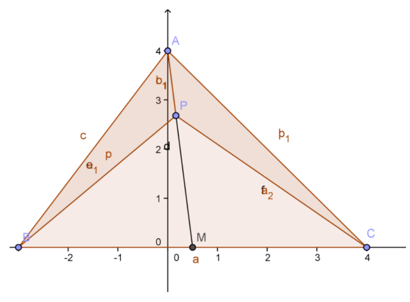
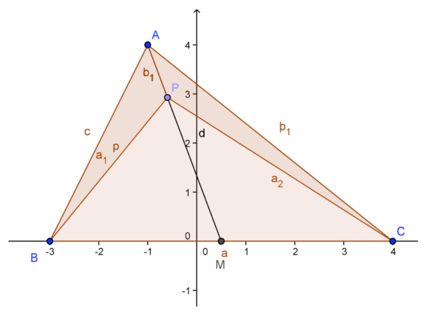
	

Figure 2: Construction with GGB of the triangle ABC and its median. Santi uses the tool polygon to construct the triangles.	**Figure 3:** He moves the vertex A to obtain a general triangle. We observe that he tries to define vertices with coordinates that are integer numbers.

The last graphic deduction marks the beginning of the search for a new strategy. He observes the figure, without dragging its elements. More than five minutes have gone without doing anything in the screen. Santi requests again the help of the teacher (Table 1, line 1) for the familiarization phase of the problem.

		Interactions
1	Santi	P has to be any point in [AM]? Isn't it the midpoint? [Santi tries to consider particular cases]
2	Teacher	P is any point in the median [AM]. The triangle ABC is also a general triangle (cognitive message of level 1 for the familiarization phase)
....	Santi	[Santi reacts to this message modifying the initial triangle. He moves again the vertices to obtain the triangle in Figure 3].
3	Santi	I think that I see it!...The triangles have a common side and the same height [the segments [BM] and [MC] (wrong deduction)]

4	Teacher	Are you sure about that?
5	Santi	[Santi reacts to this message observing the triangle without doing any action on the screen. Then he states:] No.These lines are not perpendicular! [(AM) and (BC)]. But, this was a good trial... Have they the same base? [he refers to the common side of both triangles]
6	Teacher	Yes

Table 1: How Santi tries a new solving path

For the first time, Santi tries to drag the vertices of the triangle trying to find invariants. While he drags the vertexes he looks in the algebraic window for invariants. We observe here the simultaneous use of the algebraic window and the geometric window. He observes again that the triangles have the same area in all the cases and a common side. He tries to prove that the heights are equal but he wrongly considers that the side [BM] is the height of the triangle BAP (Figure 3). The teacher gives him a message of level 0 for the validation phase (Table 1, lines 3 to 6). Santi reacts to this message constructing with GGB the perpendicular line from the vertex B to the base of the triangle (Figure 4). He tries to follow with the same strategy (proving that the heights have the same length) and he drags the vertexes A, B and C observing the constructed lines (Figure 4).

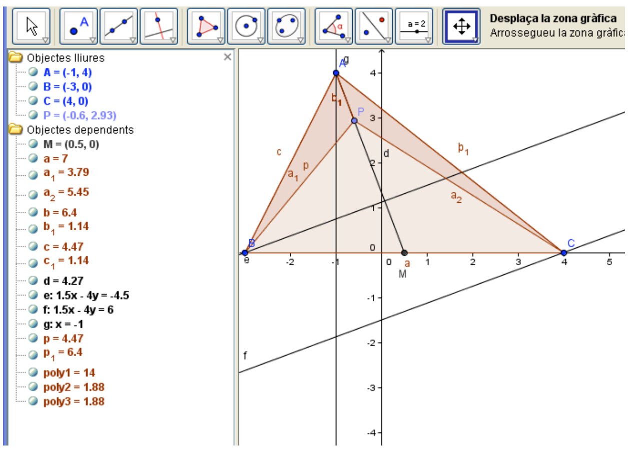


Figure 4: Construction of the height of the triangle BPA and perpendicular line through C to the median.

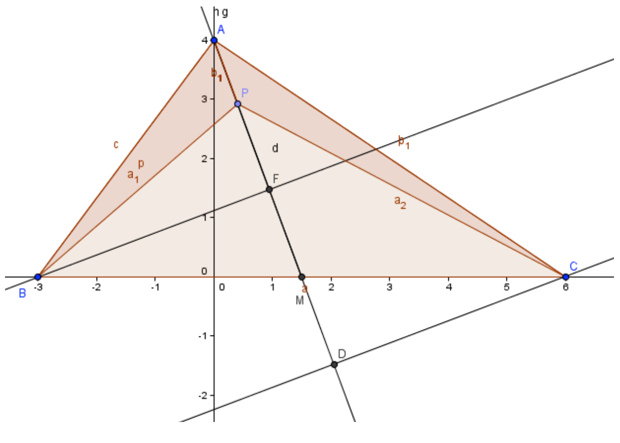


Figure 5: the heights have the same length (congruent triangles BFM and MCD)

In this time, he observes again the figure (Figure 4) without dragging. He is lost. This is the beginning of a new phase. We wonder if Santi had found a proof for his conjecture if he had constructed the heights of both triangles. Nevertheless, he does not construct the points F and D (Figure 5) and he abandons the solving strategy. Santi requests again the help of the teacher for the planification/execution phase.

- Santi: *Is it possible to solve the problem with trigonometry?*

The teacher gives him a new message: *"Could you think of some way of breaking the triangle ABC into triangles and look for invariants with the help of GGB"* (cognitive message of level 2 for the planification phase). Santi does not react to this message and try to find a resolution strategy using trigonometry. Santi does not find any trigonometric strategy and tries again with GGB. This time, he reacts to the previous message of the teacher. He erases the perpendicular lines and drag the vertices of the triangle ABC. He observes in the algebraic window the changing values looking for invariants. He observes then that the interior triangles BPM and CPM (Figure 3) have always the same area (Table 2). This observation will suggest him a new solving path based on comparing areas. He makes a new conjecture and request the help of the teacher for validating his deductions (Table 2).

		Interactions
1	Santi	Are the triangles BPM and PMC equal? (Figure 2)
2	Teacher	What do you mean by equal?
3	Santi	They have the same area
4	Teacher	Yes. You should justify this fact.
5	Santi	If I subtract two equal areas from two equal areas, do I get the same area?
6	Teacher	Yes
6	Santi	Ok! I justify this with paper and pencil.

Table 2: Strategy based on comparing areas

Finally Santi justifies his deductions with P&P. He proves that the median of a triangle divides the triangle into two triangles of same area. We wonder if the use of GGB helps Santi to find a strategy based on comparing areas.

FINAL REMARKS

We observe in this study that Santi appropriates the software in few sessions of class and he always makes constructions based on geometric properties of the figures. He also combines the simultaneous use of the algebraic window and the geometric window and he tends to reason on the figure. We consider that the affordances of the software have influenced Santi's resolution strategies (instrumentation process). In the ongoing research (longer teaching experiment) we have also observed some common heuristic strategies in both environments such as the strategy of supposing the problem solved and the strategy of particularization. We also observe that Santi tends to use more algebraic strategies when he works only with P&P than when he works in a technological environment. Moreover he tends to produce more generic resolutions, independent of numerical values, fostered by a proposal of problems that accept these kinds of solving strategies. Nevertheless, given that students have different relationships with the use of GGB and the detailed study of Santi gives us some insight of a future classification of typologies in the instrumental genesis. In our broader research we try to follow the instrumental genesis for a group of fourteen students to observe different students' profiles. Future research should help to better understand the process of appropriation of the software and to analyze the co-emergence, connectivity and synergy of computational and P&P techniques in order to promote argumentation abilities in secondary school geometry.

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INTERNET AND MATHEMATICAL ACTIVITY WITHIN THE FRAME OF "SUB14"

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In this paper we analyze and discuss the use of ICT, particularly the Internet, in the context of a mathematical problem-solving competition named "Sub14", promoted by the University of Algarve, Portugal. Our purpose is to understand the participants' views regarding the mathematical activity and the role of the technology they've used along the competition. Main results revealed that the participants see the usage of Internet quite naturally and trivially. Regarding the mathematical and technological competences elicited by this competition, evidences were found that develops mathematical reasoning and communication, as well as it increases technological fluency based on the exploration of everyday ICT tools.

A GLIMPSE OVER THE MATHEMATICAL COMPETITION "SUB14"

Sub14 (www.fct.ualg.pt/matematica/5estrelas/sub14) is a mathematical problem-solving web based competition addressed to students attending 7th and 8th grades.

It comprises two stages. The *Qualifying* consists of twelve problems, one every two weeks, and takes place through the Internet. The Sub14 website is used to publish every new problem; it provides updated information and allows students to send their answers using a simplified text editor in which they can attach a file containing any work to present their solution. The participants may solve the problems working alone or in small teams and using their preferred methods and ways of reasoning. They have to send their solution and complete explanation through the website mailing device or using their personal e-mail account. Every answer is assessed by the organizing committee, who always replies to each participant with some constructive feedback about the given answer.

The word problems are selected according to criteria of diversity and involve several aspects of mathematical thinking not necessarily tied to school mathematics. Their aim is to foster mathematical reasoning, either on geometrical notions, numbers and patterns, and logical processes, among others. There is a concern on presenting problems that allow different strategies and also some that have multiple solutions.

In Iona’s class the students had to elect a delegate and a co-delegate. Each student wrote two names in a voting sheet by order: the first for the delegate and the second for the co-delegate. There are 13 students in the class. How many ways have a student to vote if his or her own name is allowed?

Fig. 1: A problem aiming to elicit the abilities of organizing and counting

The *Final* consists of a one-day tournament where the finalists solve five problems, individually, with paper and pencil, and explain their reasoning and methods. This *Final* also provides some recreational activities addressed both to contestants and accompanying persons, namely parents and teachers.

Joanna, Josephine and Julia are all very fond of sweets. As the summer aproaches they decide to go on a diet. Their father has a large scales and they used it to weigh themsleves in pairs.
Joanna and Josephine together wheighed 132 kg
Josephine and Julia together wheighed 151 kg
Julia and Joanna together weighed 137 kg.
What is the weight of each one?

Fig. 2: A problem from the Final on identifying and relating variables and numbers

Demanding a clear description of the reasoning, methods and procedures was a strong concern of the committee. Moreover, the feedback sent to each participant had an essentially formative role (Diego & Dias, 1996), aimed at stimulating self-correction and valuing students’ own ideas. Every two weeks the Sub14 committee publishes a proposal of the solution of the previous problem, stressing the diversity of strategies that students could have applied. Hence, the committee selects noteworthy excerpts from student’s solutions, whether due to the originality of their reasoning, their creativity or the interesting usage of technological tools.

A THEORETICAL FRAMEWORK

In this paper we are addressing a part of a larger study and consequently we refer to a few theoretical aspects of the overall framework. There are four main focuses in the theoretical approach: (a) looking at mathematics as a human activity, (b) taking problem solving as an environment to develop mathematical thinking and reasoning, (c) exploring the concept of being mathematically and technologically competent and finally (d) considering the role of home ICT in out of school mathematics learning.

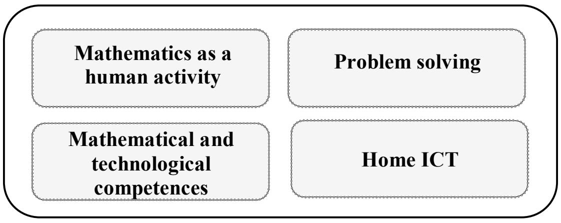

Fig. 3: Main conceptual elements of the theoretical framework

Mathematics as a human activity

Doing mathematics may be recognized as a human activity based upon a person's empirical knowledge, in search of a formalized understanding of the everyday problematic situations. From this point of view, Freudenthal (1973, 1983) states that human activity, which comprises empirical knowledge, guides oneself from the simple observation and interpretation of phenomena – horizontal mathematizing – to its abstract structuring and formalization – vertical mathematizing.

One of the criteria observed in launching a problem in Sub14 refers to the expectation that participating students will be able to activate their empirical knowledge and their experience to tackle mathematical problems. This perspective on mathematical activity is shared by many authors who emphasize the importance of exploring mathematical situations starting from common sense knowledge (Hersh, 1993, 1997; Ernest, 1993; Ness, 1993; Matos, 2005). As Schoenfeld (1994) claims, easiness in the use of mathematical tools, like abstraction, representation or symbolization, does not guarantee that a person is able to think mathematically. Rather mathematical thinking requires the development of a mathematical point of view and the competence to use tools for understanding.

This is the perspective that is present in Sub14 and which expresses the prevailing concept of mathematical activity arising from the perspective of Realistic Mathematics Education: bringing student's reality to the learning situation so that he/she is the one who does the mathematics, drawing on his/her knowledge and resources.

Mathematical knowledge and problem solving

Several authors from the field of mathematics education have proposed problem solving as a privileged activity "for students to strengthen, enlarge and deepen their mathematical knowledge" (Ponte et. al, 2007, p. 6).

This view on mathematical problem solving entails a conception of mathematical knowledge that is not reducible to proficiency on facts, rules, techniques, and computational skills, theorems or structures. It moves towards broader constructs that entails the notion of mathematical competence (Perrenoud, 1999; Abrantes, 2001) and problem solving as a source of mathematical knowledge. In solving a problem there are several cognitive processes that have to be triggered, either separately or jointly, in pursuing a particular goal: to understand, to analyze, to represent, to solve, to reflect and to communicate (PISA, 2003).

According to Schoenfeld (1992), the concept of mathematical problem can move between two edges: (i) something that needs to be done or requires an action and (ii) a question that causes perplexity or presents a challenge. The educational value of a problem increases towards the second pole where the solver has the possibility of coming across significant mathematical experiences. One of the purposes of mathematical problems should be to introduce and foster mathematical thinking or adopting a mathematical point of view, which impels the solver to mathematize: to model, to symbolize, to abstract, to represent and to use mathematical language and tools (Schoenfeld, 1992, 1994).

The formative aims of the problems proposed in Sub14 are essentially in line with the perspective of giving students the experience of mathematical thinking and also the opportunity to bring forth mathematical models and particular kinds of reasoning.

Communication, home technologies and learning

Considering that mathematics is a language that allows communicating your own ideas in an accurate and understandable way (Hoyles, 1985), Sub14 intends to develop that relevant communicational aspect, as stated in the current National Curriculum: "students must be able to communicate their own ideas and interpret someone else's, to organize and clearly present their mathematical thinking" and "should be able to describe their mathematical understanding as well as the procedures they use" (Ponte et. al, 2007, p. 5). Conversely, the importance of developing the competence of mathematical communication draws on a strong connection between language and the processes that structures human thought, as it is referred by Hoyles (1985). Accordingly, language takes up two different roles in mathematical education: communicative, where students show the capacity to describe a situation or reasoning act; and cognitive, which may help to organize and structure thoughts and concepts. Hence, there is a multiplicity of capacities and competences, both mathematical and technological, which are triggered through the combination of facts and resources in order to solve each problem of the competition.

Technologies and particularly the Internet, which gave life to Sub14, had a somewhat "neutral" or "trivial" role since the main focus of students' concerns was on the actual mathematical activity involved. Noss and Hoyles (1999) used the "window image" to emphasize this phenomenon: a window allows us to look beyond, and not only at the object itself. Although every new technology tends to draw attention to the tool itself, we soon need to "forget" the tool and concentrate on the potentialities it has to offer, namely on the learning and cognition field.

Using Lévy's (1990) ideas, Borba and Villarreal (2005) claim that technology mediates the processes that are responsible for the rearrangement of human thinking. In fact, knowledge is not only produced by humans alone, but it's an outcome of a symbiotic relationship between humans and technologies – which the authors entitled *humans-with-media*: "human beings are impregnated with technologies which transform their thinking processes and, simultaneously, these human beings are constantly changing technologies" (p. 22).

Indeed, human thought used to be defined as logical, linear and descriptive. Nowadays it is hastily changing into a *hypertextual thinking*, comprising many forms of expression that go beyond verbal or written forms, such as image, video or instant messaging. These social changes allow youngsters to develop a large number of competences, which grants them the skills and sophistication required to learn outside the school barriers.

Towards the conclusions of the ImpaCT2 project, that took place in Great Britain, Harrison (2006) asserted that the model used to measure the influence of new technologies on youngster's school achievement was too simplistic and induced to settle on the absence of such influence. This author then proposed a new model that emphasized the importance of social contexts in which learning takes place. Harrison (2006) was thus able to conclude that learning at home must not be neglected, but be faced as a partner of the school curriculum.

Although knowledge gathered outside the school is frequently seen as worthless, it is clear that children are capable of watching a YouTube's video, talk to their friends through MSN, and also solve the Sub14 problems and express their thinking using an ordinary technological tool. These "digital natives" (Prensky, 2001, 2006) access information very fast, are able to process several tasks simultaneously, prefer working when connected to the Web and their achievement increases by frequent and immediate rewards.

METHODOLOGY

The purpose of this study was to identify and understand the participants' perceptions regarding the (i) mathematical activity, (ii) the competences involved and (iii) the role of the technological tools they've used along the competition.

A case study methodology reveals itself appropriated in cases where relevant behaviours can't be manipulated, but it is possible and appropriate to proceed to focused interviews, attempting to understand the surrounding reality (Yin, 1989). Since we intended to get diversity and interpret results, eleven participants were chosen intentionally, from the 120 finalists, hoping they would provide interesting data according to the research questions.

The field work began collecting data that would allow a complete understanding of the competition, in order to adjust the approach to the participants. Later on, we used other data collecting techniques: a *questionnaire* to the finalists, *video records* from the Final, *documental data* from participants (such as their solutions to the Sub14 problems, or their interactions with the Sub14 committee, using e-mail). That data allowed the planning of interviews to the eleven participants, as well as to their parents, aiming at collecting descriptive data, in their own language, hoping for an understanding on how they viewed certain aspects of Sub14 and of their involvement.

For the data analysis we used an interpretative perspective (Patton, 1990) and an inductive process (Merriam, 1988), based on content analysis. Thus, the objective was to understand the significance of the events from the interviewees' perspective, within the scope of the theoretical assumptions defined prior to the interviews.

THE INTERNET – THE SUB14 LIFE SUPPORT

The first evidence produced about students' perceptions on the problem solving environment was the fact that the Internet and the technologies used within Sub14 assumed, from the point of view of students, a neutral role in the development of their mathematical activity. However several aspects of their products and statements showed evidence of the importance and usefulness of different tools, behind their apparent indifference to technology if put in abstract terms. Therefore, we may state that the Internet undoubtedly is the technology that brings Sub14 to life; all the learning processes and the competences involved derive from the interaction provided and nourished by this tool.

Trivializing Technology

Resorting to the Internet and other technologies was seen as absolutely natural by some participants.

> "As I see it, reasoning comes from the mind; therefore I think no technology will help us to really solve a problem." [Bernardo]

Trivializing the role of the Internet and the technology involved in the competition can be found in the model proposed by Harrison (2006), which highlights the importance of the social context surrounding the learning process. These participants show all the characteristics of a digital native (Prensky, 2001), i.e., they start using computers at an early age, with a great variety of purposes, which can be related or not to school learning. Furthermore, these participants can also be considered as "humans-with-media", or particularly, "humans-with-Internet", according to the definitions proposed by Borba and Villarreal (2005), since their personality is being built, simultaneously, through the daily interaction with the Internet and other technologies.

The Role of Communication and Feedback

Essentially, the participants like the feedback sometimes provided immediately by the Sub14 committee, resulting from the analysis of their answers to each problem. The possibility of correcting little mistakes or even change the resolution completely, using the hints from the feedback, increase their self-esteem and motivation to remain in the competition. For the interviewed students, this is the characteristic that distinguish Sub14 from other similar competitions.

> "This year I also participated in another competition. We send an answer to a problem, but they don't reply to us, and the Sub14 committee keeps sending hints". [Isabel]

As students pointed out there is someone who receives their answer to the problem, their questions or even their complaints.

> "It's not something that we send and no one will care about, they are always there." [Lucia]

As mentioned above, the feedback is almost immediate and this is only possible due to the communicability that the Internet enables. The constant request for auto-correction forces the participants to reflect on their own reasoning and the mistakes given, stimulating them to submit a correct answer as quickly as possible. Some of them sent messages to Sub14 several times a day, until they get the confirmation that their answer was correct.

Another positive aspect of this bilateral communication is the request of a complete, coherent and clearly written explanation of the participant's reasoning. This way, the feedback provided by the organizing team respects and nourishes the reasoning of each participant, as well as the processes used. We have even noticed a development on the correctness of the answers that the participants submitted throughout the competition.

> "In the beginning it was somehow strange. I wasn't used to it. I'd put the calculations and that was it. But we had to present all our thinking. It was as if I had to write what I was thinking. Thus, I would think out loud and split it into parts. But from the 3rd or 4th problem I was already used to it." [Isabel]

This feedback originated a change of attitudes in some participants within their mathematics classroom when facing assessment situations. The students themselves observed they took more care while answering to questions posed by the teachers, presenting all the necessary justifications and showing a greater predisposition to interpret a problematic situation, find a reasoning path or procedure in order to explain the solution in a convincing way.

> "[…] I now pay more attention to little details that sometimes others don't, and it reflects on the tests and on the problems that the teacher gives us, some of them really tricky… but now I am tuned!" [Lucia]

"Home Technologies"

The dynamic nature of the bidirectional communication can be felt in other aspects revealed by the participants. First off all, we note the usage of the Sub14 website: the participants use it frequently and think that the available information is important and useful, they like the design, the way it is organized and the fact that it is permanently updated:

> "I like having an organized website (…) the 'Press Conference' page was always updated." [Ana]

The purpose of posting submitted solutions was to show the methods used by some of the participants, hoping to improve their performance by the positive reinforcement of seeing their works and their names posted online.

> "Yes! Sometimes I would go there to see if any of the posted solutions was mine! Once or twice I found my answer and I was very happy and shouted… 'Daddy, daddy, come here!'" [Bernardo]

Bernardo's enthusiasm, as well as many other participants', supports the pedagogical and motivational aspects of the methodology adopted in Sub14. Not only it promotes the diversification of reasoning strategies and points out the several problem solving phases, but it also increases self-esteem and improves innovation and creativity as “special” answers are selected to be published online.

Moreover, the fact that Sub14 is a digital competition allowed the participants the opportunity of communicating their reasoning in an inventive way, since they could resort to any type of attachments, particular the ones they felt more comfortable with or the ones they found adequate to the problem itself. Therefore, the participants used mainly the text editor, MSWord, but they also used drawing and spreadsheet programs, like MSPaint and MSExcel, all examples of home technology.

MSWord was used to compose text, organize information in tables, and insert images, automatic shapes, WordArt objects or Equation expressions. It was elected the favorite between the participants, since it is the one they better understand and constantly are asked to use for several school assignments.

> “[Word] is the simplest to use, it’s the one that I have more confidence on to do school tasks, and I’m used to it. It’s the one I’m good at.” [Lucia]

Using images was a strategy that seven of the interviewee used. Nevertheless, some of them only inserted images that had something to do with the problem context, more like an illustration. In this case, we may consider that resorting to images had mainly an aesthetic function, as it didn’t help presenting or clarifying the reasoning and strategy used to solve the problem. However, other interviewees sketched their own images using MSPaint in order to improve the intelligibility of their thoughts:

> “Anything that I thought that could help to improve the reasoning, I would draw it [in paper] and then I’d put it in the computer.” [Bernardo]

> “We were playing with some straws and we reached the solution by trial and error. Then [we took some pictures] with the digital camera [and] put them in the computer so that we could send them.” [Alexandra]

In this way the image usage assumed, essentially, two roles in the answers of these participants. Firstly, it was merely a visual detail, which may be influenced by the type of work done in students’ school assignments. Secondly, the creation of images within the context of their interpretation of the problems is an evidence of their efforts on expressing their reasoning in the best possible way. Moreover, we can notice their awareness of the different representations that could materialize their reasoning and even some decision ability when facing the options they had at hand.

Two interviewees used Excel to present their answers. One of them used this tool to solve every Sub14 problem, showing however a narrow usage of the program as a means to organize the information and his answer. Seldom using the function “SUM”, he essentially resorted to tables and images, considering that the spreadsheet was better than a text editor. The referred simplicity seems to come from the fact that he has been exposed to this tool from an early age:

> “Sometimes, when I was a kid – I got my first computer when I was six – I liked to get there [MS Excel] and do squares with the cells, paint them and that sort of things…” [Bernardo]

Another participant used the spreadsheet to solve five of the twelve problems, showing that he knew some of the advantages of this tool. Therefore, these participants were confident enough in using MSExcel, nonetheless not as a result of work within the school context, but rather of their domestic “findings”.

ANOTHER LOOK AT SUB14 AS A LEARNING ENVIRONMENT

Solving the Sub14 mathematical problems requires looking at a problem situation from a mathematical perspective. This can be seen as a mathematizing process, since the participant is stimulated to express the way in which thinking was organized and progressed. In this competition, the participants found a place where they could freely communicate their ideas, had someone who listened and advised them, helping to make their mathematical thinking and expression become clearer. Moreover, when solving a problem, they faced the transition from convincing themselves to convincing the others (Mason, 2001). This led participants to develop their own understanding of the problem, promoting the usage of domestic technologies to communicate, thus adding competences that sometimes school neglects or forgets.

As a learning environment, although being external to the school context, Sub14 is aligned with school mathematics, and promotes a set of competences that fit within current mathematical education purposes and curricular targets. The fact that the competition occurs in a loose institutional context allows a greater family commitment and complicity with the participant’s learning process, fostering the discussion on mathematical questions and problems outside the school environment, especially at home, maybe around dinner table.

Further work on this field shall include a future experience to investigate the possibility of allowing participants to communicate amongst them, within the website, bearing in mind the idea of a connected learning environment.

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USING TECHNOLOGY IN THE TEACHING AND LEARNING OF BOX PLOTS

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Box plots (or box-and-whisker-plots) can be used as a powerful tool for visualising sets of data values. Nevertheless, the information conveyed in the representation of a box plot are restricted to certain aspects. In this paper, we discuss both the potential and limitations of box plots. We also present a design for an empirical study in which the use of a variety of tasks explicitly addresses this duality. The activities used in the study are based on an interactive box plot applet that surpasses the currently available tools and offers new ways of experiencing box plots.

MOTIVATION

Recently, the mathematics curricula of many parts of the world were revised in order to include more statistics and data analysis. In the literature, one can find an extensive discussion about this idea under the notion of "statistical literacy" (Wallman, 1993; Watson & Callingham, 2003). This reflects the growing importance of the ability to understand and interpret data that has been collected or is being presented by others. The NCTM (2000) standards, for example, state, "To reason statistically--which is essential to be an informed citizen, employee, and consumer--students need to learn about data analysis and related aspects of probability." The global availability of data through the Internet makes it easy to access and process huge data sets. For these, it is important that students have the skills and tools to summarise and compare the data, also by using the computer.

In this paper, we focus on *box plots* as a means to visualize statistical data. Box plots are used not only in textbooks, but are also available in graphing calculators. In order to use statistical information properly, the students have to develop a clear concept of what the information means, no matter whether it is given numerically or, in this case, visually.

The situation described also applies to Germany where some states have incorporated a larger amount of statistics and data analysis into the mathematics curriculum. Our personal experience with teacher students teaching in 8th grade (14-year-olds) has shown that both teachers and learners tend to ignore the mathematical concepts behind the statistical analysis and fall back to recipes that enable them to solve the standard exercises from the text books. In a similar way, Bakker, Biehler & Konold (2004) point out that some of the features inherent to box plots raise difficulties in young students' understanding and use of them. As a remedy, we developed a series of activities that should enable students to develop a clear understanding of the statistical terms. The ultimate goal of the activities is that students can not only draw box plots for given data, but also interpret given box plots that describe real world situations.

THEORETICAL BACKGROUND

Box plots are part of the field of Exploratory Data Analysis where data is explored with graphical techniques. Exploratory Data Analysis is concerned with uncovering patterns in all kinds of data. A box plot (or box-and-whisker-plot) is a relatively simple way of organizing and displaying numerical data using the following five values: the minimum value, lower quartile[1], median[2], upper quartile, and maximum value. Considering a set of data values, like, for example,

52, 32, 29, 30, 35, 17, 42, 63

these five values are easy to calculate:

- Minimum value – 17
- Lower quartile – 29.5
- Median – 33.5
- Upper quartile – 47
- Maximum value – 63.

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[1] As there is no universal definition of a quartile, we dedicated a whole subsection of this article to this issue. Also, the original box plot uses the lower and upper hinge instead of the quartiles.

[2] The median can be defined as the number separating the lower half of a data set from the higher half in the sense that at least 50% of the values are smaller than or equal to the median.

Using these five numbers, the related box plot can be constructed on a vertical (which we use in the following description) or horizontal scale (which is used in Fig. 1) by

- Drawing a box that reaches from the lower quartile to the upper quartile,
- Drawing a horizontal line through the box where the median is located,
- Drawing a vertical line from the lower quartile (the lower end of the box) to the minimum value,
- Drawing a vertical line from the upper quartile (the upper end of the box) to the maximum value,
- Marking minimum and maximum with horizontal lines.

Fig. 1 shows the box plot corresponding to the data above, created with a box plot applet provided by CSERD.

The representation of a box plot communicates certain information at a glance: The line indicating the median illustrates the centre of the data, the height of the box demonstrates the spread of the central half of the data, and the length of the two lines above and below the box show the spread of the lower and upper quarters of the data.

In our box plot visualisation we do not use outliers, as these are not used in the standard textbooks, either.

Various authors have declared that box plots are particularly useful for easily comparing two or more sets of data values (e.g. Kader & Perry, 1996; Mullenex, 1990). In order to illustrate this idea, compare two data sets where the minimum and maximum values as well as the arithmetic mean are equal and reveal no hint of how to draw conclusions about the values as shown in Fig. 2.

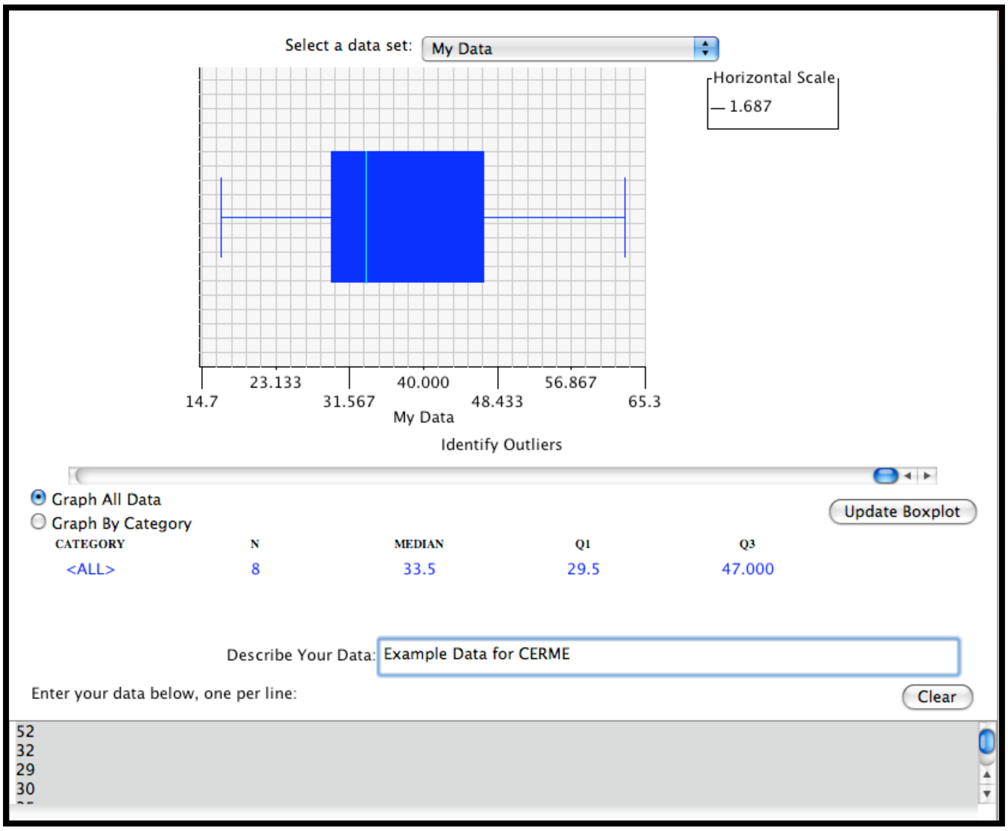

Figure 1: A box plot created online[1] for the sample data in this article

It is obvious that in the second case, the box is much smaller than in the first one, indicating that the spread of the central half of the data is lesser.

We use this technique extensively in the exercises that are part of the teaching unit.

A Useful Quartile Definition

There is no universal definition of a quartile; actually, there are at least five different definitions in use (Weisstein 2008). The situation is even worse for software packages. According to Hyndman and Fan (1996) even within a single software package several definitions might be used concurrently. A visualisation sometimes uses a different definition than a numerical calculation. One reason for this is that the original concept of box plots as introduced by Tukey (1977) used the *hinges* of a data set instead of the quartiles, which are different in one of four cases. Unsurprisingly, the concept of a quartile is obscure to most students and even teachers.

[1] http://www.shodor.org/interactivate/activities/boxplot/ . Also available in the NCTM Illumniations database (NCTM 2008)

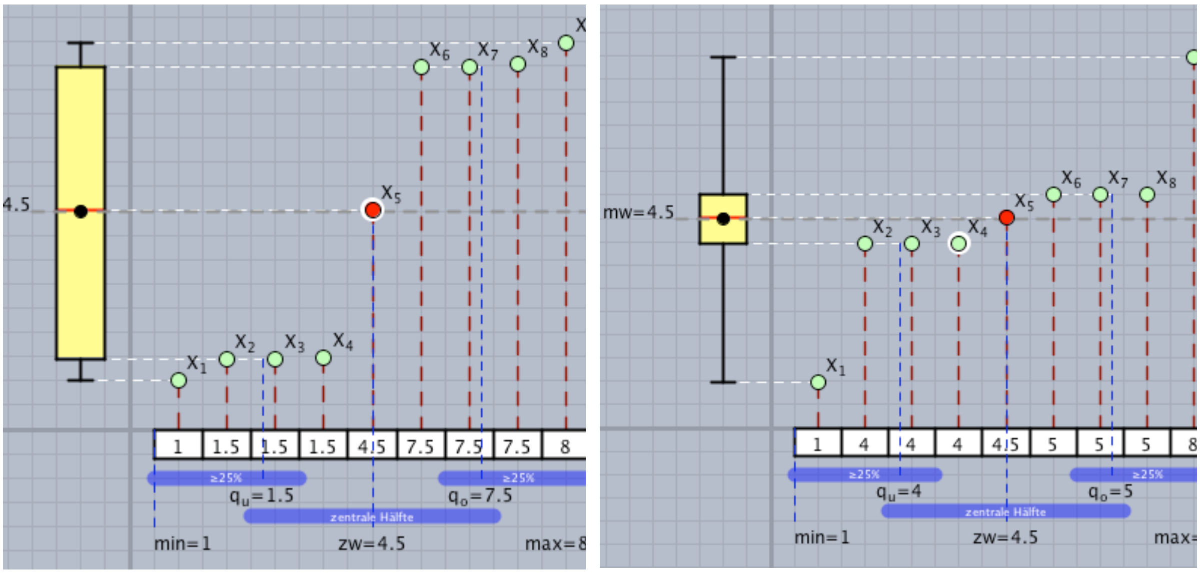

Figure 2: Two box plots with different interquartile ranges

School textbooks in Germany usually do not give an exact definition of quartiles, but combine a colloquial description with a recipe to calculate the quartiles. All definitions are not based on the desired result (i.e., "the first quartile is a value such that at least 25% of the values are less or equal, and at least 75% of the values are greater or equal"), but on a specified way to calculate them (i.e. "the first quartile is the value that is placed at position (n+1)/4, if this is an integer, else…" or similar). Unfortunately, these recipes are incompatible with the QUARTILE function as provided by Excel, which is the most common tool for data analysis in German schools, besides the availability of special purpose educational tools for statistical analysis like, e.g., Fathom (Key Curriculum Press, 2008). The documentation of the QUARTILE function in Excel[2] is similar to the text book definitions of quartiles: it lacks a formal definition or explanation of the desired properties, and focuses on examples instead. It is not possible to explain the results of Excel on that basis.[3]

Most of the critique above only applies to small data sets. With larger amounts of data the actual definition used is not as significant as with less than, say, 20 values. Still, these data sets are the ones that are accessible to hands-on manipulation in the classroom.

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[2] We used the German version of Excel 2004 on Mac OS X. There are explanations of the formulas used available, for example in learn:line NRW at http://www.learn-line.nrw.de/angebote/eda/medio/tipps/excel-quartile.htm. Excel uses a weighted arithmetic mean for the quartiles.

[3] Büchter and Henn (2005) provide a definition of quartiles that is precise and matches the expectation that the lower and upper quartile are the smallest values that cut off at least 25% of the values.

For our study, we chose a definition that is both easy to understand and easy to use. A lower quartile[4] of a set of values is a number q_u such that at least 25% of all values are less than or equal to q_u, and at least 75% of all values are smaller than or equal to q_u. In many cases, this number is a value of the data set, but we do not restrict quartiles to be chosen from the values. The definition for the upper quartile q_u is analogous. Using 50% instead of 25% and 75% we can also use it to define the median. All definitions are valid even if some values occur several times.

Finding the Median and Quartiles

A very useful and action-oriented way to *find* the median and quartiles is the following one.[5] Order all values in increasing order, and write them down in a row of equal-sized boxes. The strip of ordered values may look like this (for 8 values):

1	4	7	14	26	31	33	42

Now, fold the strip in the middle by lining up the left and right border. The crease will be between 14 and 26, in this example, as is the median. We may use any number between 14 and 26 (not including them), for example the arithmetic mean, 20.

Finding the quartiles works by iterating the procedure described above. Folding the left and right half of the strip will create creases between 4 and 7, yielding a suitable lower quartile of 5.5, and between 31 and 33, which suggests choosing 32 as upper quartile.

1	4	7	14	26	31	33	42

The appeal of this method is that it also applies to situations where the creases pass through the boxes instead of separating two of them (i.e., for odd numbers of values, or if the number is not zero (modulo 4)). In that case, the (only) suitable value for the quartile (resp. median) is the value in that box. The conditions of our definition above are fulfilled automatically.

[4] We are using the standard German notation here, instead of Q_1 and Q_3 for lower and upper quartile.

[5] A student teacher, Simone Seibold, came up with this method during her traineeship in school.

Of course, the method is not suitable for real computations with data sets of significant size, but only for the proper conceptualisation. It can easily be transferred to a formula for the quartile and medians, however.

Advantages of Using Technology

Computers are a major reason for the increasing importance of statistics, and vice versa. The whole field of *data mining* became feasible only through the computing power to analyse large sets of data easily. Actually, the first applications of mechanized computing were of statistical natures, for example in the 1890 United States census (Hollerith 1894).

In general, multimedia learning bears advantages, in particular if several representations of a situation have to be connected mentally (see Schnotz & Lowe 2003, Cuoco & Curcio 2001). Relating to suitable design for multimedia learning, we refer to the book of Mayer (2003) that details some of the guiding principles. This being said, the existing online tools for creating box plots disregard these principles. Even the online tool that is officially endorsed by the NCTM (see Fig. 1) violates most of these rules. For example, the distant placement of the data entry and the box plot is in clear contradiction to the Spatial Contiguity Principle of Mayer.

The quality of interaction is another measure for multimedia learning. The direct interaction with a simulation with *immediate* feedback supports the learner (Raskin 2000). Even if there is no such concept of a "level of interactivity," as it is not a one-dimensional scale, such interaction is considered a key ingredient of good software (Niegemann et. al 2003, Schulmeister 2007). Sedig and Sumner (2006) categorized the possible types of interaction in mathematics software. Again, the activities found on the web so far do not obey these rules.

DESIGN OF THE ACTIVITIES

Based on the theoretical analysis given above we designed a set of exercises that enables the students to experience both the power and the restrictions of box plots. In all exercises students use the same interactive applet.[6] The applet is embedded into a plain web page and can be used without prior installations using a standard Internet browser. Using this applet, students can view and manipulate data with up to 22 values (the limit is not due to technical reasons, but given by the screen size). They can add or remove data, change data by dragging the associated data point with the mouse vertically, and re-order

[6] see http://kortenkamps.net/material/stochastik/Quartile.html. The applet is based on Cinderella (Richter-Gebert & Kortenkamp 2006).

values by dragging the points in direction of the *x*-axis. Points that have been added by the students are shown in red, others that were given are depicted in green.

According to Biehler, Backers and Konold (2004), it is helpful for students if individual cases can be recognized within the box plot representation. This is granted in the applet that we use in our study. All data is visible at all times. While the students are manipulating the data, the current mean value is displayed both numerically and by a dashed horizontal line. The values that correspond to the data points are shown numerically in a white box below each point (Fig. 3).

If the values are ordered ascending the applet adds more statistical information to the visualisation. To the left of the values the corresponding *box plot* showing the minimum, maximum, quartiles and median, is drawn. Those are connected through dashed lines with the corresponding "creases" and the values that are shown below the data. The blue bars mark the lower and upper quarters of the values as well as the central half (Fig. 4).

Finally, students can easily create a new data set by changing all values randomly. This enables them to quickly create new situations.

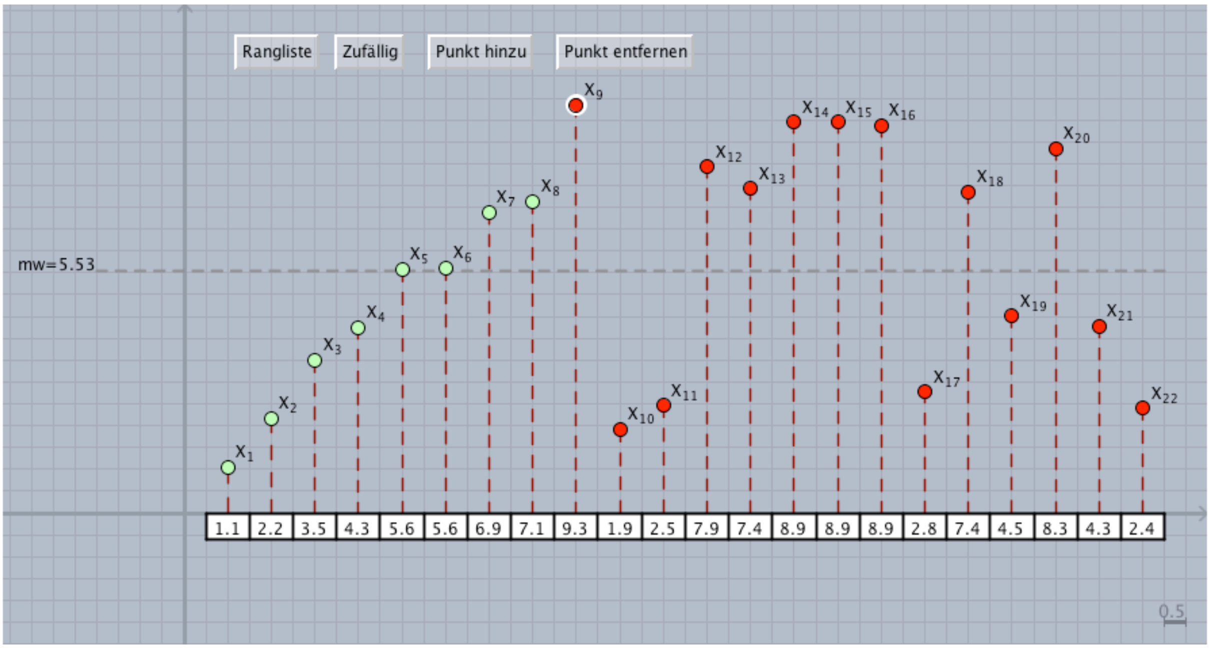

Figure 3: The interactive applet with unordered values

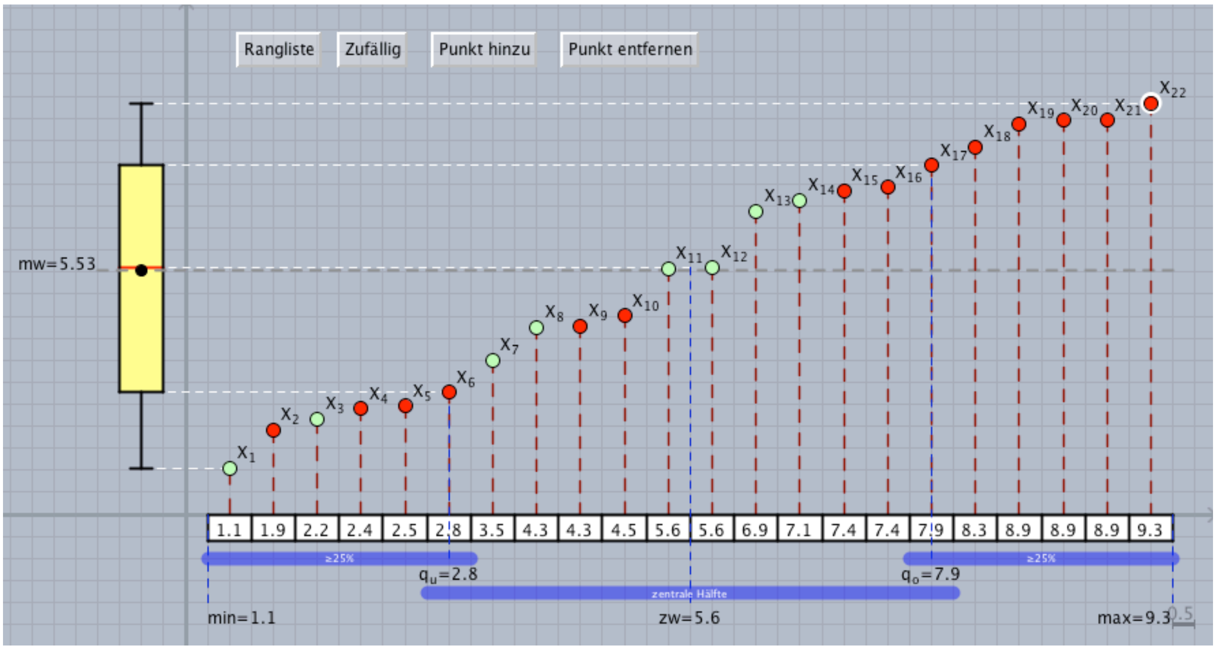

Figure 4: When the data set is ordered additional information and the box plot is shown

Exploratory Exercises

After a general introduction into the basic concepts and tools of statistical analysis, the students first work on standard textbook exercises, both using the manual approach described above and the applet. After that, we present a new set of exercises that are not based on real (or fake) data, but focus on modifying data sets in order to change or preserve the measures of variation:

a. Change *only* the arithmetic mean by changing values.

b. Change *only* the minimum or maximum by changing values.

c. Change *only* the length of the whiskers

d. Change *only* the size of the box (the interquartile range)

e. Add values without changing the box plot.

f. Remove values without changing the box plot

g. Try to move the arithmetic mean outside of the box

h. Try to move the median outside of the box

Our primary goal is that students understand that box plots are a compact visualization of five (or six, depending on the plot) statistical measures, which in turn describe the distribution of values in a data set. Based on these measures it is possible to draw conclusion about the original set. Students should be able to find as many conclusions as possible, while not over-interpreting the measures. The activities force the students to create data sets that differ only in certain aspects, while showing an interactive visualization of the data and the measures.

For example, while experimenting with (d) students will see that for a distribution with smaller box (i.e. a smaller interquartile range) the values in the central half are more densely distributed than for a distribution with a larger box.

Also, common misconceptions like a correspondence between the size of the box and the number of values in the data set are addressed. Adding or removing values does not necessarily change any of the measures of variation.

EMPIRICAL STUDY

In line with the recommendations formulated by a group of stochastic educators in Germany (Arbeitskreis Stochastik, 2003), the participants in our study are aged at least 15 years. We test the material in schools in two German states, North Rhine-Westphalia and Baden-Württemberg.

In order to ensure the comparability and reproducibility of the tests, the introduction to the necessary concepts is provided as a handout for the students with an accompanying teacher's manual. After this introduction, students work independently in groups of two with the exercises.

The planned assessment consists of a series of boxplots for *real world* data, taken from magazines or newspapers, that the students are asked to analyze and interpret. They have to write short articles summarizing the data. This design of the study – students are asked to solve a task they did not practise before – is used in order to answer our main research question: *Can students understand better which information box plots convey if they work with them interactively on abstract data sets?*

RESULTS

The study will be conducted in November 2008, and we expect to be able to show first results at CERME 6 in January.

CONCLUSION

We agree with the NCTM (2000) standards that students should also be able to create and use graphical representations of data in form of box plots as well as discuss and understand the correspondence between data sets and their graphical representations.

While box plots are an appealing tool for teaching, we still need research that proves whether working with box plots increases the ability of students to analyze and interpret data. Based on an approach that is easy to implement into the classroom, we provide a theory-based teaching unit that tries to enable students to understand box plots as a tool. As the applets used do not need further software packages, we hope for a wide reusability. At the same time, we are currently investigating the efficacy of our approach.

Biehler & Kombrink (2004) describe how they use interactive tools based on Fathom for teaching elementary stochastics at university level. In case we can prove that our activities are suitable for teaching in school, the future teachers that received a thorough education in statistics will be able to use their skills in school even if only basic ICT equipment is available.

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INTEROPERABLE INTERACTIVE GEOMETRY FOR EUROPE – FIRST TECHNOLOGICAL AND EDUCATIONAL RESULTS AND FUTURE CHALLENGES OF THE INTERGEO PROJECT

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*Abstract: In this overview article we describe the manifold achievements and challenges of Intergeo[1], a project co-funded within the eContent*plus *programme[2] of the European Union.*

THE INTERGEO PROJECT

The Intergeo project started in October 2007 and will be funded until September 2010. Its main concern is the propagation of Interactive or Dynamic Geometry Software.

Goals

Interactive Geometry is a way to improve mathematics education by using computers and Dynamic Geometry Software (DGS) and there are many advantages in comparison to "classical" geometry without DGS. Figures can e.g. be easily manipulated [see e.g. Roth 2008] and thus virtually be brought to life, comparable to what movies mean to images or to what interactive computer games mean to motion pictures.

It is therefore not amazing that Interactive Geometry obtains more and more attention in many educational institutions. Around 25 per cent of the countries within the EU refer explicitly to DGS in their national curricula or guidelines and roughly 40 per cent refer to ICT in general. And although the remaining countries do not mention ICT, some of them recommend the use of DGS in schools [Hendriks et al. 2008].

[1] http://inter2geo.eu

[2] http://ec.europa.eu/information_society/activities/econtentplus/index_en.htm

Still, the adoption of DGS at school is often difficult. Despite the fact that a lot of DGS class material exists, Interactive Geometry is still not used in classrooms regularly. Many teachers do not seem to know about the new possibilities, or they do not have access to the software and/or resources.

The Intergeo Project has identified the three following major barriers, that have a negative impact on the use of Interactive Geometry in classrooms [Intergeo Project 2007]:

- Missing search facilities
  Though many resources exist, there remains the problem of finding and accessing them. If the files were put on the internet by their developers, they are virtually scattered all over the web and it is extremely hard to retrieve them by using search engines like Google.
- Lack of interoperability
  There are many different programmes for Interactive Geometry on the market and each software has its own proprietary file format. Thus, finding a file does not automatically mean that it can be used – it must be a file for the specific software that is used.
- Missing quality information
  And even if a teacher finds a file and the file works with her DGS, it may still be unsuitable for the use in class due to a lack of quality. Lacking quality can be software-sided in the way the figures are constructed or missing (or even wrong) mathematical background.

The aims of Intergeo are to dispose of the problems stated. In other words, Intergeo will

- enable users to easily find the resources they are looking for,
- provide the materials in a format that can be used with different DGS systems, and
- ensure classroom quality.

All three facets will be dealt with in the following chapters in extenso.

Furthermore, Intergeo attends to a topic that is mostly neglected but of high importance nonetheless: the question of copyright.

Consortium

The Intergeo Consortium, the founding partners of the Project, assembles software producers, mathematicians, and mathematics educators: Pädagogische Hochschule Schwäbisch Gmünd (D), Université Montpellier II (F). Deutsches Forschungszentrum für künstliche Intelligenz DFKI (D), Cabrilog S.A.S. (F), Universität Bayreuth (D), Université du Luxembourg

(LUX), Universidad de Cantabria (ES), TU Eindhoven (NL), Maths for More (ES), and Jihočeská Univerzita v Českých Budějovicích (CZ). As the common interest of all partners is the propagation of sensible use of Interactive Geometry in the classroom, it was possible to collect both commercial, semi-commercial and free software packages. This is one of the key ingredients of the project: By building upon the joint knowledge and expertise of all parties, we hope to be able to address the needs of the teaching community.

Participation of External Partners

The participation of External Partners, as Associate Partners, Country Representatives, and User Representatives justifies the basis for assuring the sustainability of the projects' goals as mentioned above. Furthermore, gathering partners, as software developers, teachers, and persons at school administration level enables the development of a Europe-wide network that is indispensable for obtaining the projects' major achievements.

Since the project start in October 2007, several key actors in interactive geometry throughout Europe, including software producers, mathematics educators, governmental bodies, and innovative users that can provide additional content or serve as test users for the first content iterations were acquired.

Associate Partners

The role of Associate Partners implicates a variety of tasks and expectations, as the adoption of the common file format for their software, the provision of significant content to the Project, the development of ontologies, and the conduction of classroom tests. The project could successfully find several important Associate Partners, see [Intergeo Project 2008] and the following table.

Table: List of Associate Partners

Nr.	Country	Name	Nr.	Country	Name
1	Austria / USA	Markus Hohenwarter (GeoGebra)	**15**	Germany	Andreas Göbel (Archimedes Geo3D)
2	Brazil	Leônidas de Oliveira Brandão (iGeom)	**16**	Germany	Reinhard Oldenburg
3	Canada / Spain	Philippe R. Richard, Josep Maria Fortuny (geogebraTUTOR)	**17**	Germany	Andreas Meier
4	Canada	Jérémie Farret (3D Geom)	**18**	Germany	Roland Mechling (DynaGeo)
5	Croatia	Sime Suljic (Normala)	**19**	Italy	Giovanni Artico (CRDM)
6	France	Cyrille Desmoulins	**20**	Luxembourg	Daniel Weiler
7	France	Odile Bénassy (OFSET)	**21**	México	Julio Prado Saavedra (GeoDin)
8	France	François Pirsch (JMath3D)	**22**	Portugal	Arsélio Martins
9	France	The Sesamath association	**23**	Portugal	José Francisco Rodrigues (CMAF)
10	France	EducTice - INRP / Luc Trouche	**24**	Slovakia	Dusan Vallo
11	France	IUFM - Jacques Gressier (Geometrix)	**25**	United Kingdom	Albert Baeumel

12	Germany	Jürgen Roth (Universität Würzburg)
13	Germany	Heinz Schumann
14	Germany	René Grothmann (C.a.R. / Z.u.L.)
26	United Kingdom	Nicolas van Labeke (Calques 3D)
27	United States	Joshua Marks (Curriki)

Country Representatives

For each EU country a Country Representative serves as a contact person in their respective country. They come from ministries of education, preferably, and enable the Project to easily contact the relevant persons at school administration level. Based on these contacts, the project develops ways to map curricula into the ontology for geometry that suits all countries of the EU. The project could successfully find several Country Representatives, and a list is available at [Intergeo Project 2008].

User Representatives

User Representatives, as teachers and software partners, build the basis for the sustainability of the project. They are a contact point with their associations, in order to support the relationship with potential Intergeo-users [Intergeo Project 2008].

- Selected teachers ease experimentations in the classroom of educational content gathered by the project, promote the use of the Intergeo-platform and the philosophy of resource sharing and quality control.
- Selected Software-partners promote the uploading of content to the Intergeo-platform.

Among others, the selection of external partners will be performed at several local user meetings during the project period. The local user meetings have a central role in gathering the community of practice. They intend to help providing a complete European coverage:

- The Local User Meetings present Intergeo to the users: The need of a common file format for interoperability, the need of a web platform to share resources, the need of the ontology and the curriculum mapping to share resources across all European countries.
- The Local User Meetings are a good way to reach power users and engage them into the project to improve the projects' dissemination.
- Local User Meetings identify suitable schools for the Quality Assessment.

MAJOR ACHIEVEMENTS

Content Collection

The consortium promised to offer a significant amount of content for use in the database. Before the project started in Oct. 2007 we identified more than 3000 interactive resources to be used. All these and more[3] have been collected through the Intergeo platform by September 2008, first as *traces*, and now being converted to real assets that are searchable and tagged with meta-data. The available content ranges through all ages and educational levels, and also mathematical topics and competences. See http://i2geo.net to access and use the content.

Copyright/Licence issues

A major issue with content re-use and exchange is the handling of intellectual property rights. This affects not only the copying of resources, but also the modification and the classroom use. Without being able to process the data, it is also impossible to offer the added value of cross-curriculum search, for example.

Thus, all content that is added to the Intergeo portal has a clear license, usually of the creative commons type allowing for modification and free (non-commercial) use. See http://creativecommons.org for details.

Theoretical Foundation For Cross-Curriculum Categorization and Search

Interactive geometry has one quality that makes it very particular among learning resources: it is often multilingual. This led us naturally to propose a search tool for interactive geometry resources that is not just a textual search engine but a *cross curriculum* search engine.

A simple scenario can explain the objective of cross-curriculum search: a teacher in Spain contributes a Cabri construction which is about the intercepting lines theorem (the *Teorema de Tales*) and measuring segment lengths; a teacher in Scotland looks for a construction which speaks about the *enlargement* transformation, *segment lengths*, and the competency to *recognize proportionalities*. They should match: the Scottish teacher should find the Cabri construction of the Spanish teacher (and be able to convert it to his preferred geometry system). No current retrieval system can afford such a matching process: there is no common word between the annotation and the query.

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[3] On September 30th, 2008, there was a total amount of 3525 traces available.

For cross-curriculum matching to work, a language of annotations is needed that encompasses the concepts of all curriculum standards and that relates them. Careful observation of the current curriculum standards (see [Laborde et al. 2008]) has shown that topics, expressed as a hierarchy, and competencies are the two main type of ingredients that are needed. To this end the Intergeo project has built an ontology of topics, competencies, and educational levels called GeoSkills. This OWL ontology [McGuinness et al. 2004] has been structured and is now being populated by a systematic walk through the national curriculum standards; a report of this encoding is at [Laborde et al. 2008]; completeness for several school-years has been reached in French, English, and Spanish curriculum standards. Because the edition of an ontology using a generic tool can be difficult, a dedicated web-based tool is under work which will make it possible for the complete German, Spanish, Czech, and Dutch curriculum standards to be encoded by the Intergeo partners and its associates.

For the match to happen, the input of topics or competencies has to be cared for. We use the auto-completion paradigm for this purpose: the (textual) names of each topic and competency are searched for in this process and the user can thus choose the appropriate node with sufficient evidence, maybe browsing a presentation of the topics and competencies. An alternative approach proposed is to browse curriculum standards, being documents that teachers potentially know well, in order to click a paragraph to choose the underlying topics and competencies.

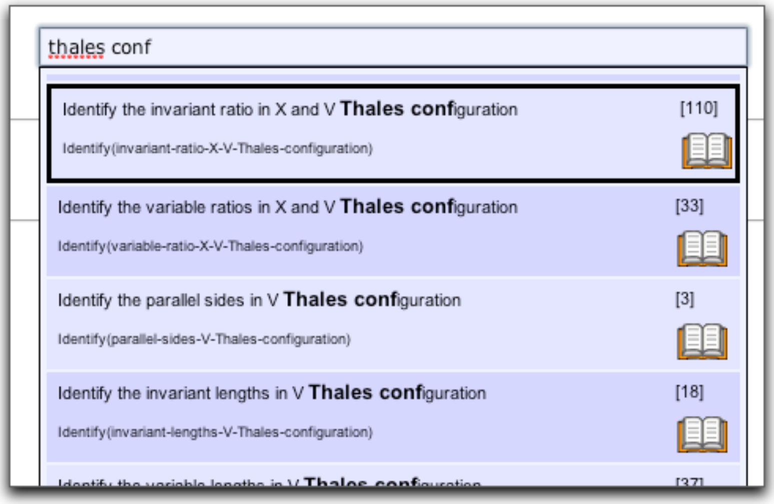

Figure 5: The skills textbox

Quality Assessment Framework

A Quality Assessment Framework for the Intergeo project was set up based on a questionnaire filled freely by the teachers themselves [Mercat et al. 2008]. This assessment has two different aims:

- To rank the resources so that, in response to a query, "good" resources are ranked before "bad" resources, at equal relevance with respect to the query.
- To help improve resources by identifying criteria to work upon in order for the author to revise his resource according to the user's input.

The questionnaire is both easy and deep; it can provide a light 2 minutes assessment as well as a deep pedagogical insight of the content. This is achieved by a top-down approach: The quick way just asks for 8 broad statements that can be answered on a scale from "I agree" to "I disagree":

- I found easily the resource, the audience, competencies and themes are adequate
- The figure is technically sound and easy to use
- The content is mathematically sound and usable in the classroom
- Interactivity is coherent and valid
- Interactive geometry adds value to the learning experience
- This activity helps me teach mathematics
- I know how to implement this activity
- I found easily a way to use this activity in my curriculum progression

These broad questions can be opened up by the reviewer to give more detailed feedback on issues of interest for him, such as "Dragging around, you can illustrate, identify or conjecture invariant properties" in the "Interactive geometry adds value to the learning experience" section.

Of course a thorough questionnaire is weighted more than a quick reply in the averaging of the different answers. The questionnaire is to be taken twice, as an a priori evaluation, before the actual course, and as an a posteriori evaluation, after the teaching has taken place. This second variant is being more weighted than the first one.

Different users are weighted differently as well: seasoned teachers with a lot of good activity, or recognised pedagogical experts, will have a high weight: their reviews are taken into account more than the average new user. Negative behaviour like steady bashing or eulogy will, on the contrary, lower user's weight. We are thinking as well about a social weight: teachers could flag some of their colleagues as "leaders", users whose past choices they liked, because they are teaching at the same level for example, and the weight of these leaders would increase.

The I2Geo Platform

The central place of exchange of interactive geometry constructions is a web-platform; the i2geo.net platform is becoming a server where anyone with interest to interactive geometry can come to search for it and to share it.

The i2geo.net platform is based on Curriki, an XWiki-extension tuned for the purpose of sharing learning resources: strong metadata scheme, quality monitoring system and self-regulated groups. Being based on a wiki platform, Curriki offers an online editing and inclusion facility and thus also makes collaborative content construction possible.

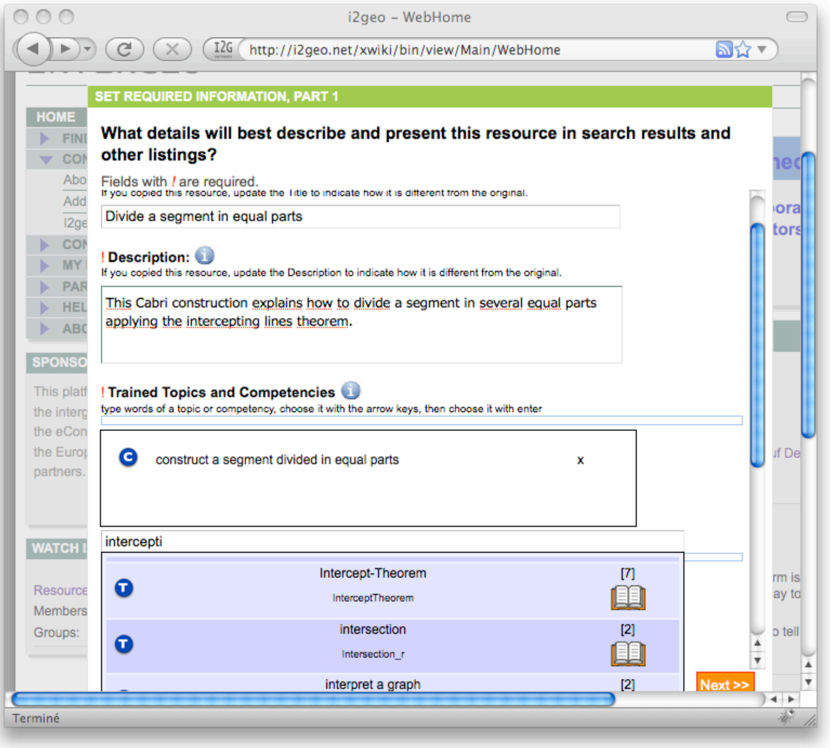

Figure 6: Editing metadata

The i2geo platform has three major adaptations compared to the tools provided by Curriki: the search and annotation tools, the review system, and the support for interactive geometry media.

The i2geo search and annotation tool uses the GeoSkills ontology described above: this allows the trained topics and competencies, the required ones, and the educational levels to be all entered using the input methods described above (auto-completion and pick-from-document).

Such elaborate methods are needed if one wants to honour the rich set of educational levels in Europe and the diversity of curriculum standards sketched in [Laborde et al. 2008].

The i2geo search tool uses the GeoSkills ontology as well: queries for any concept are generalized to neighbouring concepts which thus allows the match of the intercepting-lines-theorem when queried for the concept of enlargement.

The i2geo platform is under active development and can be experimented with on http://i2geo.net. Its current development focus is the input of metadata annotated resources and the review system described in the previous sections.

The services specialized to the geometry resources, enabling easy upload, preview, and embedding of interactive geometry resources will be provided later.

A Common File Format

A wide variety of Dynamic Geometric Systems (DGS) exist nowadays. Before this project, each system used incompatible proprietary file formats to store its data. Thus, most of the DGS makers have joined the project to provide a common file format that will be adopted either in the core of the systems or just as a way to interchange content.

The Intergeo file format aims to be the convergence of the common features of the current DGS together with the vision of future developments and the opinion of external experts. Its final version based on modern technologies and planed to be extensible – to capture the flavour of the different DGS – could serve as a standard in the DGS industry.

The specification of the first version of the Intergeo file format has been released by the end of July as deliverable D3.3 [Hendricks et al. 2008] after intensive collaboration between DGS software developers and experts. At present, the file format is restricted to the geometry in the plane, although it does not seem difficult to extend it, in the future, to the space. Besides it specifies only a restricted subset of possible geometric elements, which however lead to an agreement on the structure and basic composition of the format.

The general framework was clear from the outset: to design a semantically rich format that could be interpreted by at least all DGS in the consortium. One main design decision in this respect consists of the choice of constructions, as opposed to constraints, because in general, it is very difficult to give any particular solution for a set of constraints. Besides constraints of a strictly classical geometric nature do not say anything about the dynamic behaviour of a figure. A natural way to shed light on both of these problems is a more precise specification of how the objects depend on each other, stipulating first which objects are free and then proceeding step by step. Such a specification is called a construction. This decision implies less interoperability with constraint-based systems, since some of their resources will not be encodable into this format. But it ensures that construction-based DGS – the majority of the existing systems – will be able to interpret the resources.

As stated in the Description of Work, OpenMath Content Dictionaries are used to specify the symbols – the main ingredients used to describe a construction – of the file format. The XML schema can be generated automatically with some knowledge of how the atoms are expressed in XML. The complete list of official symbols defined so far can be found at http://svn.activemath.org/intergeo/Drafts/Format/.

As soon as version 1 of the file format got more concrete, some software developers started to investigate its practical usage by integrating it (partially) into their software. It was possible to move simple content between several of the packages in the project. For more information on the file format we refer to [Hendriks et. al 2008], which also lists the relevant URLS to see the progress.

NEXT STEPS AND CHALLENGES

Metadata Collection

With the arrival of the first curriculum-aware beta version of the i2geo.net platform we are now able to attach metadata to the existing content. This includes information about the authors, but also about the intended audience for a resource, the skills and competences that can be acquired through the resource, the prerequisites, and, of course, the topic – categorized according to the ontology.

While some of this information can be extracted automatically, there is still need for a lot of manual intervention. At the same time, the curricula available on the platform have to be revised and extended to accommodate all the content.

Quality Testing

The partners in the Quality Assurance work package will conduct small-scale experimentations in the classroom during the period January-April 2009. Teachers, whether alone or in homogeneous teams, will

- Use the platform in order to identify content suitable for their course,
- First fill an a priori questionnaire,
- Teach the resource in the classroom,
- And finally report on its use by updating the a posteriori questionnaire.

We will have to agree on a modus operandi, recruit volunteers, especially among the teachers that were contacted during the users meetings, instruct them and have them conduct the experimentations.

Then these assessments will be analyzed. The analysis will be used to iteratively improve the quality assessment framework according to the users' feedback on usability and relevance of the different items and of the online platform.

It is a primary concern that all resources receive at least basic testing. Thus, we will check the overall coverage in the project and, if necessary, identify resources to be tested.

As the quality assessment primarily aims to make it possible to improve ranking and quality of the resources, we can use this as a performance indicator. For this, the changes in ranking due to the quality evaluation will be measured. Additionally, selected examples will be analysed in order to understand whether authors can infer improvements of their resources.

Via interviews with selected authors we try to understand how they perceived quality assessment and how we can improve its perception as positive, constructive and scientific more than negative, useless and personal.

In the final year of the project, mass scale experimentations will take place. More countries and more parts of the curriculum shall be covered.

File format

As for version 1 of the file format some decisions that should be made with the help of other developers of DGS have been postponed, those experts are invited to join the discussion and propose solutions or give remarks, see [Hendriks 2008]. Thus, substantial modifications of this specification are expected to solve all practical issues that might arise.

Better Visibility

The ultimate goal and a measure of success is the visibility of the Intergeo platform in Europe as a whole. After the first year was devoted to setting up the technical prerequisites and administrative processes, as well as clearly describing how we can measure and improve the standards for successful interactive resources, we can now offer a *usable* platform with substantial content. We now have to make the platform more visible and raise interest within the didactical community, the teachers, and the governments throughout Europe.

Today, the websites of the individual software packages from the project still have much more visits a day than the i2geo.net portal. So a first step will be to announce the portal on the websites of the software packages and on the websites of (associate) partners using banners and an i2g-compliance badge that shows the compatibility of the software with the i2g file format.

CONCLUSIONS AND CALL FOR PARTICIPATION

In this article, we can only highlight the basic structure of the project. We invite everybody to visit the project website at http://inter2geo.eu, submit their own content on http://i2geo.net, join as an Associate Partner or become a User or Country Representative.

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REALISATION OF MERS (MULTIPLE EXTERN REPRESENTATIONS) AND MELRS (MULTIPLE EQUIVALENT LINKED REPRESENTATIONS) IN ELEMENTARY MATHEMATICS SOFTWARE

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Assumptions of multiple mental representations lead to the presumption of an enhanced mathematical learning, especially of the process of internalization, due to MERs (Ainsworth 1999) and MELRs (Harrop 2003). So far, most educational software for mathematics at the primary level aims to help children to automatize mathematical operations, whereby symbolical representations are dominating. However, what is missing is software and principles for its design that support the process of internalization and the learning of external representations and their meaning themselves – in primary school these are in particular symbols. This paper summarizes the current state of research and presents a prototype that aims to the above-mentioned purpose.

INTRODUCTION

In this article we describe the theory and new achievements of a prototypical educational software for primary school arithmetic. After developing the guiding principles that are based on multimedia learning models, we present DOPPELMOPPEL[1], a learning module for doubling, halving and decomposing in first grade.

THE COGNITIVE THEORY OF MULTIMEDIA LEARNING (CTML)

In the 1970s and 80s it was assumed that comprehension is limited to the processing of categorical knowledge that is represented propositionally. Nowadays, most authors assume the presence of multiple mental representation systems (cp. Engelkamp & Zimmer 2006; Schnotz 2002; Mayer 2005) – mainly because of neuro-psychological research findings. With regard to multimedia learning the Cognitive Theory of Multimedia Learning (CTML) of Mayer is to emphasize (Fig. 1).

[1] see http://kortenkamps.net/material/doppelmoppel for the software

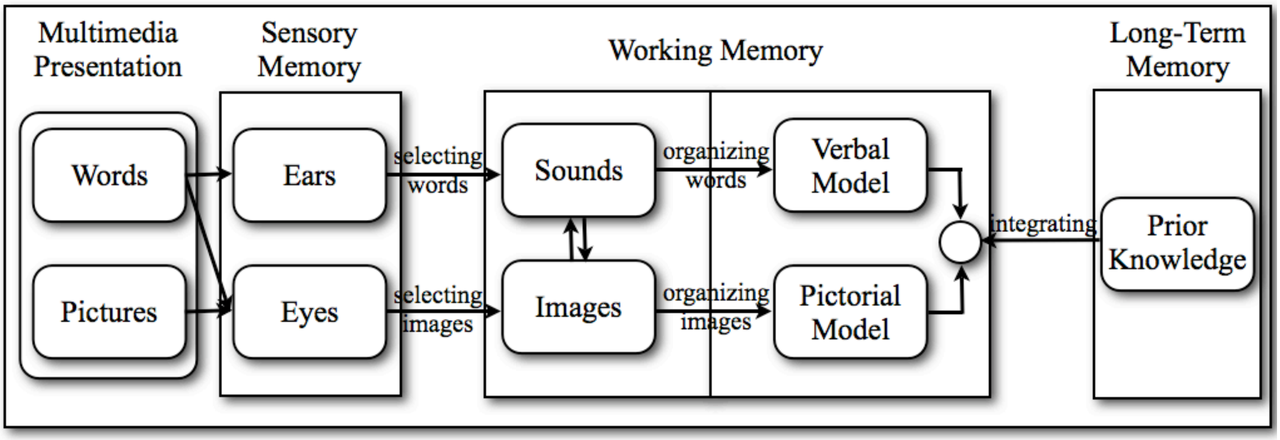

Figure 1: The Cognitive Theory of Multimedia Learning (CTML) of Mayer

Mayer (2005) acts on the assumption of two channels, one for visually represented material and one for auditory represented material. The differentiation between the visual/pictorial channel and the auditory/verbal channel is of importance only with respect to the working memory. Here humans are limited in the amount of information that can be processed through each channel at a time. Besides the working memory Mayer assumes two further types: the sensory memory and the long-term memory. Furthermore, according to Mayer humans are actively engaged in cognitive processing. For meaningful learning the learner has to engage in five cognitive processes:

(1) Selecting relevant words for processing in verbal working memory

(2) Selecting relevant images for processing in visual working memory

(3) Organizing selected words into a verbal model

(4) Organizing selected images into a pictorial model

(5) Integrating the verbal and pictorial representations, both with each other and with prior knowledge (Mayer 2005, 38)

Concerning the process of internalization the CTML is of particular importance. The comprehension of a mathematical operation is not developed unless a child has the ability to build mental connections between the different forms of representation. According to Aebli (1987) for that purpose every new and more symbolical extern representation must be connected as closely as possible to the preceding concrete one. This connection takes place on the second stage of the process of mathematical learning where the transfer from concrete acting over more abstract, iconic and particularly static representations to the numeral form takes place (Fig. 2). A chance in the use of computers in primary school is seen in supporting the process of internalization by the use of MELRs. This is the main motivation for the research on how the knowledge about MERs and MELRs in elementary mathematics and educational software is actually used and how it can be used in the future.

TO THE REALISATION OF MERS AND MELRS IN ELEMENTARY MATHEMATICS SOFTWARE

Despite the fact that computers can be used to link representations very closely, it is hardly made use of in current educational software packages. Software that offers MERs and MELRs with the aim to support the process of internalization is very rare. This is also the reason why tasks are mainly represented in a symbolic form (Fig. 2).

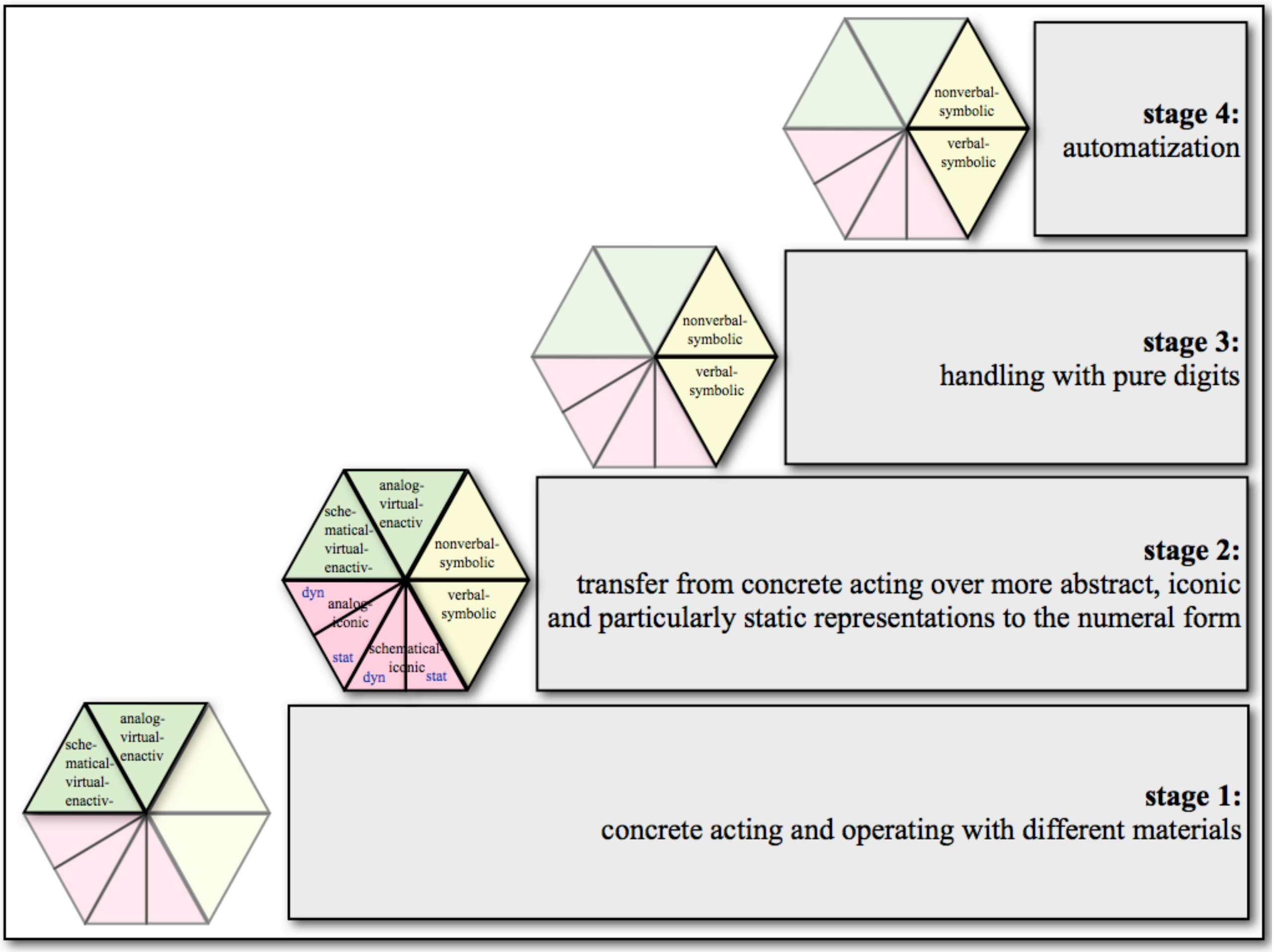

Figure 2: Forms of external representations combined with the four stages of the process of mathematical learning

Nevertheless, most software offers *help* in form of visualizations and thereby goes backward to the second stage. This is realised in different ways, which is why a study of current software was done with regard to the following aspects:

- Which forms of external representations are combined (MERs) and how are they designed?
- Does the software offer a linking of equivalent representations (MELRs) and how is the design of these links?

After this analyse, a total of sixty 1^{st}- and 2^{nd}-grade-children at the age of six to eight years were monitored in view of their handling of certain software (BLITZRECHNEN 1/2, MATHEMATIKUS 1/2, FÖRDERPYRAMIDE 1/2). Beside this own exploration – which will not be elaborated at this point - there is only a small number of studies that concentrates on MERs and MELRs on elementary mathematics software. In 1989, Thompson developed a program called BLOCKS MICROWORLD in which he combined Dienes blocks with nonverbal-symbolic information. Intention was the support of the instruction of decimal numeration (kindergarten), the addition, subtraction and division of integers (1^{st} – 4^{th} grade) as well as the support of operations with decimal numbers (Thompson 1992, 2). Compared to activities with "real things", there were no physical restrictions in the activities with the virtual objects to denote. Furthermore the program highlighted the effects of chances in the nonverbal-symbolic representation to the virtual-enactive representation and reverse. In his study with twenty 4^{th}-grade-children Thompson could show that the development of notations has been more meaningful to those students who worked with the computer setting compared to the paper-pencil-setting. The association between symbols and activities was established much better by those children than by the others.

Two further studies that examined multi-representational software for elementary mathematics are by Ainsworth, Bibby and Wood (1997 & 2002). The aim of COPPERS is to provide a better understanding of multiple results in coin problems. Ainsworth et al. could find out, that already six-years-old children do have the ability to use MERs effectively. The aim of the second program CENTS was the support of nine- to twelve-years-old children in learning basic knowledge of skills in successful estimation. There were different types of MERs to work with. In all three test groups a significant enhancement was seen. The knowledge of the representations themselves as well as the mental linking of the representations by the children were a necessary requirement. The fact that a lot of pupils weren't able to connect the iconic with the symbolic representation told Ainsworth et al. (1997, 102) that the translation between two forms of representations must be as transparent as possible.

The opinions about an automatic linking of multiple forms of representations vary very much. Harrop (2003) considers that links between multiple equivalent representations facilitate the transfer and thus lead to an enhanced understanding. However, such an automatic translation is seen very controversial. Notwithstanding this, it is precisely the automatism that presents one of the main roles of new technologies in the process of mathematical learning (cf. Kaput 1989). It states a substantial cognitive advantage that is based on the fact that the cognitive load will be reduced by what the student can concentrate on his activities with the different forms of representations and their effects. An alternative solution between those two extremes – the immediate automatic transfer on the one hand and its non-existence on the other hand – is to make the possibility to get an automatic transfer shown to a decision of the learner.

PRINCIPLES FOR DESIGNING MERS

The initial point and justification of multimedia learning is the so-called multimedia principle (cf. Mayer 2005, 31). It says that a MER generates a deeper understanding than a single representation in form of a text. The reason for this is rooted in the different conceptual processes for text and pictures. In being so, the kind of the combined design is of essential importance for a successful learning. The compliance of diverse principles can lead to an enhanced cognitive capacity. Thus Ayres & Sweller (2005) could find a *split-attention-effect* if redundant information is represented in two different ways because the learner has to integrate it mentally. For this more working space capacity is required, and this amount could be reduced if the integration were already be done externally. Mayer (1995) diversifies and formulates besides his *spatial contiguity principle* the *temporal contiguity principle*. According to this principle, information has not only to be represented in close adjacency but also close in time. If information is also redundant, the elimination of the redundancy can lead to an enhanced learning (*redundancy-effect*). The *modality principle* unlike the split-attention principle does not integrate two external visual representations but changes one of it into an auditory one. Hence an overload of the visual working memory can be avoided.

In addition to the modality principle Mayer recommends the segmenting principle as well as the pretraining principle to enhance essential processes in multimedia learning. As a result of the *segmenting principle* multimedia information is presented stepwise depending on the user so that the tempo is decelerated. Thus the learner has more time for cognitive processing. The *pretraining principle* states that less cognitive effort will be needed if an eventual overload of the working memory is prevented in advance through the acquisition of previous knowledge. Finally, the abidance of the *signaling principle* allows a deeper learning due to the highlighting of currently essential information. Extraneous material will be ignored so that more cognitive capacity is available and can be used for the essential information.

In elementary instruction the children first of all have to learn the meaning of symbolic representations and how to link them with the corresponding activities. So the above-described principles cannot be adopted one-to-one. Based on an empirical examination of the handling of six- to eight-years-old pupils with MERs and MELRs in chosen software, we could identify new principles and the above-described ones could be adapted, so that their compliance supports the process of internalization. These principles are demonstrated and realized in the following example of the prototype DOPPELMOPPEL.

THE PROTOTYPE DOPPELMOPPEL

Didactical concept and tools

The function of the ME(L)Rs in DOPPELMOPPEL is the construction of a deeper understanding through abstraction and relations (fig. 3). The prototype was built using the Geometry software Cinderella (Richter-Gebert & Kortenkamp 2006) and can be included into web pages as a Java applet.

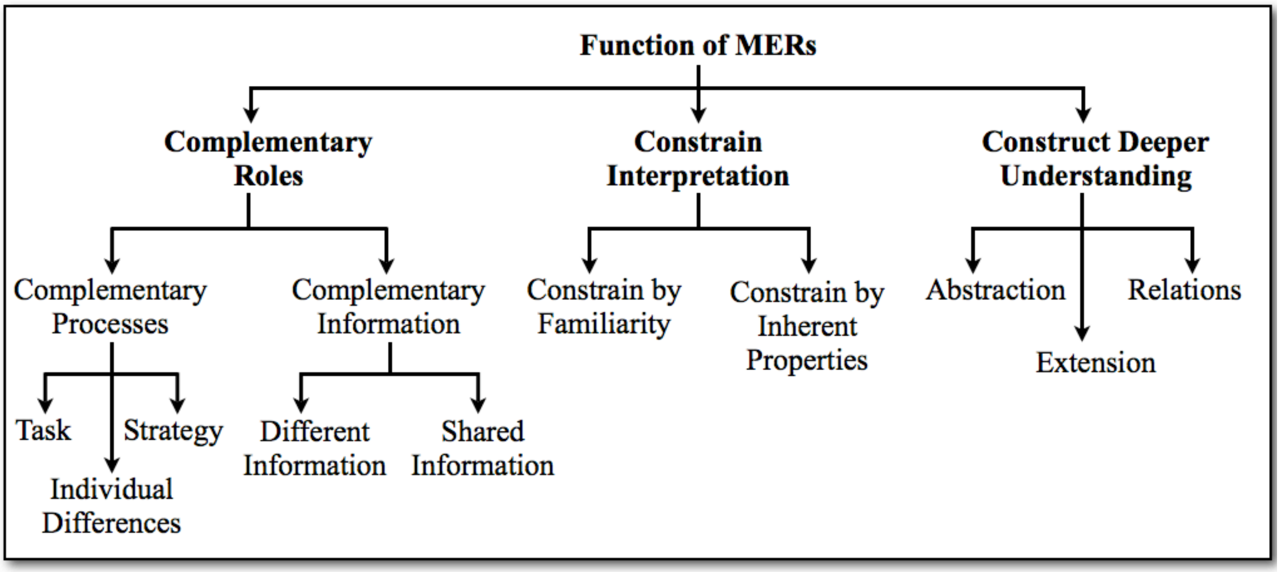

Figure 3: Functions of MERs according to Ainsworth (1999)

Using the example of doubling and halving the children shall – in terms of internalization – link their activities with the corresponding nonverbal-symbolic representation and they shall figure out those symbols as a log of their doing. The mathematical topic of doubling and halving was chosen because it is a basic strategy for solving addition and subtraction tasks. In addition, DOPPELMOPPEL offers to do segmentations in common use.

The main concern of the prototype is to offer a manifold choice of forms of representations and their linking in particular (MELRs). Two principles that lead the development are the *constant background principle* and the *constant position principle*. The first one claims a non-alteration of the design of the background but an always-constant one. Furthermore the position of the different forms of representations should always be fixed and visible from the very beginning so that they don't constrict each other.

DOPPELMOPPEL provides the children with the opportunity to work in many different forms of representations. On the one hand there is a zone in which the children can work **virtual-enactive**. Quantities are represented through circular pads in two colours (red and blue). To enable a fast representation (*easy construction principle*) and to avoid "calculating by counting" there are also stacks of five next to the single pads. According to our reading direction the five pads are laid out horizontally. The elimination of pads happens through an intuitive throw-away gesture from the "desk" or, if all should be cleaned, with the aid of the broom button. A total of maximal 100 pads fit on the table (10x10). The possible activities of doubling, halving and segmenting are done via the two tools on the right and the left hand side of the desk (fig. 4).

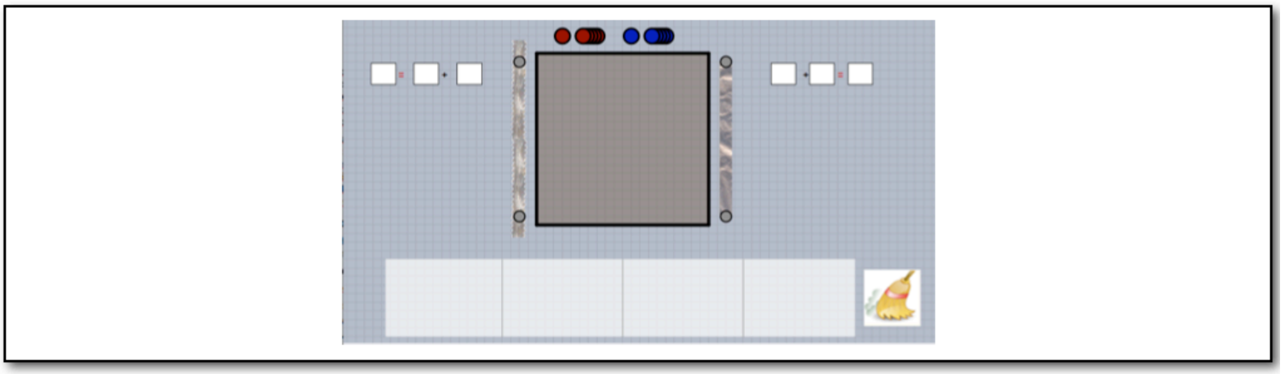

Figure 4: Screenshot of the prototype DOPPELMOPPEL

The doubling-tool (to the right) acts like a mirror and doubles the laid quantities. The saw (to the left) divides the pads and moves them apart. Both visualisations are only shown for a short time after clicking on the tools. Afterwards, the children only see the initial situation and have to imagine the final situation (mirrored resp. divided) themselves. The pupils can use the mouse to drag the circular points on the doubling-tool and the saw to move them into any position. A special feature of the saw is that it also can halve pads. At this point the program is responsive to the fact that already six-years-olds know the concept of halves because of the common use in everyday life.

The children can do **nonverbal-symbolic** inputs themselves in the two tables on the right and the left hand side. The left table enables inputs in the form _=_+_, the right one in the form _+_=_. The table on the right is only intended for doubling and halving tasks. That's why the respectively other summand appears automatically after the input of one. In the table on the left any addition task can be entered.

If the pupils don't fill in the equation completely they have the possibility to get their input shown in a **schematic-iconic** representation. Depending on the entered figures, the pads appears in that way that the children can't read the solution directly by means of their colour. The doubling-tool respectively the saw are placed according to the equation so that the children – like in the virtual-enactive representation – are able to act with the tools (fig. 5).

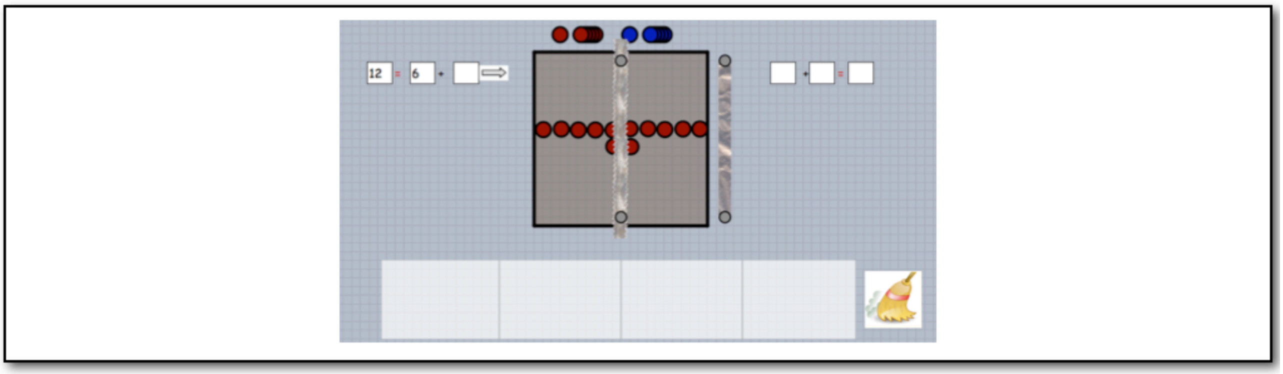

Figure 5: Schematic-iconic representation of a task

According to the *signaling principle* an arrow is highlighted when the pupils enter numbers in the free boxes. A click on this arrow initiates the **intermodal transfer**. A similar arrow appears below the desk after every activity done by the children (click on the doubling-tool respectively the saw). Here, the pupils have the possibility to let the software perform the intermodal transfer from the virtual-enactive and the schematic-iconic representation to the nonverbal-symbolic one. This is another special feature of DOPPELMOPPEL that is rarely found in current educational software. If external representations are linked, the linking is mostly restricted to the contrary direction. Depending on the activity the equation appears again in the form _=_+_ or _+_=_. Those equations aren't separated consciously, however a coloured differentiation of the equal and the addition sign (as in the tables above) point to pay attention.

Besides the forms of representations there are two more functions available. Both –the broom to clean the desk and the exclamation mark for checking answers – take some time in order to encourage considerate working and to avoid a trial-and-error-effect. If the equation is false the program differentiates on the type of error. In case of an off-by-one answer or other minor mistake the boxes are coloured orange otherwise red. If the equation is correct a new box appears below.

This prototype doesn't already respond to *modalities* but the concept already incorporates auditory elements.

Testing of DOPPELMOPPEL

For the testing of DOPPELMOPPEL four versions of the prototype were created. Two of those feature multiple representations; the other two only offer single representations. One of the multiple representations provides an additional linking, that is an intermodal transfer in both directions (fig. 6).

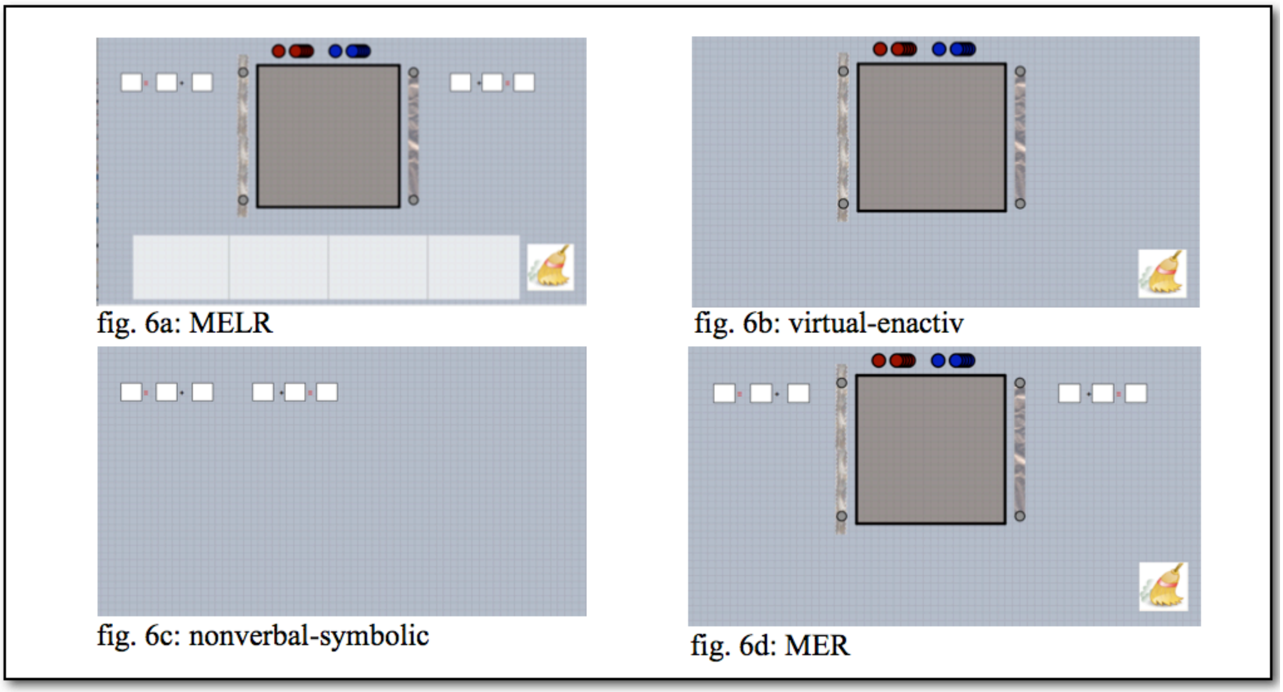

Figure 6: 4 versions of the prototype

The dedication of those four versions is to make sure that it is neither the medium computer nor the method of instruction that causes results of the testing.

28 pupils of a 1st class worked about 20 minutes per five terms with the program. During their work there was one student assistant who observed and

took care of two children. In addition, the activities of the children were recorded with a screencorder-software. Furthermore a pre- and a posttest were done.

To the current point of time the data interpretation is still in progress but first results should be available to the end of January.

CONCLUSION

Educational software that is based on the primacy of educational theory, as claimed by Krauthausen and others, has to take both mathematics and multimedia theory into account. Carefully crafted software however, is very expensive in production. We hope to be able to show with our prototype that this investment is justified.

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THE IMPACT OF TECHNOLOGICAL TOOLS IN THE TEACHING AND LEARNING OF INTEGRAL CALCULUS

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There is still a tendency to see that mathematics is not visual. At University education, it´s evident in several ways. One of them, is an algebraic and reductionist approach to the teaching of calculus.

In order to improve educational practices, we designed an empirical research for the teaching and learning of integral calculus whith technological tools as facilitator resources of the process of teaching and learning: the use of predesigned software that enables to get the conceptualization in a visual and numeric way, and the using of a virtual platform for complementary activities and new forms of collaboration between students, and between teachers and students.

Keywords: Predesigned software – virtual enviroments – registers of representation - social infrastructure - epistemological infrastructure

INTRODUCTION

The ideas, concepts and methods of mathematics presents a visual content wealth, which can be geometrically and intuitively represented, and their use is very important, both in the tasks of filing and handling of such concepts and methods, and for the resolution of problems.

Experts have visual images, intuitive way of knowing the concepts and methods of great value and effectiveness in their creative work. Through them, experts are able to relate, most versatile and varied, often very complex, constellation of facts and results of their theory and, through such significant networks, they are able to choose from, so natural and effortless, most effective ways of solving the problems they face (Guzman, 1996). Viewing, in the context of teaching and learning of mathematics at the university, has to do with the ability to create wealthy images that individuals can handle mentally, can pass through different representations of the concept and, if necessary, can provide the mathematic ideas on a paper or computer screen (Duval, 2004). The creative work of mathematicians of all times has had "the visualization" as its main source of inspiration, and this has played an important role in the development of ideas and concepts of the infinitesimal calculus.

However, there is a tendency to believe that mathematics is not visual. At university education, it´s evident, particularly through an algebraic and reductionist approach of the teaching of calculus. One of the didactic phenomena which is considered essential in the teaching of Mathematical Analysis, is the "*algebrización*", that is: the algebraic treatment of differential and integral calculation. Artigue (in Contreras, 2000) expresses this fact in terms of an algebraic and reductionist approach of the calculation which is based on the algebraic operations with limits, differential and integral calculus, but it treats the thinking and the specific techniques of analysis in a simplistic way, such as the idea of instantaneous rate of change, or the study of the results of these reasons of change.

We believe that the problems with Mathematical Analysis learning, in the first year of college, have to do with this context. These difficulties are associated with the formalism in dealing with the concepts and the lack of association with a geometric approach. Anthony Orton has worked for a long time about the difficulties in learning calculus. His research work at the University of Leeds confirmed that students had difficulty in learning the concepts of calculus: the idea of exchange rate, the notion of a derivative as a limit, the idea of area as the limit of a sum (Orton, 1979). Cornu (1981) arrived at similar conclusions regarding the idea of "unattainable limit" and Schawarzenberger and Tall (1978) regarding the idea of "very near". Ervynck (1981) not only documented the difficulties of the students in understanding the concept of limit but he also remarked the importance of viewing the processes by successive approximations. In this sense, wue can see that usual graphs met in textbooks of calculus have two problems: they are static, which can not convey the dynamic nature of many of the concepts, and also they have a limited number of examples, usually one or two, which leads to develop, in students, a narrow image of the concept in question. (Tall and Sheath, 1983). In this sense, taking into account our previous exploratory research (Milevicich, 2008), we can say that students can not understand the concept of definite integral of a function as the area under the curve, because they do not visualize how to build this area as a sum, usually known as Riemann Sum.

In terms of the educational processes, it should be noted that teachers usually introduce the concept of integral in a narrative way, avoiding the real purpose, which is to obtain more precise approximations. A simplistic approach to the concept is usually done, disconnected from integral calculus applications, which hinders the understanding of students, and consequently, the resolution of problems relating to calculation of areas, length of curves , volume of solids of revolution, and those dealing with applications to the engineering work, pressure, hydrostatic force and center of mass.

JUSTIFICATION

Innovation in educational processes including th use of multimedia means demands not only on teachers´ professionalism but also new activity managing. Research work is currently being carried out at different universities aiming to find out what use teachers make of these tools and the specific competencies that they have to acquire for making effective use of them. From a didactic point of view, the usage of multimedia in teaching-learning process, presumably, should increase students motivation, and, in that sense, we ask ourselves: What should be the goals of education aimed at improving the university today? and How can we make it easier through the use of technological tools?. The answers to these questions are not clear for us. Students, nowadays, have more and more information than they can process, so that one of the functions of the university education would be to provide them with cognitive and conceptual tools, to help them to select the most relevant information. University Students should try to get skill and develop attitudes that enable them to select, process, analyse and draw conclusions. This change in the goals represents a departure from traditional learning. In this sense, the use of a predesigned software in the classroom, designed within the group research, can be a teaching facilitator resource of the process of teaching and learning:

- to convey the dynamic nature of a concept from the visualization,
- to coordinate different registers of representation of a concept,
- for the creation of personalized media best suited to the pedagogical requirements of the proposal.

RESEARCH CHARACTERISTICS

Population and sample

The population is made up of Engineering students from Technological University and the specimen is a Electrical Engineering commission of about 30 students. Regarding the characteristics of the population, some considerations can be made about their previous knowledge of integral calculus. Some students come from the Mechanic School of a known automotive Company and others, from a technical electricians school. Based on a detailed analysis of library materials used by teachers in these institutions, and the students' writings, we infer that integrals are taught as the reverse process of derivation, with the focus on the algebraic aspects. These students study the concept of integral associated with a primitive, practice various methods of integration, transcribe or solve hundreds of exercises in order to calculate integrals, and some of them even achieve a considerable level of skill in the use of tricks and recipes that help to be more effective in getting results. Another group of students come from near schools where geometric concepts are little, essentially the calculating of areas studied during primary and middle school. However, the largest group, is made up of students studying Mathematical Analysis for the second or third time. Some of them have completed the course in previous years but failed in the exams. It may be that those students have some ideas about integral calculus and its applications, or not. It is possible that those ideas interfere with the getting of new knowledge or hinder it (Bachelard, 1938), primarily on those students who associate the integral exclusively to algebraic processes. That is why it was very important to carry out a diagnostic test (pretest) that would allow exploration on the previous skills and students ideas about definite integral and thus, categorize according to the following levels of the independent variable:

Level 1: associate the concept of integral to the primitive of a function and calculates easy integrals.

Level 2: associate the concept of integral to the primitive of a function, calculates easy integrals and links the concept with the area under the curve.

Level 3: associate the concept of integral to the primitive of a function and links the concept with the area under the curve.

Level 4: has no specific pre knowledge associated with the topic.

Focus

The general purposes of our research work were:

to determine if students understand the concept of integral through the implementation of a proposal that would allow its teaching in a approaching process, using different systems of representation, according to the processes man has followed in his establishment of mathematical ideas,

to analyze, in a reflective learning context, the ways in which students solve problems related to integral calculus,

and the specific purposes were:

to categorize the students, involved in the experience, according to his integral calculation preconceptions, at the beginning of the intervention,

to implement a proposal that provided, on the one hand, the use of different systems of representation in the development of individual and group activities, and on the other, to promote conjeturación, experiment, formalization, demonstration, synthesis, categorization, retrospective analysis , extrapolation and argumentation, with the help of specific software, and feedback on students' early productions so they could reflect on their own mistakes,

to review progress achieved after the implementation of the didactic proposal, to *analyze* the impact of using a virtual platform for complementary activities.

Methodology

The design is pre-experimental type of pretest - treatment - postest with a single group. The independent variables in this study are: the design of teaching and pre knowledge of students on the definite integral. The dependent variable is: the academic performance.

Regarding these previous knowledge, a pretest at the beginning of the intervention allowed to place each student in one of the preset categories. After 8 weeks of intervention, a postest allowed to determine the levels of progress made in learning the concepts of integral calculus in relation to the results obtained in the past three years cohorts (2003, 2004 and 2005). In addition, an interview at the end of the experience was implemented, in order to gather qualitative information.

In order to improve educational practices, we designed a proposal for teaching and learning integral calculus according to the proposal of using a pre designed software as indicated in the goals. In this sense:

We designed a software package allowing the boarding of integral calculus from the concept of definite integral associated with the area under the curve, from a geometric point of view.

We selected the problems students should solve, in a way, that their approach would allow to establish a bridge between conceptualization of integration and problems related to engineering. In that sense, the use of the computer allowed to have a very wide range of problems, where the choice was not conditioned by the difficulty of algebraic calculus.

The students used pre designed software for:

a) The successive approximations to the area under a curve, considering left and right points on each of the subintervals. The software allows to select the function, the interval and the number of subdivisions. (See Figure 1).

b) The successive approximations to the area under a curve through the graph of the series which represents the sum of the approach rectangles (See Graphic 1) and the table of values (See Table 1).

c) The visualization of the area between two curves, it also allows to determine the points of intersection.

d) The representation of the solid of revolution on different axes when rotating a predetermined area. (See Figure 2)

e) The numerical and graphical representation (through table of values) of the area under the curve of an improper integral.

It was designed a set of activities with the purpose students conjecture, experience, analyze retrospectively, extrapolate, argue, ask their peers and their teachers, discuss their own mistakes and evaluate their performance. Assessment techniques were redesigned, so that the analysis of students productions would provide feedback about their mistakes.

We incorporated a Virtual Campus using Moodle supporting design, as an additional element, in order to keep continuity between two spaced weekly meetings. According to Misfeldt and Sanne (2007), communication on mathematical issues is difficult using computers and a weekly meeting is insufficient. In response to this problem, we used the virtual campus for communication, flexibility and cooperation, but the use of it was not a learning objective in itself. Instead, we used it to publish texts and exercises guides and also, students made active use of the forum for discussion groups.

We also had in mind that the challenges in creating an online learning environment might be different when working with mathematics than in other topics (see also: Misfeldt et. al, 2007 & Duval, 2006). Many of the signs that goes into building mathematical discourse is not available on a standard keyboard, and the way that mathematical communication often is supported by many registers and modalities that are used simultaneously, as writing and drawing various representations on the blackboard or paper is also not avalilable. Students, using the Virtual Campus, had the possibility to upload archives showing the solving process and using every symbol they needed.

Implementation of the proposal

Students were distributed in small groups no more than three, who worked in several sub-projects. Each of them included a significant number of problems.

Subproject No. 1: The concept of integral.

Subproject No. 2: Fundamental theorem of Calculus.

Subproject No. 3: Improper integrals.

Subproject No. 4: Area between curves.

Subproject No. 5: Applications of Integral Calculus.

Guidelines for systematic work for each of the meetings were made. In the first part, it was discussed the progress and difficulties of the previous practice, where the essential purpose was to ensure that students analyze their own mistakes, and the second part, teachers and students worked on new concepts at the computer laboratory. The first part of each meeting was guided by the teacher, but a assistant teaching and a observer teacher were present in the class. The second half had the same staff and an extra assistant teaching.

The assesment took place during the whole experience through:

- weekly productions of students reflected in their electronic folders and notebooks. These ones allow cells to keep comments, observations, etc.; very valuable material in assessing the level of understanding achieved by students.
- students interaction in classes and into working groups.
- Students participation in the discussion forums of the virtual campus.

In that sense, spreadsheets were used for monitoring activities, which proved to be an effective tool to assess different aspects relevant to student´s performance. Summary notes taken by the observer teacher along the 8 weeks allowed us to infer the change of attitude in an important group of these students. From the initial population, made up of 30 students, 24 of them showed increased commitment to the development of activities.

Some of these activities were:

Subproject 1: Evaluate the following integrals by interpreting each in terms of areas

a) $\int_1^3 e^x dx$ *b)* $\int_0^3 (x-1)dx$

Case a: because $f(x)=e^x$ is positive the integral represents the area. It ca be calculated as a limit of sums and a computed algebra system can be used to evaluate the expression.

Case b: The integral cannot be interpreted as an area because f takes in both positive and negative values. But students should realize that the difference of areas works.

Subproject 3:Sketch the region and find its area (if it is possible)

a) $S=\{(x,y)/\ 0 \le x \le \pi,\ 0 \le y \le Tan(x)Sec(x)\}$
b) $S=\{(x,y)/\ x \ge 0,\ 0 \le y \le e^{-x^2}\}$

Case a: Probably students confuse the integral with an ordinary one. They should warn that there is an asymptote at x= π/2 and it must be calculated in terms of limits. At this point students must bear in mind that whenever they meet the symbol $\int_a^b f(x)dx$ they must decide, by looking at the function f on [a,b], whether it is an ordinary definite integral or an improper integral.

Case b: The integral is convergent but it cannot be evaluated directly because the antiderivative is not an elementary function. It is important students look for a way to solve the problem and although it is impossible to find the exact value, they can know whether it is convergent or divergent using the Comparition Test for Improper Integrals.

Both examples above show activities where students need to find out solutions and get conclutions without teacher telling them.

RESULTS

The pretest was done by 30 students, the results allowed us to locate them as follows: 15 at Level 1, 1 at Level 2 and 14 at level 4. It should be noted that those who came from technical schools had achieved a considerable level of skill in the calculation of integrals but they didn´t know about the links with the concept of the area.

The postest consisted of 6 problems related to the sub projects students had worked on, each of which was formed by several items. It was provided to the 24 students remaining at the end of the experience, and took place at the computer laboratory, where students usually worked. In general, the level of effectiveness was above 50%, except in the case where they were asked to determine the area between two curves and then the volume to rotate around different axes. The difficulty was to get the solid of revolution from a shift in the rotation axis. Although the students had no difficulty in getting the solid geometrically, they could not get an algebraic expression for it.

In a comparison with the three previous year cohorts, it was possible to emphasize the following differences:

a) There were no important difficulties in linking the concepts of derivative and integral.
b) An important group of students (83% of them) successfully used Fundamental Theorem of Calculus.
c) In general, there were no difficulties in algebraic developments, however it is possible to associate the lack of such obstacles to the use of the computer.
   All of students tested, could associate the concept of solid revolution with the concept of integral, and even more, they were able to correctly identify the area to rotate.

d) The 74% of the students tested could identify improper integrals, but only 43% of them, correctly, applied the properties.
e) Most of the students tested succeeded in establishing a bridge between the conceptualization of integration and problems related to engineering: 89% of them correctly solved problems relating to applications for work, hydrostatic pressure and force.

The written interviews at the close of the experience reflects the importance that students attribute to the use of virtual campus as an additional resource: most of students were very keen on having prompt responses from the teacher when asking questions in the forum and the help offered by other students.

One of the questions was:

"How did teachers interventions at the forum helped, when you had difficulties in the development of practices? (A: they were decisive, B: they helped me to understand, C: they were not decisive. I managed without them, D: they did not contribute at all. Please explain your choice)."

12 students selected A , 8 puplis selected B, 4 students selected C and D was not selected.

Some of the explanations given by students were:

Student a: *"...They helped me because teachers answered quickly and clearly"*

Student b*: "...Excellent, clear and concise answers that helped with the resolution of the problems."*

Student c:*"...There were many situations where I managed to solve a problem just reading the doubts of my fellow students. I have not done a lot of questions at the forum because someone asked my doubt before me..."*

It is worth mentioning that there were no substantial differences between the students belonging to different categories, according to the pretest. An analysis of results in relation to the initial categorization, suggests that pre conditioned ideas did not influenced the acquisition of new knowledge. There were no significant differences among the largest groups of students ranked in levels 1 and 4.

CONCLUSIONS

The failure of the students in understanding the concepts of calculation, more generally, and the definite integral, in particular, is one of the most worrying problems in the learning of Mathematical Analysis, in the first year of Engineering, as this hinders the understanding and resolution of problems of application. The way to search for the causality of this failure led us to raise the need for a change in the point of view. This is a change in the processes and representations through which students learn, in this case, the concept of integral.

Focusing our attention on the problem how students can understand more deeply the concepts using tools and technology, we can conclude that the recent evolution of digital materials leads to devote a specific interest to the change of activities induced by virtual learning environments which allow new forms of collaboration between students, and between teachers and students. Besides, the use of the computer is a valuable strategy with the aim of achieving significant learning. While learning the concept of definite integral, the computer facilitates making the important amount of calculations and displays the successive approximations, contributing to the concept of area under the curve. In that sense, the use of a predesigned package software allowed students to view the alignment between the smaller and smaller geometric rectangles and curvilinear area to be determined.

The carrying out of the activities required the use of the predesigned package software, specifically adapted to the needs of the experience. Students had to make numerous graphs, edit their guesses, propose new solutions, test, and analyze retrospectively the achieved results. Dynamic graph was valued for making student work with figures easier, faster and more accurate, and consequently for removing drawing demands which distract them from the key point of a problem. Various aspects of making properties apprehensible to students through dynamic manipulation were expressed in CERME V Plenaries (see: Ruthven, 2007). *"When a dynamic figure is dragged, students can see it changing and see what happens, so that properties become obvious and students see them immediately*" (Ruthven,2007: 56). In that sense, technology is seen as supporting teaching approaches based on guiding students to discover properties for themselves. We agree on suggesting that teachers might guide students towards an intended mathematical conclusion, but students could find out how it works without us telling them so that they could feel they are discovering for themselves and could get a better undestanding.

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APPENDICES

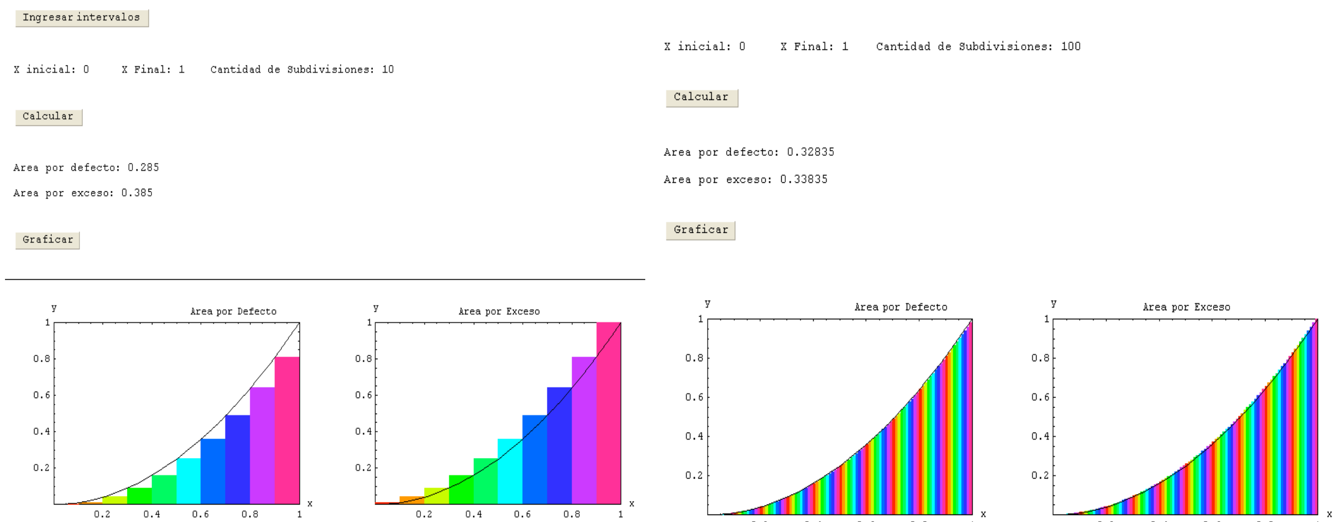
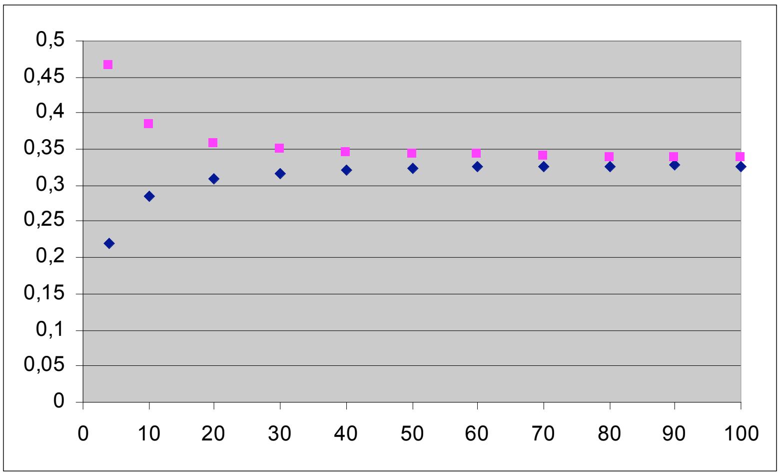


Figure 1. Capture screen from the predesigned software about conceptualization of definite integral. Estimation of the area of $y=x^2$ using 10 subdivisions and 100 subdivisions, $0 \leq x \leq 1$

number of subdivisions	default sums	excess sums
4	0,219	0,467
10	0,285	0,385
20	0,308	0,358
30	0,316	0,35
40	0,321	0,346
50	0,323	0,343
60	0,325	0,342
70	0,326	0,34
80	0,327	0,339
90	0,328	0,339
100	0,327	0,337

Table 1. Sums for different subintervals increasingly small under the curve $y= x^2$ on the interval [0,1]



Graphic 1. the series which represents the sum of the approach rectangles, default sums are in blue and excess sums are in pink.

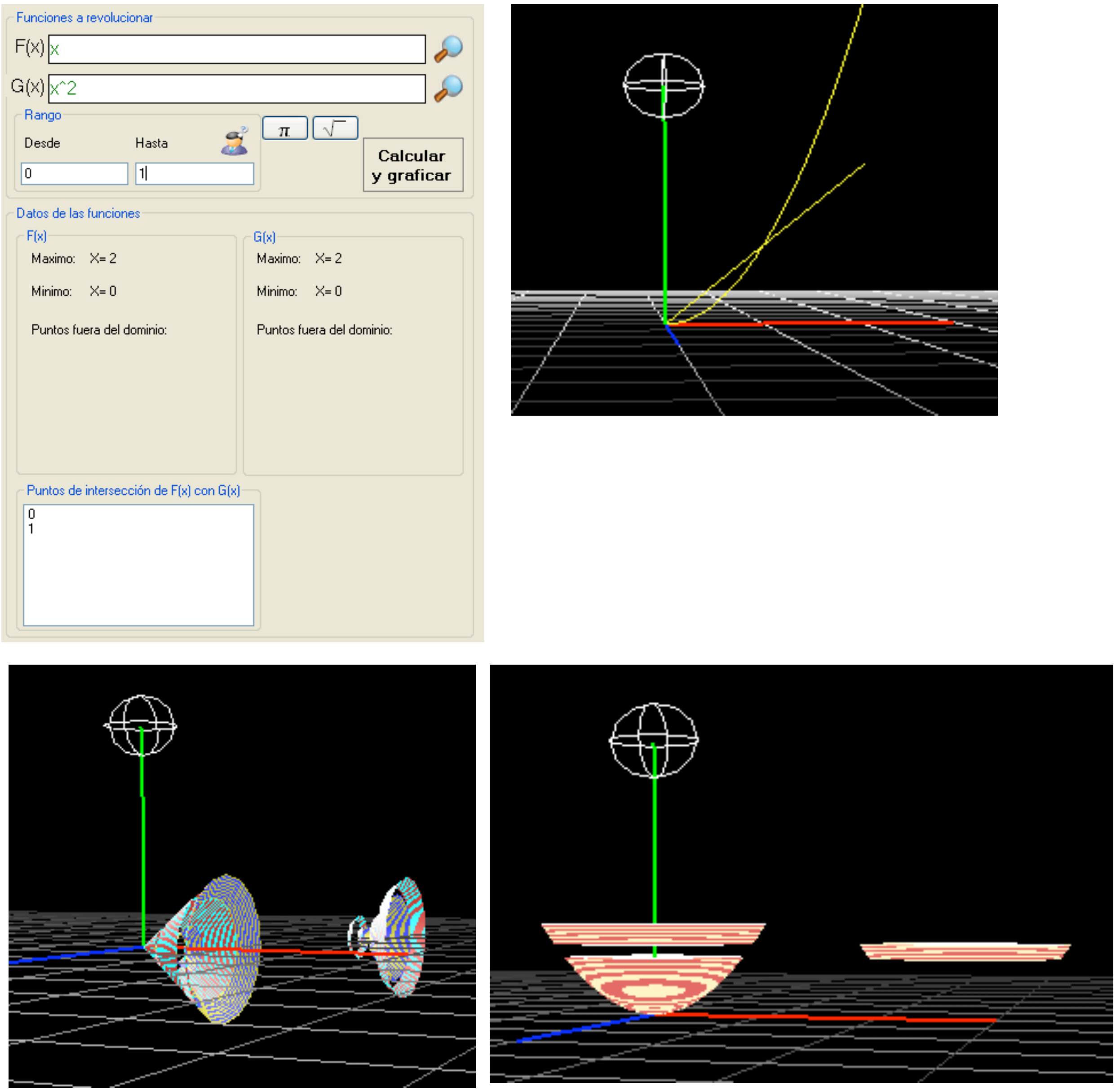


Figure 3. Captured screen from the predesigned software about Solid of revolution.

Area between the functions y=x and y=x^2 , and the solid of revolution that is generated to rotate on the x-axis and the vertical axis.

LINKING GEOMETRY AND ALGEBRA: ENGLISH AND TAIWANESE UPPER SECONDARY TEACHERS' APPROACHES TO THE USE OF GEOGEBRA

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The idea of the integration of dynamic geometry and computer algebra and the implementation of open-source software in mathematics teaching underpins new approaches to studying teachers' thinking and technological artefacts in use. This study opens by reviewing the evolving design of dynamic geometry and computer algebra; teachers' conceptions and pioneering uses of GeoGebra; and early sketches of GeoGebra mainstream use in teaching practices. This research has investigated English and Taiwanese upper-secondary teachers' attitudes and practices regarding GeoGebra. More specifically, it has sought to gain an understanding of the teachers' conceptions of technology and how their pedagogies incorporate dynamic manipulation with GeoGebra into mathematical discourse.

INTRODUCTION

Algebra and geometry are two core strands of mathematics curricula throughout the world and are considered the 'two formal pillars' of mathematics (Atiyah, 2001). It is therefore not surprising that they have been specifically targeted by the field of technology (Sangwin, 2007). Many researchers consider mathematics education as one of the earlier education fields to introduce technology as an assistant tool in classrooms (Papert, 1980; Noss and Hoyles, 1996).

The major application of technology in mathematics education is the integration of mathematical software in teaching practices. In respect of geometry, the most widely used computer applications, known as Dynamic Geometry Software (DGS) and include, Cabri-géomètre and Geometer's Sketchpad (GSP), etc. One important feature of DGS is the drag mode, encouraging interactions between teachers, students and mathematics (Jones, 2000). The drag mode can be used to explore and visualise geometrical properties by dragging objects and transforming figures in ways beyond the scope of traditional paper-and-pencil geometry (Laborde, 2001; Ruthven, 2005). DGS also has options to visualise the paths of objects as they move. For algebra, the most widely used applications are known as Computer Algebra Systems (CAS) and include programmes such as Mathematica, Maple and Derive. Some graphical visualisation and symbolic representations of algebraic expressions are implemented in CAS. Using the metaphor of the two 'formal pillars' of mathematics, geometry and algebra are afforded prominent positions especially at the secondary level (Hohenwarter & Jones, 2007). However, the connection between geometry and algebra, namely 'the beam connecting the two pillars', is apparently missing, as evident that in some countries geometry and algebra are entirely separate in their curricula (ibid). Ruthven (2008) researches the specific examples of computer algebra and dynamic geometry, and highlights 'three important dimensions- interpretative flexibility, instrumental evolution and institutional adoption-of the incorporation of new technologies into educational practices'. Although research into current technology use of computer algebra and dynamic geometry in teaching practices separate each sphere into distinct areas for study; I argue against this separation as there are areas overlapping algebra and geometry such as functions and graphs (Dubinsky and Harel, 1992). Examining both together has great educational implications and the connections between the two should not be ignored (Edwards & Jones, 2006). However, there is a gap in the literature dealing with this linkage between both fields and the use of technology. Despite an awareness of the need for a combination of DGS and CAS (Hohenwarter & Fush, 2004), software designers struggle to combine them as there are completely different constructs in software design. GeoGebra could be seen as pioneering software, although whether or not it is successful in linking DGS and CAS still needs research as the supporting evidence is limited at present.

Linking Geometry and Algebra

Since CAS and DGS are two completely different mathematical constructs, the 'beam' of the two pillars is weakly constructed within current mathematical software. Historically, CAS programmes have mainly provided algebraic and numerical computations while DGS have provided graphical and dynamic demonstrations. Hohenwarter and Jones (2007) point out that 'forms of CAS have begun to include graphing capabilities in order to help to visualise mathematics; likewise, DGS have begun to include elements of algebraic symbolisation in order to be useful for a wider range of mathematical problems'(p. 127) . In recent years, the need to integrate CAS and DGS has become apparent as Schumann and Green (2000: 337) claim that '[t]here is a need for further software development to provide a single package combining the desired features [of DGS and CAS]'. The recently published software GeoGebra by Markus Hohenwater (2004) explicitly links the two (as evidenced by the name Geometry and alGebra). This integration aims to provide unprecedented opportunities for mathematics education (Sangwin, 2007). GeoGebra affords a bidirectional combination of geometry and algebra that differs from earlier software forms. The bidirectional combination means that, for instance, by typing in an equation in the algebra window, the graph of the equation will be shown in the dynamic and graphic window. Similarly, by dragging the graph, the equation changes accordingly (Hohenwarter and Fuchs, 2004). A closer connection between the visualisation capabilities of CAS and the dynamic changeability of DGS is therefore offered by GeoGebra (ibid).

The Case of GeoGebra

One problem is that most mathematical software in mainstream use is commercial, which means the availability of software is subject to the school or student's finances. Therefore, some teachers or students who cannot afford to buy commercial software would search for free software for their own purposes. There is positive potentiality and improvement offered by encouraging a collaborative community of open-source software users and voluntary software developers (Suzuki, 2006). GeoGebra is one of these open-source softwares.

My rationale behind carrying out this inquiry into GeoGebra is not only due to its being open-source software with freely available support and online materials, but also due to its unique capacity to integrate geometry and algebra. The significance of this research is not only the investigation of how GeoGebra usage can be incorporated into the teaching of either geometry or algebra alone, but more importantly, how the teaching of geometry and algebra can be linked using GeoGebra, thus contributing to a better understanding for students of their interrelationships. Studies such as this one will contribute to knowledge of GeoGebra-mediated teaching and the future pedagogical development.

Comparative Study

Recent research has indicated that culture influences the ways that teachers behave and inter-culture differences appears to be stronger than intra-culture differences (Schmidt et al., 1996; Givvin et al., 2005; Andrews, 2007). In particular, comparing eastern and western traditions with their respective Confucian and Socratic underpinnings can be enlightening as there are great differences in teacher beliefs and practices (Leung, 1995; Tweed and Lehman, 2002; Andrews, 2007). There is little comparative research of technology use in mathematics education, especially between Eastern Asian and Western countries (Graf. and Leung, 2001). Consequently, seeing how culture influences technology-mediated mathematics teaching is a pertinent issue.

There are large-scale quantitative studies such as TIMSS and PISA and small-scale qualitative studies. These studies highlight both similarities and differences between mathematics education in different cultural contexts in depth and in breadth. Large scale surveys are limited, however, by the fact that they often compare students' academic achievements without taking cultural and social factors into consideration (Prais, 2007). Quantitative studies such as TIMSS have also been reproached for their uncritical evaluation and for promoting globalisation over curricular and cultural diversity (Andrews, 2007). In contrast, small qualitative studies acknowledge cultural differences without attempts for generalisation. Particularly, when comparing East Asian and Western traditions with their respective Confucian and Socratic underpinnings, there is a significant difference between what are classically designed with the educational traditions (Leung, 1995; Kaiser et al., 2005; Tweed and Lehman, 2002). In particular, Kaiser et al. (2005) proposed a framework analysing East Asian and West European cultural traditions in mathematics education. The framework by Kaiser et al. (2005) is adapted partially in terms of teaching styles as I undertake a small-scale qualitative study in countries that exemplify East and West with a focus on teachers' perspective and their use of technology in mathematics teaching. The Eastern country chosen is Taiwan since it is viewed as 'the one most often cited admiringly by educators in the West for the level of its students' educational achievements (Broadfoot, et al., 2000: 13)' and a high mathematics performing country in international comparative studies such as TIMSS and PISA (Mullis, 2003; OECD, 2004; 2007). The Western country chosen for the study is England due to its contrasting educational system (Broadfoot et al., 2000). A cross-cultural study between Taiwan and England helps obtain a sense of the commonalities and discrepancies of teachers' conceptions and practices in relation to GeoGebra use. I have chosen to research at the upper-secondary level (students aged 15-18) as this level is less researched but is a crucial step for bridging students' secondary mathematics learning and higher education. Therefore, the overarching research questions are: (1)What are the upper-secondary mathematics teachers' conceptions of technology in relation to GeoGebra in England and Taiwan? (2) In what manner is GeoGebra used for the teaching of geometry and algebra by Taiwanese and English teachers? (3)How are the teachers' conceptions of technology and GeoGebra related to their teaching practices in both countries?

METHDOLOGY

Since there is little research into GeoGebra usage to date, this study is exploratory (Marshall and Rossman, 2006; Creswell, 2007). In brief, exploratory and multiple-case studies are my chosen methodology as the research focuses on this particular mathematical software, requiring specific teachers who utilise GeoGebra to teach upper-secondary level mathematics. Comparing and contrasting cases of teachers with interest in using GeoGebra from Taiwan and England provide a comprehensive understanding of how GeoGebra can be used in two very different cultural traditions, pedagogies and curricula.

I define mathematics teaching with the use of GeoGebra in Taiwan and England as the two main units of analysis. These have embedded two cases of teachers who use this software. Moreover, within the units, four cases of English and Taiwanese teachers are studied to obtain evidence of their views on GeoGebra teaching practices. To achieve the comparability between cases and units, pre-determined themes: *teacher background, views on technology and GeoGebra, software comparisons* and *ways of using GeoGebra* have been set for research design and data collection. A complete set of data was collected from four school visits. All of the interviews were audio and video-recorded, lasted for approximately an hour each and took place in classrooms using either a laptop or a computer connected to an interactive whiteboard. During the interviews the teachers demonstrated ways they utilised the software. The interview data were collated and summarised for each of the four cases.

THE CASES

Jay

Jay has been teaching mathematics for twelve years in two senior high schools in Taiwan (students aged 15 -18). Jay's views about the incorporation of technology into teaching practices are generally more negative than positive. He inferred that both students and teachers viewed computers as a tool for entertainment rather than a learning or teaching tool. On the contrary, he held positive attitudes only with regard to GeoGebra. He claimed GeoGebra to be a convenient tool, which can be used for demonstrations, checking and visualisation as well as research. He mentioned that GeoGebra provides powerful capabilities that other software packages cannot offer: 'It is actually very good, especially when you want to do addition and subtraction in the grid coordinate system.' In general, Jay was discouraged by the current educational environment regarding technology and both students' and teachers' attitudes toward mathematical software in Taiwan. He also asserted that support from mathematical software was limited as human brains do the logical deduction. However, he emphasised that GeoGebra provides quality functionalities that encouraged his use of this software in his teaching practice. The salient categories emerged from the data are listed as follows:

Tool use	Graphing, calculations, visualisation, demonstration, dragging, checking, test and verify, teaching and research
Mathematics topics	Cartesian coordinate systems, both algebra and geometry
Teaching style	Textbook-oriented
Infrastructure	Laptop demonstration in the classroom

Li

Li has thirteen years of teaching experience at the upper-secondary level in Taiwan. Since his first degree was in applied mathematics, he gained an interest in IT during his undergraduate study. He was enthusiastic about new technologies and volunteered to translate the Traditional Chinese version of GeoGebra. Moreover, he had been creative in using different software packages, free software in particular, and trying to use a combination of different open-source software to make teaching materials. He has written some journal articles comparing new, open-source software packages detailing how they might be incorporated into mathematics teaching for Taiwanese teachers. In addition, he conducted GeoGebra training courses and workshops for teachers in senior high schools in Taipei. He had also set up his website and school website and uploaded his up-to-date GeoGebra materials and step-by-step tutorial materials for students or teachers. Li had a similar opinion to Jay on students' and teachers' attitudes towards the use of computers. However, he was positive that exploiting GeoGebra can change students' attitude towards mathematics. Some of his designed teaching materials and tutoring examples of using GeoGebra in solving examination problems were displayed on the websites. He also encouraged students to use the websites for reference and discussion. The salient categories are listed as:

Tool use	Graphing, calculations, demonstration, problem-solving, revision, investigation, and interaction
Mathematics topics	Geometrical topics and algebraic calculations
Teaching style	Curriculum-based, textbook-oriented and exam-driven, self-developed teaching materials and website with GeoGebra
Infrastructure	Home, IT room or computer and projector in classroom

Richard

Richard has taught secondary and A-level mathematics for twelve years in England. He is skilled in computer programming and is in charge of the school mathematics website where a combination of GeoGebra, Yacas and JavaScript are used for developing online mathematics materials and tests. He designed a piece of DGS and used it to teach before starting to use GeoGebra. Previously, he was working as a software developer and cooperated with the NCETM GeoGebra project. Richard has an ambivalent view of GeoGebra. He expressed that he was not convinced that GeoGebra links geometry and algebra but then stated that: ‘it does the connection between algebra and geometry much better than other programmes - anywhere you can enter a number you can also enter a formula’. He asserted that GeoGebra had changed the way he taught as he had been taking students to IT rooms more often and some students liked the revision with GeoGebra as it sped up some processes of preparation for examinations and for accuracy. He stressed ‘the fact that you can animate any variable by turning it into a **slider** is a very powerful feature’. The salient categories emerged from the data are listed as follows:

Tool use	Graphing, calculations, demonstration, revision, student activities, investigation with the slider
Mathematics topics	Mainly geometrical topics, gradients of a curve and transformations
Teaching style	Activity-based, a combination system of paper-and-pencil and computer environments
Infrastructure	Home, IT room or computer and projector in classroom

Tyler

Tyler has taught mathematics to 11-16 year olds in a college for twelve years. He has also acted as an AST[1] supporting schools and as a part-time school consultant, cooperated with the NCETM GeoGebra project and hosted a GeoGebra training workshop at his college. Tyler’s utterances reflected a view of GeoGebra as an environment for exploring dynamic geometry rather than algebra. He viewed GeoGebra as a **replacement** to Cabri, which he used before GeoGebra. However, he mentioned that his experience with GeoGebra was approximately half a year, which meant that there were areas of using GeoGebra that were under-explored and underdeveloped, such as using GeoGebra in teaching algebra.

Some criticisms about current usage of technology in schools were brought up in terms of the IT rooms and school websites. He described his intention to change the way his pupils work from being passive to actively involve in

[1] Advanced Skills Teacher

learning through software. Moreover, he did not expect that students would not undertake much thinking in the IT room. In addition, some school mathematics websites have mathematics tests for pupils to log on to at home with their personal passwords which, in his view, allowed no room for discussion and interaction. He pointed out that GeoGebra is interactive and intuitive so he could set up diagrams and activities for students to interact with easily: **'This is different. This is maths by interacting; this is maths by trying things out, by conjecturing, by having a go.'** He emphasised that GeoGebra could not only be used as a presentation tool by teachers but also as an investigation tool for pupils. An enthusiasm for GeoGebra was apparent in Tyler's strategies of using GeoGebra in mathematics teaching.

Overall, Tyler was reflective and explorative about different practices with GeoGebra, and eager to find out possible areas where GeoGebra could be useful in mathematics teaching. He also drew a distinction between '*knowing how'* to use it and '*getting used to'* using it in relation with GeoGebra. This inferred that he acknowledged the differences between *using* GeoGebra and *teaching* with the use of GeoGebra. The salient categories emerged from the data are listed as follows:

Tool use	Demonstration, interaction, investigation, exploration, testing hypothesis, creation, projection capability and the slider
Mathematics topics	Mainly geometrical topics
Teaching style	A whole-class teaching activity
Infrastructure	Home, IT room or computer and projector in classroom

FINDINGS

Analysing the data thematically across the case studies revealed four salient dimensions in relation to GeoGebra-assisted teaching: **educational tools, teacher transition, mathematical scope and infrastructural change**. The findings are introduced in the following, which indicate that understanding the linkage between teachers' conceptions and practices is crucial. Firstly, the teachers' conceptions of GeoGebra seemed to be strongly rooted in their conceptions of the effectiveness and infrastructure of technology. The English teachers imbued a more positive attitude towards technology than their Taiwanese counterparts. However, teachers in both countries expressed favourable opinions regarding GeoGebra's agreeable contribution to their teaching. Secondly, GeoGebra was commonly used as a tool for visualisation, demonstration and interaction of mathematical topics, whereas for algebraic topics it was rarely utilised in England. It appeared that the English teachers associated GeoGebra primarily with geometric topics. Conversely, Taiwanese teachers worked with GeoGebra on both geometric and algebraic topics as they did not consider algebra and geometry to be necessarily separate; possibly as a result of the structure of Taiwanese curriculum and textbook-oriented culture. Thirdly, there were three different environments where teachers engaged with GeoGebra: - preparation of teaching materials at home, presentation and interaction in classrooms and activities for pupil investigation in IT rooms. Teacher transitions evolved from and were influenced by the infrastructure as they moved from preparation to presentation, incorporating interaction with pupils and finally encouraging investigation.

In effect, GeoGebra can be implemented in upper-secondary mathematics teaching as a network of preparation, presentation, interaction and investigation whereby teachers mediate their practices with flexibility. Based on the findings above, I present the general schema of this thesis (Fig.1). Arguably, there is a conceptual change in accordance with infrastructural change when technology is introduced in mathematics teaching. Teachers are the first to encounter this re-conceptualisation of pedagogical practices. They not only experience changes in their conceptions but also modification of their practices when they experience the transition. This transition would possibly alter teachers' choices of the mathematical scope and their uses of GeoGebra as an educational tool in light of their new pedagogical practices.

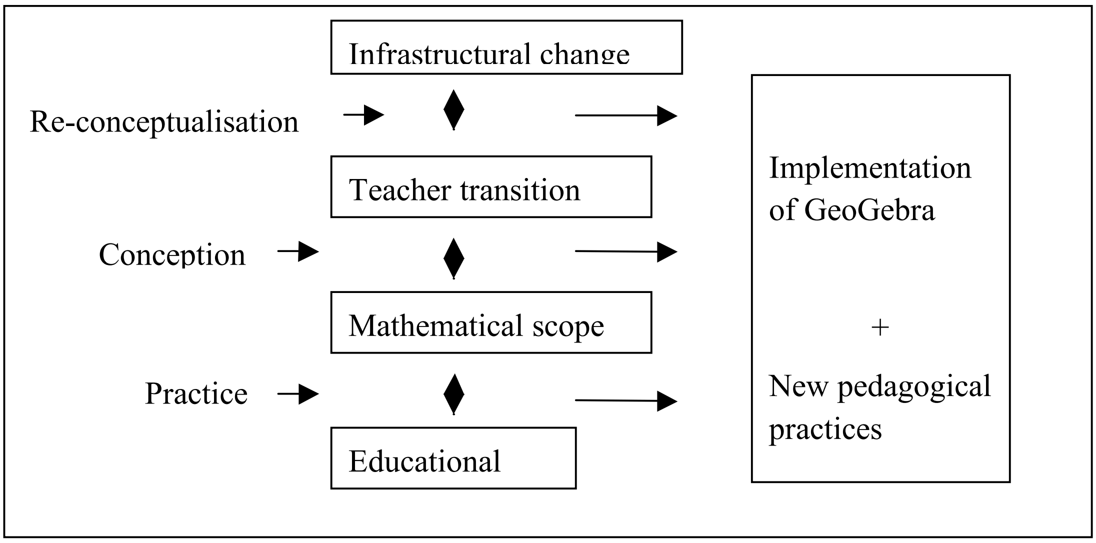

Figure 1: The general schema of teachers' conceptions and practices of GeoGebra

CONCLUSION

There are several areas with respect to the use of GeoGebra in Taiwan which are different from England. However, ascertaining the commonalities and differences of the use of GeoGebra between Taiwan and England is not particular easy as cultural influence is a complex issue. In addition, the presentation of four cases cannot offer a broad understanding or generalisation of what is happening in both countries. What this study offers is an exploration into teachers' use GeoGebra in England differently from their Taiwanese counterparts according to their personal characteristic, conceptions and practices.

There are three aspects generated from the data that could be seen significantly different between the cultures in England and Taiwan. Firstly, teachers' attitudes towards technology in both countries varied. The participated Taiwanese teachers held negative conceptions of technology use for teaching practices, whereas the English teachers were positive about it not only because they were confident and comfortable about using ICT but also students seemed to have higher level of acceptance. Secondly, the Taiwanese teachers experienced greater difficulties pertaining to infrastructure as the classroom settings were not particularly designed for technology use in Taiwan whilst the English classroom settings implemented interactive whiteboards and projectors which offered convenience for teachers. Finally, in terms of pedagogy, the Taiwanese teachers tended to follow a curriculum based teaching strategy and mostly related GeoGebra exercises to textbooks; therefore, GeoGebra was used specifically for assistance of visualisation of textbooks examples. Again, the English teachers appeared to be more creative and flexible in choosing their teaching methods. As the Taiwanese educational system has an examination-driven culture, there are several areas being used extensively such as problem solving for university entrance examinations and proof of theorems as well as revision for examination preparation. In contrast with Taiwan, the English educational system has a focus on individual learning, therefore, there seemed to be a stress on students' individual investigation and interaction with GeoGebra.

Teachers' practical elaboration of GeoGebra can be seen interrelated within the four dimensions. The infrastructure of ICT has a great impact on the ways in which teachers regard GeoGebra as an educational tool since if ICT facilities are not available or advanced, it would definitely influence the way teachers use it. Given ICT provision, teachers' mathematical content knowledge and conceptions may affect their choices of mathematical topics utilising GeoGebra. Certainly, providing sufficient support for the use of GeoGebra, teachers might start experiencing changes in their behaviour with GeoGebra. This teacher transition will move them from beginners to advanced users of GeoGebra as well as help them develop their pedagogical practices in teaching practices. In spite of these common dimensions between Taiwan and England, there are substantial discrepancies in technological artefacts and adaptation of curricular resources which underpin English and Taiwanese teachers' decisions and practices with GeoGebra applications. These significant differences could be explained by the two opposed Eastern and Western cultural traditions.

Despite the potentiality of GeoGebra, teachers have not fully discovered its capability to link geometry and algebra but acknowledged that it offers pervading possibility in teaching practices. As Markus Hohenwarter puts it, 'GeoGebra is free software because I believe education should be free. This philosophy makes it easy to convince teachers to give this tool a try, even if they haven't used technology in their classrooms before'.

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LEARNER ASSISTANCE BY WORKED EXAMPLE VIDEOS IN INTERACTIVE LEARNING ENVIRONMENTS

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With Worked Example Videos (WEVs) we present a new, innovative format for instructional material. These videos are on-screen recordings of expert solutions of geometrical tasks which are provided to learners within interactive learning environments. In this paper we report on a framework for the implementation of this format. We further expose some results of an empirical study we conducted with secondary school students. The students were supported by WEVs while working individually with a web-based learning environment on geometrical tasks. The results show that the format of WEVs has high potentials, especially to support low achieving learners.

AIMS & BACKGROUND

In our research project concerning students' learning with interactive, web-based work sheets for geometry we developed an Interactive Learning Environment (ILE) with the focus on learner support by interactive videos. In the first phase of our study we analysed the usage and the acceptance of the ILE in a qualitative way. We discovered individual learning styles of students and found high acceptance and an intuitive usage of our environment (Mann & Ludwig, 2007; Mann, 2008).

This paper reports on the second phase of our empirical studies. In this phase we wanted to quantify students' learning outcomes after working with the ILE. Thus we were especially interested in the efficiency of example-based learning videos, the so called *Worked Example Videos* (*WEV*s). These videos demonstrate experts' solutions and the solution *processes* of geometrical tasks. They are on-screen recordings of these processes and can be used in an interactive way.

THEORETICAL FRAMEWORK

The solution of a construction task is always a process that consists of a sequence of single construction steps. Such a process, where every step follows a previous one until the solution is found can perfectly be visualised by video, which is nothing else but a sequence of images. If such a solution is demonstrated by an expert it fits the idea of (animated) worked examples. Thus a WEV for learning with digital tools like DGEs is the screen recording of an expert's solution of the task and provides instructional support to learners. These can use WEVs in an interactive way, namely they can use all functions of a media player like 'play', 'pause' or the search bar.

Worked examples in general are considered to be successful in a number of empirical studies (Renkl, 2002). An explanation for positive learning outcomes by learning with worked examples is given by Sweller's Cognitive Load Theory (Sweller, 1988;

Sweller, van Merrienboer & Paas, 1998; Sweller, 2005). It explains the learning effects in terms of a reduction of extraneous load, an ineffective load of working memory: the delivery of worked examples can lower extraneous cognitive load, so that cognitive capacity can be used for meaningful learning (Renkl, 2002; Kirschner, 2002). Reduced cognitive load enables learners to better construct schemata and automation. E.g. Tarmizi & Sweller (1988) show that worked examples are effective, when a learner's attention is drawn to the aspects relevant for learning. This form of attention guiding reduces cognitive load:

> "On the other hand, extraneous cognitive load might be decreased by providing learners with informative examples of data and [...] by guiding their attention to relevant aspects of the simulation." (Bodemer & Ploetzner, 2004, p. 4)

But worked examples are not effective in themselves. In an example-based learning environment it is important to activate the learners' cognitive activities (Atkinson & Renkl, 2007). Besides positive learning outcomes it is remarkable that worked examples are a method that is preferred by learners (Renkl, 2005).

Guidelines for Interactive Work Sheets

> "When the message is poorly designed, learners must engage in irrelevant or inefficient cognitive processing; when it is well designed, extraneous cognitive load is minimized." (Mayer, 2001, p. 50).

The design of the ILE and its integrated *Interactive Work Sheets* is an outstanding factor for learning success. Though we propose five empirically based guidelines for the development of interactive, web-based work sheets for geometry according to results of research concerning learning from worked examples and learning with digital media (Mann 2008b). We formulated the following research based guidelines and developed our ILE according to these five principles:

(G1) Reduce the interface and ensure that the ILE has elements the learner is familiar with; include an "Undo"-function.

(G2) Provide feedback to learners for their self monitoring and performance control.

(G3) Reduce extraneous load by offering learners support in form of worked examples.

(G4) Increase germane load by activating meaningful cognitive processes, e.g. by offering a high degree of interactivity (Schulmeister, 2003).

(G5) Support learners in their usage of the new ILE, especially in their tool usage, e.g. by offering animated demonstrations.

A fundamental idea behind these guidelines is the stimulation of the active processing of information (Roy & Michelene, 2005). So it is found to be important to activate and stimulate learners' cognitive activities to avoid superficial processing and to support meaningful learning (Atkinson & Renkl, 2007). Furthermore learners need free space to become cognitive active, but also need guidance that these activities lead to the construction of knowledge: "Children seem to learn better when they are active and when a teacher helps guide their activity in productive directions." (Mayer, 2004, p. 16)

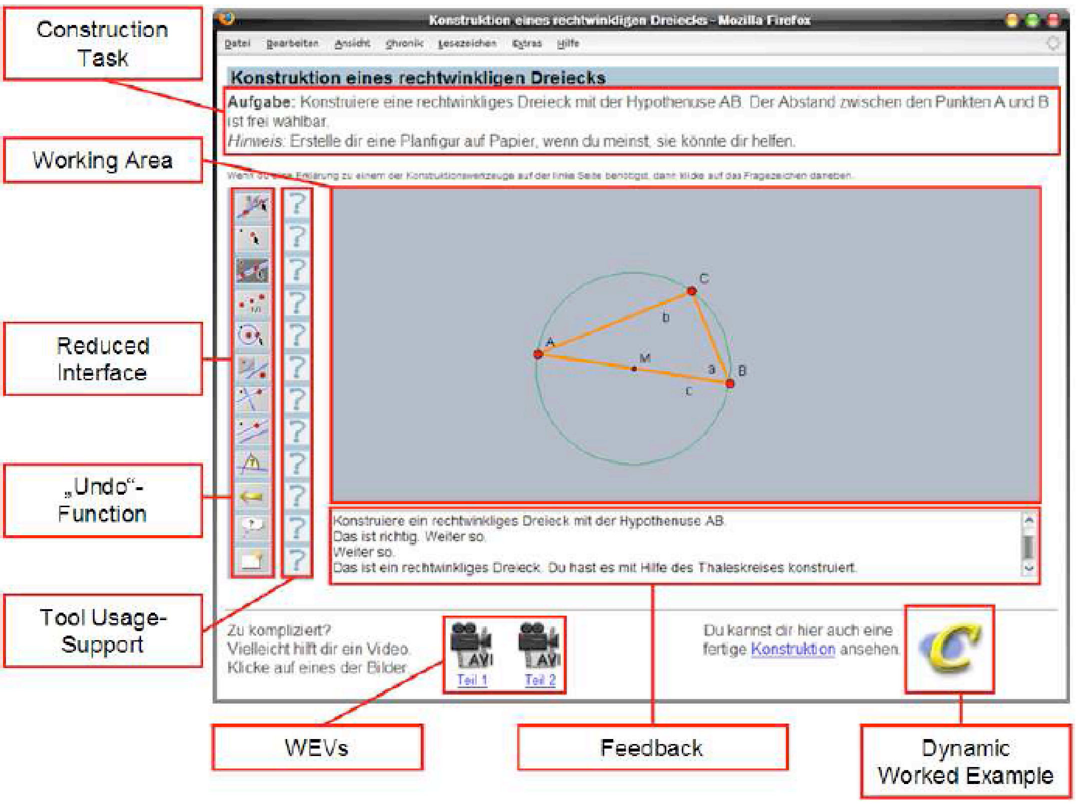

Figure 1: Interactive Work Sheet (designed according to the guidelines)

Besides guidance Heintz (2002) states that a reduced user interface and direct feedback is important and supports learners (guideline G1). Thus with the new kind of tool technical problems no longer appear and do not hinder the learning process. Moreover feedback enables students to work self directed with the ILE (Mann & Ludwig, 2007). In the frame of learning with Dynamic Geometry Environments Laborde et al. (2006) also stress the importance of feedback. According to these findings the Interactive Work Sheets we integrated in the ILE provide intelligent feedback to learners.

Learning with Interactive Videos

The focus of our research is set on learner support by WEVs. To help learners through animations and videos interactivity plays an important role (Riempp, 2000). By using interactive elements to control the video the learner can use this form of support in an individual way and it fosters self regulated learning (Salomon, 1994). McNeil & Nelson (1991) argue that learning with interactive videos is an effective form of acquisition of knowledge. Morrison et al. (2000) discuss, that learning with animations is highly effective in cases, where animations comprise additional information or interactive elements. In particular novices can profit when operations and effects are visualised (Salomon, 1994). Plaisant & Shneiderman (2005) argue that animated demonstrations can be an alternative to active and practical help by experts. In their study Palmiter & Elkerton (1993) show that learners who are supported by animations, solve tasks faster and better than learners who get support in text form.

We integrated videos in the form of WEVs in our environment to offer support for the solution of tasks and to help students to get along with the environment. The integration of videos and the implementation of the five guidelines result in an ILE which includes interactive work sheets and learner support by worked examples (cp. Figure 1).

Research questions

In our empirical study we were firstly interested in the efficiency of the ILE and of WEVs. We wanted to know if they are a suitable format for the acquisition of new content knowledge and for learning complex construction processes. Do learners who can use WEVs take more profit than learners who use Dynamic Worked Examples or who don't use example based support at all? And in more detail: can especially low-achieving learners profit from this format as this group often is at a disadvantage when learning with digital media.

METHOD

The participants of our study were 110 students from five German secondary schools (49 male, 61 female, 13 to 14 years of age). 69 of them had some low level experiences with Dynamic Geometry Software, 101 had watched videos on a PC before. The students were randomly assigned to one of the following subgroups:

- Group 1: *"Worked Example Video"* (*WEV*). The members of this group could use WEVs, which showed experts' solutions of construction processes.
- Group 2: *"Dynamic Worked Example"* (*DWE*). Members of this group could make use of support in the form of dynamic constructions that showed them a worked example (without the construction process).
- Group 3: *"No Worked Example"* (*NWE*). The members of this group did not use support by worked examples while working with the ILE.

Prior knowledge of all participants was quantified by a Pre-test test consisting of 15 content-related items. A Post-test with identical items was used to identify learning outcomes after the students' work with the environment. Additional data was collected by two questionnaires: a first one was used to get information about the participants' ICT competences; with the second questionnaire we asked for students' attitudes towards the ILE and its components.

The intervention was accomplished in regular classroom situations. Students of five different classes worked with the ILE in their regular math lessons in a computer room that was familiar to them. The introduction to the treatment and to the ILE was very short. So the students only got some general information and the advice to look for help (if needed) inside the ILE.

To measure the efficiency of the different kinds of worked example based learner assistance we calculated the increase from Pre- to Post-test. As every item was rated with “1” (correct) or “0” (false) the difference between both tests could be divided the number of items to get the relative increase of knowledge. The test problems are concerned with secondary school geometry. Five of the items are construction tasks, five variation tasks and five tasks are affiliated to other types of tasks.

RESULTS

The mean length of the intervention was 43.75 minutes (SD = 13.43). The learning periods were in a broad range between 19 minutes and 80 minutes. Figure 2 shows the results of all students and the results of the research groups in the Pre- and Post-Test. The whole group of 110 students could gain almost ten percent of the test items (total difference d = 1.50 Items; 9.97%). This improvement is significant ($t = 6.38 > 2.36$; $p < .01$) [1].

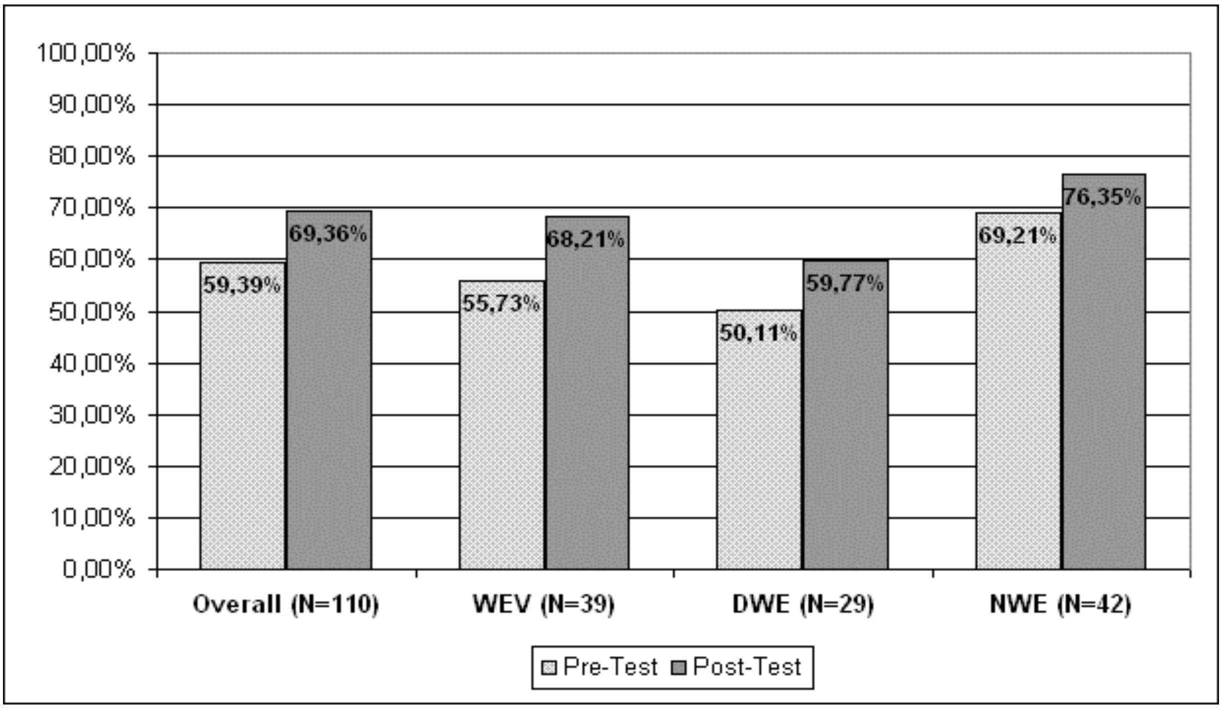

Figure 2: Results of Pre- and Post-Tests

The greatest improvement could be detected for group *WEV* and was 12.81% (1.92 Items), which was highly significant ($t = 4.13 > 2.43$; $p < .01$). But there were also significant improvements within the other groups: group *DWE* could gain 9.66% ($d = 1.45$ Items; $t = 3.07 > 2.47$; $p < .01$), group *NWE* gained 7.30% ($d = 1.10$ Items; $t = 3.96 > 2.42$; $p < .01$). Although the improvement of group *WEV* was the most considerable, it was not significantly higher than the improvements of the other groups, consequently the kind of example based assistance was not found to be a significant factor.

Low-achieving vs. High-achieving learners

To compare low- and high-achieving students we first split up the whole group into two subgroups by their performance in the Pre-tests: the high-achieving subgroup (L+) and the low-achieving subgroup (L-). The comparison of these two groups shows a significant difference in their improvements: while the high-achievers only gain 2.7% (+0.42 Items), the low-achieving learners could improve by 16.1% (+2.41 Items). The difference between the subgroups is highly significant ($t = 4.81 > 2.36$; $p < .01$).

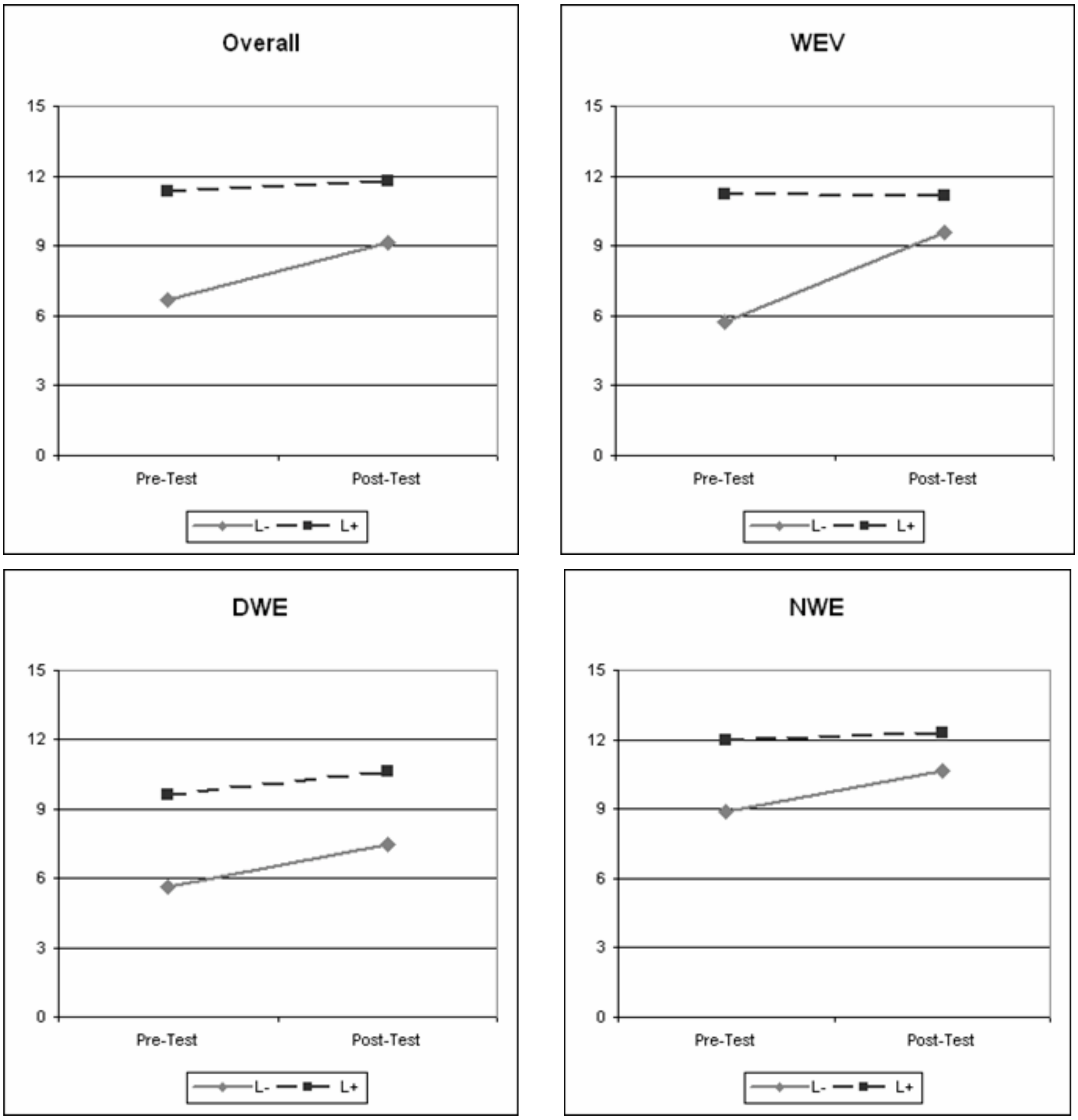

Figure 3: Comparison of the low- and high-achivers of the research groups

Even more striking is the difference between the subgroups of the first research group *WEV*. The better performing learners of this group (WEV_{L+}) couldn't even reach their Pre-test results in the Post-test (-0.06 Items, -0.4%) while the low-achieving subgroup (WEV_{L-}) gained 23.7% (+3.55 Items). Again the difference between the subgroups is highly significant ($t = 5.20 > 2.43$; $p < .01$).

For group NEW the difference between the subgroups is significant, too ($t = 3.54 > 2.42$; $p < .01$). In contrast the difference between the subgroups of group DWE is not significant ($t = 0.90 < 2.48$; $p < .01$).

Research Group	N	L– Mean	SD	N	L+ Mean	SD	Difference
Overall	59	2.41	2.50	51	0.41	1.82	2.00
WEV	21	3.55	2.26	18	-0.06	2.04	3.61
DWE	16	1.81	2.90	13	1.00	1.91	0.81
NWE	21	1.95	1.53	21	0.19	1.69	1.76

Table 1: Mean and standard deviation of differences between Pre- and Post-Test

Detailed analysis

For a more detailed view on the results we could compose item groups out of similar tasks and analyse the test results. So we found that students when they had to work on tasks concerning "tangents" could highly benefit from WEVs (29.9%) whereas students without example based support only gained 11.9%. The difference in the increase between both groups is significant ($t = 3.05 > 2.38$, $p < .01$).

Gender and ICT competences

The result of the comparison of male and female students shows an improvement of 1.45 Items (9.7%) for the male group and of 1.50 Items (10.0%) for female learners. These values are obviously nearly identical. The factor "Gender" does not have significant influence on learning outcomes ($t = 0.11 < 2.36$, $p < .01$, ns).

We obtained a comparable result for the factor "ICT-Competences". The comparison of the student group with high skills with the group with low competences did not show a significant difference ($t = 0.09 < 2.38$, $p < .01$, ns).

DISCUSSION

The above results show that students who worked with our ILE improved significantly. We ascribe their learning success to the ILE, especially to its interactive elements and the example based learner support. As all students benefit (as well students who did not have example based support) we see that the interactive elements of the environment foster students' cognitive activities. But we also see that learners who use WEVs improve the most. So we state that WEVs have the potential to help students learn more or better.

Our most striking result lies in the difference between low-achievers and high-achievers. Especially when working with WEVs the low-achieving learners take much more benefit from the ILE which leads to greater increase of knowledge. This result is contrary to earlier findings, where high-achieving learners can take more profit of learner support, because their metacognitive help seeking skills are on a higher level (Aleven & Koedinger, 2000). So we argue that WEVs are a format that is well suited for lower performing learners and for early learning phases, e.g. when learning new content. Moreover we state that our environment due to its design does not demand special or high ICT competences nor prefers learners of one gender.

CONCLUSION

With our project we could uncover some potentials of Worked Example Videos in special and of video-based learner support in general. WEVs have great potential to support learners in learning geometry. Especially low-achieving learners strongly benefit from this format. Future research work will have to look into three directions: (1) Do WEVs have potentials of which high-achieving learners can benefit? (2) Are the potentials of WEVs task- or domain-specific? (3) How can the integration of WEVs into regular classroom activities be facilitated?

NOTES

1. For the comparison of our three research groups we conducted a repeated measures ANOVA. The differences between subgroups and between Pre- and Post-test were tested by t tests.

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CASYOPÉE IN THE CLASSROOM: TWO DIFFERENT THEORY-DRIVEN PEDAGOGICAL APPROACHES

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The ReMath project is a European project that addresses the task of integrating theoretical frames on mathematical learning with digital technologies at the European level. A specific set of six dynamic digital artefacts (DDA) has been currently developed, reflecting the diversity of representations provided by ICT tools. Here we considerer the DDA Casyopée which was experimented in two different countries: Italy (Unisi team) and France (Didirem team). The paper focuses on the influence of the theoretical frames in the design of these Teaching Experiments.

PROBLEMATIC OF THE REMATH PROJECT

The project focuses on the primary and secondary school level giving a balanced attention to both teachers and students and incorporating a range of innovative and technologically enhanced traditional representations. Specific attention is given to cultural diversity: seven teams from four countries are involved in this project. The work is based on evidence from experience involving a cyclical process of

a) developing six state-of-the-art dynamic digital artefacts for representing mathematics involving the domains of Algebra, Geometry and applied mathematics,

b) developing scenarios in a common format for the use of these artefacts for educational added value,

c) carrying out empirical research involving *cross-experimentation* (Cerulli et al. 2008) in realistic educational contexts, aiming at enhancing our understanding of meaning-making through representing with digital media, in particular by providing new insight into means of using technologies to support learning, and into learning processes in relation to the use of technologies.

Many recent studies highlight the existence of a multiplicity of theoretical frameworks for addressing those themes, and there is a shared increasing need of overcoming the resulting fragmentation (Artigue, 2008). This need is also felt within ReMath project, in which a variety of educational paradigms is present. The issue is addressed through the development of specific methodological tools, some of which are drawn and re-elaborated from the experience of TELMA project (Cerulli et al., 2008).

In this paper we present two different Teaching Experiments designed and carried out within ReMath project, respectively by Didirem team of the University Paris 7 (France), and by Unisi team of the University of Siena (Italy). Both the TEs were designed around the use of the software Casyopée (partly developed within the project). After describing the main features of Casyopée (exploited in the Teaching experiments) we will focus on the design of the Teaching Experiments, and we will compare them relying on the construct of Didactical Functionality (Cerulli, Pedemonte and Robotti, 2006). Though it would be interesting, a discussion on the actual implementation of the plans in classroom is out of the goals and of the possibilities of the present paper.

THE CONSTRUCT OF DIDACTICAL FUNCTIONALITY

The construct of Didactical Functionality is meant to provide a minimal common perspective, hopefully independent from specific theoretical frameworks, to frame diverse approaches (possibly depending on theoretical references) to the use of ICT tools in mathematics education, as well as the theoretical reflections regarding the actual use of ICT tools in given contexts.

By Didactical Functionality of an ICT tool, one means the system constituted by three interrelated poles: a set of features of the tool, a set of educational goals, and the modalities of employing the specified features of the tool for achieving the envisaged educational goals.

Trivially, through the construct of Didactical Functionality one intends to acknowledge that an ICT tool (or part of it) can be used in different ways for achieving different educational goals, that is one can design or identify different Didactical Functionalities of a given tool. In particular different theoretical perspectives can lead to designing different Didactical Functionalities of a given tool.

THE DDA CASYOPEE

The DDA Casyopée (Lagrange and Chiappini, 2007) is built as an open problem-solving environment with the aim of giving students a means to work with algebraic representation, progressively acquiring control of the sense of algebraic expressions and of their transformations. Functions are the basic objects in Casyopée. Using this tool, students can explore and prove properties of functions. Casyopée takes into account the potentialities that Computer Algebra Systems offer to teaching and learning: going beyond mere numerical experimentation and accessing the algebraic notation; focusing on the purpose of algebraic transformations rather than on manipulation and connecting the algebraic activities. It is expected that students will make sense of algebraic representations by linking these with representations in these domains. See below a screen copy on the algebraic representations provided by Casyopée, it splits into two windows: a symbolic one and a graphical one.

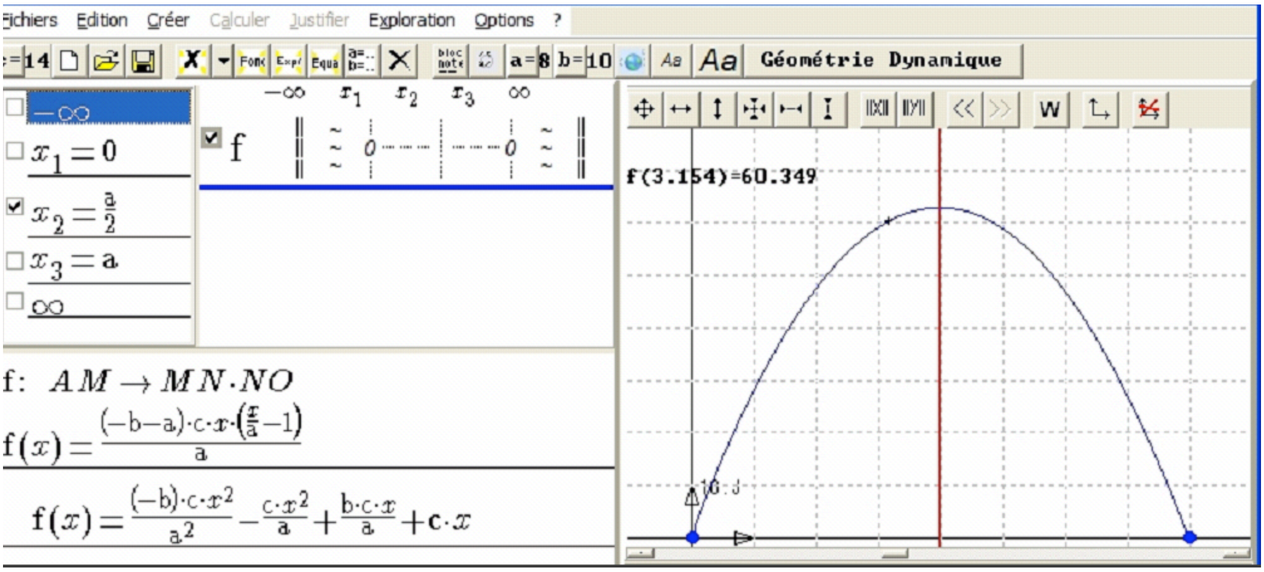

Figure 1: the algebraic setting in Casyopée

In the Remath project, Casyopée has been extended with a geometrical module. The aim is to explore what can be an interesting cooperation between a geometrical problem and its analytic treatment. The goal is not to develop a whole geometric dynamic environment but rather to see how geometric and analytic environments can articulate each other. For instance, a geometrical figure can be a domain to experiment with geometrical calculations. In the screenshot below, students can ask for the measure of the area of the rectangle MNOP. Then an algebraic model can be built choosing one of the measures as an independent variable and the other as a dependant variable. Properties of the dependency can be conjectured and proved: they take sense both in the algebraic and in the geometrical settings.

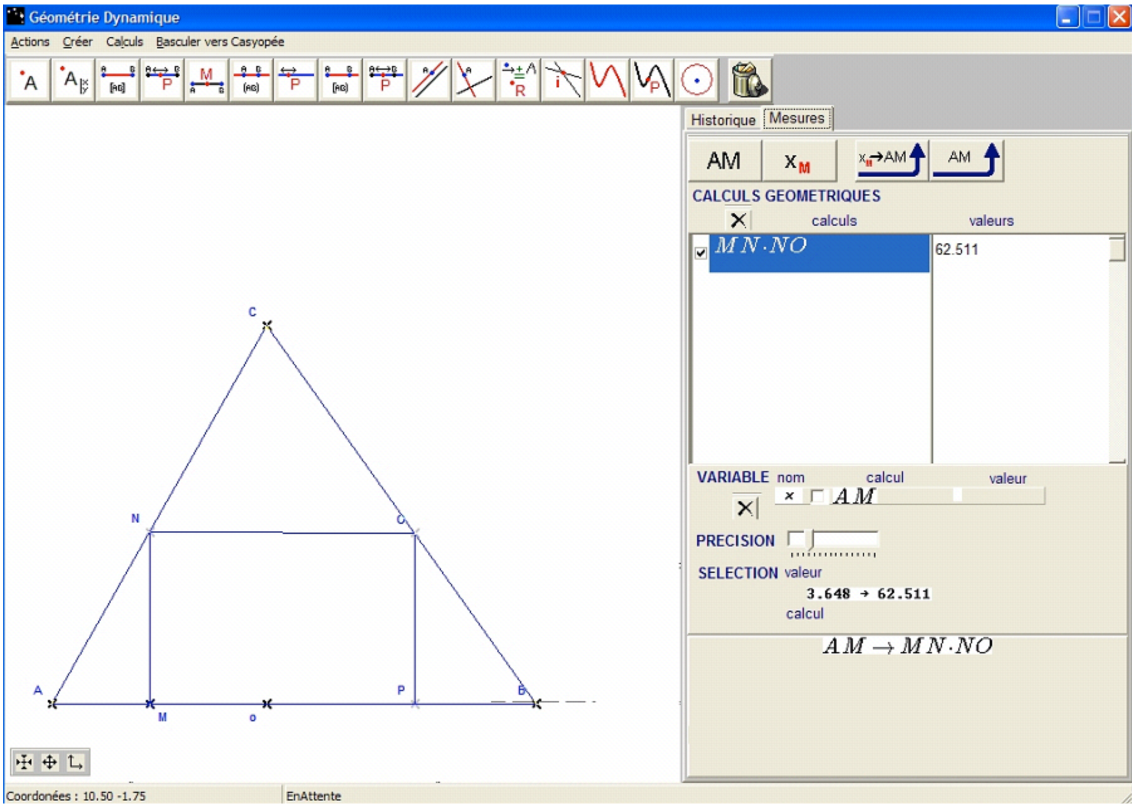

Figure 2: the geometrical window in Casyopée

The main specificity of Casyopée among other dynamic geometrical artefacts is to connect geometric and algebraic approaches. More precisely, the geometrical frame allows one to consider a geometric calculation and to export it in the algebraic environment. This transfer is allowed by choosing an adequate variable for the geometrical situation. At this point, Casyopée gives a feedback on the choice of this independent variable.

The representations offered by Casyopée have been thought to be close to institutional ones. Casyopée allows students to work with the usual operations on functions such as algebraic operations, analytic calculations and graphical representations. The geometric environment offers commands usually available in other dynamic geometry environments such as creating fixed and free geometrical objects (points, lines, circles, curves)

UNISI AND DIDIREM PEDAGOGICAL PLANS

In the introduction we recalled that different specific methodological tools have been developed within ReMath for fostering the comparability of studies dealing with the use of ICT tools in mathematics education. A new conceptual model of the pedagogical scenario, called Pedagogical Plan (Bottino et al. 2007), is one of those methodological tools. A Pedagogical Plan has a recursive hierarchical structure: each pedagogical plan is conceived as a tree whose nodes and leaves are pedagogical plans themselves. Several components are attached to each pedagogical plan: including the articulation of the educational goals, of the class activities, the specification of the features of the ICT tool used and how they are used, and of the rationale underpinning the whole pedagogical plan and of the theoretical frames inspiring it. A web-based tool (Pedagogical Plan Manager, PPM) has been also developed for supporting teams in designing their pedagogical plans.

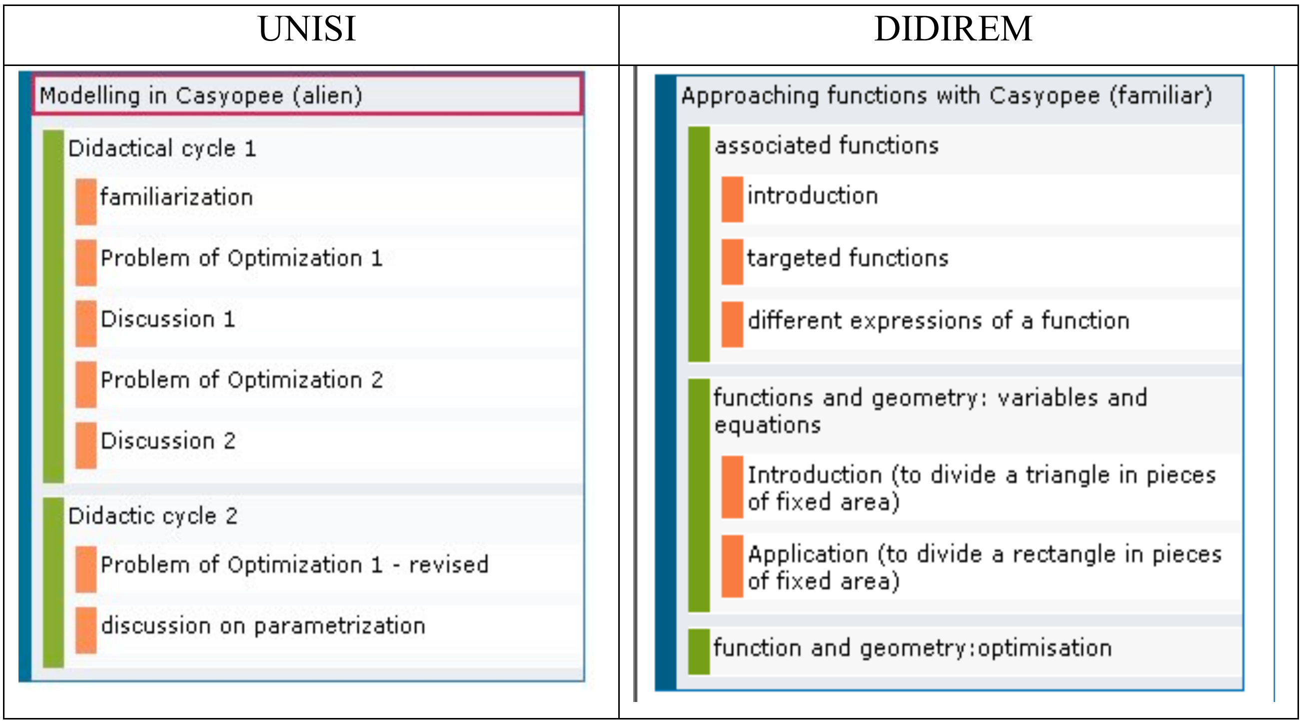

Figure 3: synthetic view of Unisi and Didirem pedagogical plans in the PPM

Figure 3 displays a screenshot from the PPM, and it is meant to provide an overview of the structures of the pedagogical plans designed by the Unisi and Didirem teams.

Details of the Unisi pedagogical plan

The Unisi pedagogical plan is inspired by the Theory of Semiotic Mediation (Bartolini Bussi and Mariotti, 2008) drawn from a Vygotsijan perspective. This theory guided both the specification of the educational goals (starting from an analysis of Casyopée) and the overall structure of the planned activities.

The designed educational goals are (a) to foster the evolution of students' personal meanings towards the mathematical meanings of function as co-variation. That regards also the notions of variable, domain of a variable… and (b) to foster the evolution of students' personal meanings towards mathematical meanings related to the algebraic modelling of geometrical situations.

Students are expected to have already received some formal teaching on the notions of variable, function and graph of a function, and on its graphical representation in a Cartesian plane. Moreover, a common experience of researchers and teachers is that meanings related to those notions are rarely elaborated in depth. The aim is to mediate and weave those meanings in the more global frame of modelling.

Hence, the pedagogical plan is not meant to help students become able to use Casyopée for accomplishing given tasks, but instead to foster the students' consciousness-raising of the mathematical meanings at stake.

The whole pedagogical plan is structured in cycles entailing: students' pair or small group activity with Casyopée for accomplishing a task, students' personal rethinking of the class activity (through the request to students of producing individual reports on that activity), class discussion orchestrated by the teacher.

The familiarization session is designed as a set of tasks aims at providing students with an overview of Cayopée features and guiding students to observe and reflect upon the "effects" of their interaction with the tool itself, e.g.:

> Could you choose a variable acceptable by Casyopée and click on the "validate" button? Describe how did the window "Geometric Calculation" change after clicking on the button. Which new button appeared?

Besides familiarization, the designed activities with Casyopée consist of coping with "complex" optimization problems formulated in a geometrical setting and posed in generic term, e.g.:

> Given a triangle, what is the maximum value of the area of a rectangle inscribed in the triangle? Find a rectangle whose area has the maximum possible value.

The aim is to elaborate on those problems so to reveal and unravel the complexity and put into evidence step by step the specific mathematical meanings at stake.

According to the designed pedagogical plan, the teacher plays the delicate role of guiding students to unravel such complexity and to make the targeted mathematical meanings emerge. The main tool for the teacher to achieve this objective, is the orchestration of the class discussions. The development of a class discussion cannot be completely foreseen *a priori*, it should be designed starting from the analysis of students' actual activity with Casyopée and of the reports they produce, and it would still depend on extemporary stimuli. Nevertheless in the design Unisi team tried to anticipate possible development of the pedagogical plan and to plan some kind of possible canvas for the teachers for managing class discussions.

The pedagogical plan is intended for scientific high schools or technical institutes, grade 12 or 13, and can be implemented over approximately 11 school hours.

Details of the Didirem pedagogical plan

The Didirem pedagogical plan aims to help students construct or enrich knowledge in the following areas: meaning of functions as algebraic objects and meaning of functions as means to model a co variation in geometric and algebraic settings. It is intended for scientific high schools grade 11 or 12 and has been implemented in ordinary classes during approximately 10 school hours. It is inspired both by the

Instrumental Approach (Artigue, 2002), the Theory of Situation (Brousseau, 1997) and the Theory of Anthropologic Didactic (Chevallard, 1999).

Specific importance is given to the construction of tasks with an adidactical potential, where students can choose different variables for exploring functional dependencies, and to the connection between algebra and geometry. This connection is supported in Casyopée by geometric expressions that allow expressing magnitudes in a symbolic language mixing geometry and algebra.

The pedagogical plan is built around three main types of tasks:

- First session: finding target quadratic functions by animating parameters (five different tasks according to the semiotic forms used for these functions):

  Lesson 1: Introducing associated functions (a function g is associated to a function f if it is defined by a formula like $g(x)=af(x)+b$ or $f(ax+b)$ or similar)

  Lesson 2: Target Functions (functions that can be graphed but whose expression is not known; each student have to guess the function graphed by his/her partner)

  Lesson 3: Different expressions of quadratic functions

So students should consolidate: the meaning of variable, the distinction between variable and parameter, the meaning of function of one variable with several registers of semiotic representation and the fact that a same function may have several algebraic expressions. The new notion of associated function is worked-out during this session.

- Second session: creating a geometrical calculus as a model of a geometrical situation to solve a problem of relationships between areas, manipulation to experiment co variation between two geometrical variables:

  Lesson 4: To divide a triangle in pieces of fixed area

  Lesson 5: Application; dividing a rectangle into figures of fixed area

This way students can enhance their knowledge on co variation and develop the ability to experiment and anticipate in a dynamic geometrical situation, and the ability to model a geometric situation through geometric calculations.

- Third session: creating a function as a model of a geometrical situation to solve an optimization problem.

  Lesson 6: solving a problem of optimisation in geometric settings by way of algebraic modelling.

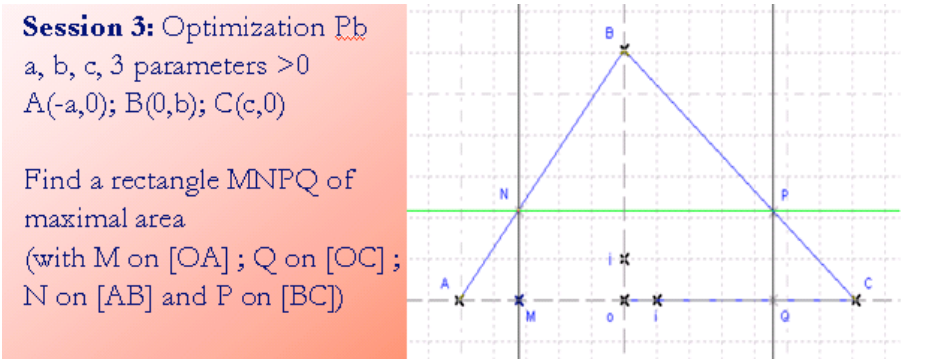

Figure 4: statement of the session 3 in Didirem pedagogical plan

This problem allows both to reinvest abilities to use the DDA, previous knowledge on associated functions and to introduce the notion of optimum in a geometrical situation.

COMPARISON OF THE UNISI AND DIDIREM APPROACHES USING THE CONSTRUCT OF DIDACTICAL FUNCTIONALITY

The two pedagogical plans, described in the previous sections, evidently share some characteristics but also have apparent deep differences. In this section we use the frame provided by the construct of Didactical Functionality to develop a more systematic comparison between the two pedagogical plans.

Tool Features

The two pedagogical plans are not generally centred on the use of the same DDA, but more specifically on the use of the same DDA features. In fact both exploit especially

(a) features of the dynamic geometry environment: the commands for creating fixed, free or constrained points, for dragging free or bonded points, for creating points with parametric coordinates, and the corresponding feedbacks of the DDA;

(b) features of the geometric calculation environment: the commands for creating "geometric calculation" associating numbers to geometrical objects, for choosing (independent) variables, for creating function between the selected variable and calculation, and the corresponding feedbacks;

(c) features of the algebraic environment, including the commands for displaying and exploring graphs of functions, for creating and manipulating parameters, for manipulating the algebraic expressions of functions, and the corresponding feedbacks.

Educational Goals

Different educational goals are associated to the use of those features. More precisely, one can recognize that both pedagogical plans share a common focus on some mathematical notions: function (in particular, conceived as co-variation), variables (independent and dependent) and parameters. Moreover the two pedagogical plans present, among other tasks, two optimization problems sharing the same mathematical core (see sections…). But, besides those surface similarities, there are profound differences.

Other Unisi educational goals are to mediate and weave meanings, related to the notions of function, variable and parameter. With that respect the Unisi team assumes, on the one hand, that those notions are familiar for students, and, on the other hand, that those notions are not elaborated in depth. Hence the Unisi pedagogical plan aims at helping students gain a deeper consciousness of the mathematical meanings at stake and re-appropriate them in the more global frame of modelling. In addition the Unisi objective includes the shared and decontextualized formulation of the different mathematical notions in focus.

The Didirem objectives are mainly to use potentialities of representations offered by Casyopée to introduce some new mathematical knowledge. This knowledge has been chosen for two main reasons: its affordance to the French curriculum and the importance to be studied in several frames of representations.

Modalities of employment

In accordance with the different objectives and the different pedagogical culture, the modalities of use are different as well.

The Unisi pedagogical plan has an iterative structure. Students' activity with Casyopée alternates with class discussions, after each session students are required to produce individual reports on the performed activities. This structure is meant to foster students' generation of personal meanings linked to the use of the DDA and their evolution towards the targeted mathematical meanings together with the students' consciousness-raising of the mathematical meanings at stake. That process is constantly fuelled by the teacher, whose role is crucial. Accordingly the teacher's role is explicitly taken into account in the design of the pedagogical plan, which provides with hints for the possible actions. The tasks used are optimization problems set in a geometrical frame. Their solution and the reflection on these solutions are fundamental steps towards the achievement of the designed educational goals. Also the familiarization with the DDA has to be considered within that perspective: as already mentioned, it aims at making students observe and reflect upon the "effects" of their interaction with the DDA itself. *Ad hoc* tasks are designed for that purpose.

Instead, the Didirem team pays specific attention to a progressive use of the DDA combining artefact and mathematical knowledge. Indeed, students work only in the algebraic window during section 1, then only in the geometrical windows in section2; finally section 3 gives an opportunity to reinvests the knowledge in the two environments. Moreover, all the tasks proposed are mathematical ones and are elaborated in order to allow students make progress alone working on the problem and to construct their new knowledge thanks the feedbacks.

CONCLUSION

Those differences can be strongly related with the different theoretical perspectives adopted by the two teams.

The Unisi team has mainly structured its pedagogical plan according to the Theory of Semiotic Mediation which inspired both the specification of the educational goals and the organization of the activities in iterative cycles. In particular the Theory of Semiotic Mediation led the Unisi team to devote attention towards the design of the teacher's action in the pedagogical plan. In fact, the teacher plays a crucial role throughout the whole pedagogical plan, especially for fostering the evolution of students' personal meanings towards the targeted mathematical meanings and facilitating the students' consciousness-raising of those mathematical meanings.

Instead, the Didirem team splits its theoretical approach into several theoretical frames which shape their pedagogical plan: the Instrumental Approach (Artigue, 2002), the theory of Situation (Brousseau, 1997) and at last the theory of anthropologic didactic (Chevallard, 1999). The first frame aims to go further than a simple familiarization with the DDA and to help the students constructing a mathematical instrument. This process goes hand in hand with the learning process. The last optimization problem is used to evaluate the progress of this process. The process is accurately designed through a careful choice of mathematical tasks, with an adidactical potential, whereas the definition of the teacher's actions and role escapes the design of the PP. Finally, the TAD is called upon to manage instrumental distance between institutional and instrumental knowledge.

No doubt that these approaches are complementary. Each team might benefit from this collective work to improve its pedagogical plan in the future. For instance, the Didirem team plans to pay more attention to the teacher's role during the pedagogical plan conception. Nevertheless, the objective is not to elaborate a wide common consensual theoretical frame, but rather to go in depth in the clarification of didactical functionalities, in a shared language.

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