

New technological environment, new resources, new ways of working: the e-CoLab project (Collaborative Mathematics Laboratory experiment)

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This article reports the first year of a French collaborative research on the latest symbolic calculator from Texas Instruments – TI-nspire CAS (Computer Algebra System). The experiment is based on a partnership between the INRP (*Institut National de Recherche Pédagogique*) and three IREM (*Institut de Recherche sur l'Enseignement des Mathématiques* -Lyon, Montpellier and Paris) and involves six 10th grade classes.

The concept of bringing calculators into the mathematics lesson is not new. However, examining the integration of this very novel machine with its specific features such as its dynamically intertwined applications (Calculations, Graphics & geometry, Spreadsheet and lists, Mathematics Editor and Data and statistics), brought into light new elements that previous research on the use of ICT in mathematics classroom have not yet taken into account.

This article is articulated around 4 major points.

Firstly, it describes the emergence of a common resource model. After briefly offering a classification of the types of resources produced in the different groups we describe their evolution since their first uses. By retracing the genesis of the resources we attempt to demonstrate how this development, which was far from simple and to a large extent reflects the challenges in designing mathematical activities incorporating TI-nspire, led all the groups to construct a true *resource model*.

Secondly, the article describes the actual situation in the classes. In one hand we examine the articulation between mathematical and instrumental progression and on the other hand the use of new possibilities for dynamic interaction between applications offered by the calculator

The third section of the article is devoted to the examination of the pupils' views of this new mathematics experience. Based on questionnaires and interviews, we present the most striking opinions of students that give us feedback about the instrumentalisation and instrumentation processes.

Finally, the article examines the difficulties and benefits of the collaborative work. The three groups in the e-CoLab project worked jointly using a workspace on the EducMath site. We describe the operation, organisation and evolution of the workspace and we consider the extent to which it facilitates communication, exchange and sharing of documents.

This paper show to what extent the profoundly new nature of this calculator and its complexity raises significant and partially new instrumentation problems both for pupils and teachers and that making use of the new potentials on offer will require specific constructions, not simply an adaptation of the strategies which have been successful with other calculators, and that these constructions will need to continue to be thought about.

New technological environment, new resources, new ways of working: the e-CoLab project (Collaborative Mathematics Laboratory experiment)

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Abstract: this article describes the collaborative research conducted by three groups (Lyon, Montpellier and Paris IREM (Research Institute on Mathematical Education) and INRP (National Institute on Education Research) on a new calculator from Texas Instruments which offers new possibilities for mathematics work. The research examines the conditions for mutual sharing of educational resources designed by the three groups on learning by pupils and on the pupils' investment in the experiment.

Key words: calculator, community of practice, educational resources, resource model, collaborative work.

Introduction

The concept of bringing calculators into the mathematics lesson is not new: the *Repères* journal has regularly described work conducted on this subject in the IREM network using, for example, graphical calculators (Trouche 1994) and then symbolic calculators – equipped with a Computer Algebra System (Canet et al 1996). The constraints and potentials of successive generations of calculators have been studied on many occasions (for example see [Trouche et al 2007](#)). IREM conferences have enabled advances to be made in the understanding of learning processes and the complexity of the role of the teacher in these environments (Guin 1999; Lagrange et al 2003). Finally, one work (Guin and Trouche 2002) put these studies into perspective from a practical and theoretical viewpoint.

Is the research conducted since September 2006 on the latest calculator from Texas Instruments just an extension of previous research? This is a symbolic calculator called TI-*n*spire CAS (Computer Algebra System). At first sight it undoubtedly looks like a highly refined calculator, but also just a calculator. However, it is a very novel machine for several reasons:

- its nature: the calculator exists as a “nomad” unit of the TI-*n*spire CAS software which can be installed on any computer station;
- its directory, file organiser activities and page structure, each file consisting of one or more activities containing one or more pages. Each page is linked to a workspace corresponding to an application: Calculations, Graphics & geometry, Spreadsheet and lists, Mathematics Editor, Data and statistics;
- the selection and navigation system allowing a directory to be reorganised, pages to be copied and/or removed and to be transferred from one activity to another, moving between pages during the work on a given problem representing an activity;

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- connection between the graphical and geometrical environments, which is used in a Graphics & geometry application, the ability to animate points on geometrical objects and graphical representations and to create and move lines and parabolae and to deform parabola;
- the dynamic connection between the Graphics & geometry and Spreadsheet & lists applications through the creation of variables and data capture and the ability to use the variables, once created, in any of the pages and applications for an activity.

It may be assumed that these new developments offer new possibilities for pupil learning as well as teachers' actions. They could enable interactions to develop between different areas and between representation registers, didactic research into which has shown their importance in the conceptualisation processes: they could enrich the experimentation and simulation methods: they could enable storage of far more usable records of pupils' mathematics activity with a calculator until now. However, we can also assume that the profoundly new nature of this calculator and its complexity will raise significant and partially new instrumentation problems both for pupils and teachers and that making use of the new potentials on offer will require specific constructions, and not only an adaptation of the strategies which have been successful with other calculators and therefore these constructions will need to continue to be thought about.

For this reason, the first year of the experiment which was also conducted using a prototype which we rapidly became aware, needed major improvements, was both an exploratory and delicate year for the groups involved in the project. The exchanges, sharing of tasks between the groups and collaboration were even more essential for this first phase of the experiment to succeed.

The experiment was based on a partnership between INRP and three IREM (Lyon, Montpellier and Paris) involving six 10th second grade classes, all of the pupils of which were provided with the TI-*nspire* calculator. The groups on the three sites were composed of the pilot class teachers, IREM facilitators and university research teachers. They met regularly on site for review meetings although the exchange also continued distantly through a common workspace on the [EducMath](#) site, which allowed work memories to be shared and common tools required for the experiment (questionnaires, class resources, etc.) to be designed as the process proceeded. The name chosen by the group, e-CoLab (for experimentation Collaborative de Laboratoires Mathématiques– Collaborative Mathematics Laboratory experiment) highlights the importance of this collaboration, the nature of the mathematics work undertaken and the hybrid nature of the exchanges, both local and distant.

Additional possibilities for cross-fertilisation of the experiment were added to this very close collaboration:

- the first with a group of around ten classes experimenting on the same environment and run by the General Inspectorate;
- the second with ongoing European research on the same calculator (two meetings took place, in Brussels in March 2007 and Turin in October 2007).

We report here the first year of the experiment, describing several of its features: the emergence of a common resource model, the actual situation in the classes, the views of the pupils and finally the difficulties and benefits of the collaborative work. We have the feeling that this lies within the spirit of this special edition of *Repères* dedicated to “working together” both locally and distant in a context of both enthusiastic and cautious exploration of new technological environments for teaching and learning mathematics. This article is the fruit of work by the three groups involved in the project. The different contributions were discussed in advance and then compared throughout the writing process in order to produce

an article reflecting the *community in practice* (Wenger 1998) which the three local groups established during their exchanges.

1. New resources, genesis of a model

There were three challenges in designing mathematical activities incorporating the TI-nspire calculator²:

- clearly, before anything, this involved designing educational resources supporting strategies to use the new potentials of the TI-nspire calculator in the context of French secondary school education and, especially, making use of the inter-relationship of the different applications;
- however, in view of the short time – a few days– separating receipt of the new tool by teachers and the start of the new term, the project also involved supporting its still developing instrumentation and constructing mathematics activities incorporating the TI-nspire, *at the same time learning to use* this new tool. Many of the special features of which (potentials but also technical complexities) had no equivalent with other calculators used until that time;
- finally, in view of the particular nature of this experiment intended to be the product of collaboration between three groups, we had to design resources for the classes which would also be liable to be used by these different groups.

These three particular features had a significant impact on resource design. When considered either in isolation or together they inform both the definition of what we call a “resource”, the structure and specific features of its different components, but in particular they identify the *evolutionary* nature of the resources produced, both in form and content.

After briefly offering a classification of the types of resources produced in the different groups we shall describe their evolution since their first uses. By retracing the genesis of the resources we shall attempt to demonstrate how this development, which was far from simple and to a large extent reflects the three specific features described above, led all the groups to construct a true *resource model* based on the experience of the Montpellier group in SFoDEM (Guin et al 2007; Guin and Trouche, 2008).

1.1 Resources: objectives, forms and contents

Two types of resources were produced during the first year of the e-CoLab project: those created and used at the start of the year designed essentially to familiarise pupils with the new technological instrument provided to them (presentation of the artifact – keys, keyboard, mouse, etc. – and introduction of some of its potentials) and the larger number in which the instrumental learning, although still present, was not the main part of the activity. In contrast to the former of these resources these were constructed around (and we should add “for”) the mathematics activity itself. It is useful to distinguish amongst these the resources designed for *learning*

² For simplicity and hereafter in the article we shall call the TI *nspire* the TI *nspire* CAS calculator.

mathematical concepts³ and those constructed in order to *assess* this learning. Some resources involved a “research activity”, articulated around several concepts of the curriculum and others were designed as activities to introduce a new concept. Resources such as “The helicopter” (which proposes an approach to the absolute value concept, “The shortest path (which involves an optimisation problem using geometrical transformations) and “Sangaku” (which involves the application of the concept of similar triangles introduced elsewhere) are articulated around a very different resource structure than “You(r) bet!” or “The Sign” intended to introduce the concept of sample variation and the concept of a function respectively.

Overall, around twenty resources were designed during the year 2006-2007. We will present several examples of these as we proceed through this article to support and inform an analysis of their genesis.

1.2 Two components of a resource and their evolution – the genesis of an inseparable duo

Whilst there was a clear willingness to incorporate the new tool into mathematics activity from the start of the experiment, the first resources constructed were often reduced simply to a pupil sheet containing the problem statement (resolution of which nevertheless implied the use of a calculator) or only a computer file loaded onto the TI-*nspire*⁴ handhelds (fig. 1). When a pupil sheet and a computer file co-existed within the resource (fig. 2), these two documents could be used quasi-independently.

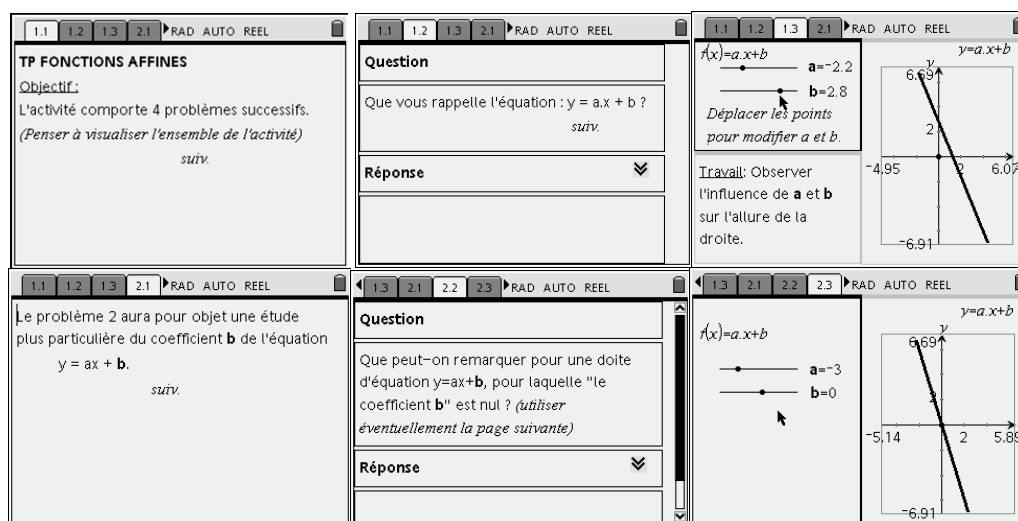


Figure 1. Screen captures showing the resource computer file

³ In the broad sense of the term. This learning is clearly not restricted only to mathematical concepts: learning of mathematics techniques, para-mathematics concepts and also the development of a certain mathematics “attitude” or a scientific process were also intended in the resources which were produced.

⁴ Referred to hereafter as “tns file” referring to the extension (.tns) of this computer file.

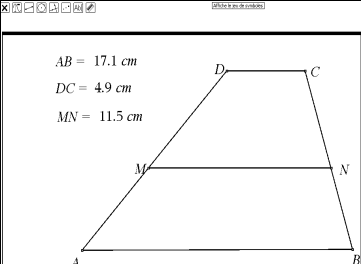
TI-nspire – Session 2 “Menus File – Problem 1	Closure problem, page 1, discussion aid <i>Mastering the tool: pointer, Graphics and Geometry Application</i>
	<p>To close the two parallel sides of a trapezoidal shaped field a farmer bought a wooden barrier of a certain length. He was able to close the two parallel sides of the field and still have half of the length remaining. He then decides to use the remaining part of the barrier to divide the field into two trapezoidal parts. Where does he have to position it in order to use the exact length remaining?</p>
<p>On opening the file does the position of point M answer the problem? Explain your answer. Determine the position of the point M solving the farmer’s problem. This then produces: $MN =$</p>	

Figure 2. An independent pupil sheet and computer file

The potentials to relate different applications of the calculator together however led to the development of a unit “pupil sheet – tns file” in which a genuine duo emerged between mathematical activity and the instrument (fig. 3).


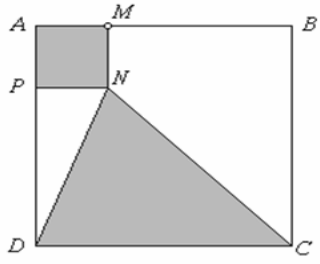
TI-nspire Séquence 3 Dossier : Fonctions Classeur : L’enseigne	<h3 style="text-align: center;">L’enseigne lumineuse</h3> <p style="text-align: center;"><i>Quelle est la fonction de l’enseigne ?</i></p> 
Première partie	
Index	Consignes
Page 1 <i>Graphiques et géométrie</i>	<p>Pour attirer plus de clients, le magasin de jeux vidéo « Aire de Jeux » a commandé une nouvelle enseigne lumineuse. Elle comporte une forme géométrique en mouvement, composée d’un carré et d’un triangle ayant un sommet commun.</p> <p>Ouvrez le classeur « L’enseigne » du dossier « Fonctions » où est représentée la figure de l’enseigne. Observez.</p> <p><i>Cette séquence propose d’étudier l’aire de cette figure et ses variations au cours du mouvement.</i></p> <p>Passer à la page 2 du classeur.</p>
Page 2 <i>Graphiques et géométrie</i>	<p>Les données de la figure</p> <p>ABCD est un carré de côté 8 cm, M un point du segment [AB]. Le carré AMNP et le triangle DNC dont les intérieurs ont été grisés constituent l’enseigne.</p> <p>Déplacez le point M sur [AB] et observez les variations de l’aire de la surface grisée qui en résultent. Décrivez ci-dessous les variations observées.</p> <div style="text-align: right;">  <p>(Figure reproduite à échelle réduite)</p> </div> <p>On note x la distance AM et $A(x)$ l’aire de cette surface.</p> <p>Précisez ci-dessous l’intervalle dans lequel x varie. $\Rightarrow x$ varie dans l’intervalle</p>

Figure 3. Extract from the resource pupil sheet “The Sign” illustrating the “pupil sheet – tns file” duo

The different components of the pupil sheet particularly well illustrate the articulation between the pupil sheet and the tns file⁵ envisaged when the resource was designed.

The column on the left entitled “Index” gives the related page in the corresponding TI-*nspire* file for each phase of the pupil sheet. In other cases (Annex 1) it can be seen that mathematical activity stated in the central column of the file can also refer to syntax learning for the calculator. The column located on the right entitled “Handling and instructions” (which appears from page 2 of the “The sign” pupil sheet, for example) usually gives the technical assistance to manipulate the calculator (Annex 2). This was found to be essential to develop pupil independence and concentrate their activity on the mathematical challenges set by the teachers (§2).

Not only did the pupil sheet evolve in terms of being inseparably integrated with its associated computer file but some parts of the pupil sheet also underwent a few changes as the technical instrument became more familiar. In particular it seemed helpful to display the mathematical knowledge around which the activity was designed within the pupil sheet itself as the files became more complex, increasing the new targeted knowledge by the teacher. In order to make the learning object more “transparent” to the pupil but also to increase pupil independence, knowledge institutionalisation for example was incorporated. This was presented gradually within the pupil sheet as knowledge emerged from the activity (the *institutionalisation areas*). One important didactic variable is then the choice of whether or not to complete these areas (fig. 4).

Choisissez une autre position du point B sur $[Ax]$.
 Votre conjecture se confirme-t-elle ? Sinon modifiez-la.
 ⇨

Justifiez cette dernière conjecture :
 ⇨

Théorème

Figure 4. Institutionalisation area to be completed at the end of a joint discussion

The evolution of these resources which is described very briefly here, must be seen as a continuous process, as a result of exchanges between the group members usually locally in each other’s presence and globally on distance using the work space created on EducMath (§4): it appears above all as a local temporary record of the activities being performed in class. Pooling of resources to the different e-CoLab groups and experimenting with pupils made them evolve (and they are still evolving) in such way (§ 2 shows the possible clues for extension of some resources).

⁵ tns is the extension for the TI *nspire* files

1.3 Combining resources: towards a resource model

Working in collaboration with other groups and the need to exchange and share work also made essential the development of ancillary components to the “pupil sheet –tns file” duo. As Guin and Trouche say (Gin, Trouche 2008):

In order to be usable by teachers, a resource cannot be reduced simply to a description of a learning situation: it must also explain the benefit of using ICT for acquiring the intended knowledge and skills, include a description of the technological environment in which it can be used and *proposals* in terms of organisation of time and space to facilitate it being used. It must then *show evidence* of effective use. [...] This is also a prerequisite for *combining* resources within a community.

Teacher sheets were therefore created based on the SFoDEM experience (Annex 3), in particular allowing the writers of the resource to describe the objectives of the sequence and to explain their educational choices. Similarly, scenario files (Annex 4) were constructed, designed to inform any teacher who wished to experiment with a resource in class for which he/she had not made the didactic choices, the didactic variables with which he could “juggle”, the expected answers from the pupils and the different stages through which the activity is conducted etc.

Pupil productions records (written or computerised) were also collected and were valuable aids for better retrospective interpretation of the progress of the activities (described in the observation report) and for the evolution of the resources themselves (§ 2).

All of these different resource “ancillary” components were equally essential in a collaborative work such as e-CoLab. These documents were essential to share resources and enrich the experience of all of the people involved (teachers in the different sites and pupils).

In the following section we shall demonstrate in particular the role of the observation reports in the evolution of the resources by examining the experiment conducted around two resources, created by the Montpellier group and adapted by the Paris group.

2. Use of resources in classes

In this section we shall examine the collaborative work conducted between the different groups and within each group, through the use of resources in classes. As highlighted previously, this collaboration was essential in developing progressions and the sessions themselves, the discussed proposals from the former and their observation reports informing reflection the latter, adapting the proposed resources for their own didactic project and context. This was also done to overcome some difficulties experienced due to the fact that the calculator used during the first year was still a prototype. As we had to make choices we decided to illustrate this collaborative work concentrating on two important dimensions which appeared to us to have been productive in this collaboration and which were described in the introduction to the previous section: firstly the articulation between mathematical progression and instrumental progression and secondly the use of new possibilities for dynamic interaction between applications offered by the calculator. Each of these dimensions is illustrated by an example. In both cases the situations proposed to the pupils originated from a proposal by the Montpellier group and the observations were made by the Paris group after the proposal was adapted.

2.1 Articulation between mathematical progression and instrumental progression: the “Descartes” resource

The **Descartes** resource designed by the Montpellier group appeared to us in principle to be useful as an introduction into the dynamic geometry of the calculator, articulated with an application of the main geometrical notions and theorems introduced in Junior High School. The use of historical sources to organise the meeting with technological modernity (fig. 5) was attractive. It also offered the advantage of linking the work which had just been performed on numbers and geometry. The Paris group which was very aware of instrumentation questions, the articulation between paper-and-pencil and calculator work and the possible sharing of responsibilities between teachers and pupils, assessed the Montpellier proposal, trying to optimise it from this perspective. We do not have space here to go into the details of this assessment and the changes it resulted in (this forms the CV of the resource– cf. Guin and Trouche, 2008) but we would like to highlight a few features of the resource which resulted from it:

- a progressive and intentionally limited acquisition of the Geometry application;
- interaction between the work on different supports with a desire to keep usable records on each support;
- attention to the problematisation of the pupil work and progressive devolution of responsibilities for proof.

We shall describe each of these points below before returning to the actual process.

Progressive and intentionally limited acquisition of the Geometry application

In this situation, several geometrical constructions are involved, enabling products and quotients of length to be produced and also the square root of a given length to be constructed. For the first construction proposed, the product, the geometrical figure is given to the pupils together with displays of the measurements required to confirm experimentally that it does provide the stated product (fig. 5). The pupils simply had to use the pointer to move the mobile points and test the validity of the construction.

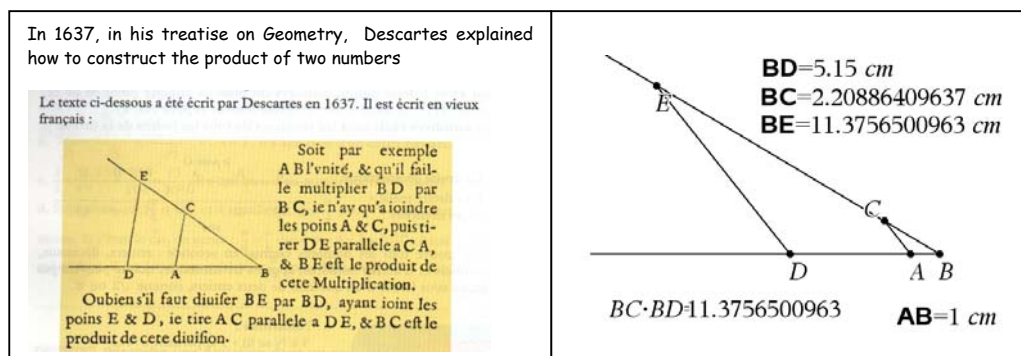


Figure 5. First part of the Descartes resource (extracted from the pupil sheet and the associated tns file)

Secondly, for the quotient, the figure provided only contained the support for the two rays [BD) and [BE). The pupils were required to complete the construction and were guided stepwise in the successive use of the “Point on”, “segment”, “Parallel”, “intersection point”, “measurement” and “calculation” tools. Thirdly, they were asked to adapt the construction to calculate the reciprocal of a length. Finally for the square root they had the Descartes figure and were required to organise the construction themselves (fig. 6). Instructions were simply given for the two new tools: “midpoint” and “circle”.

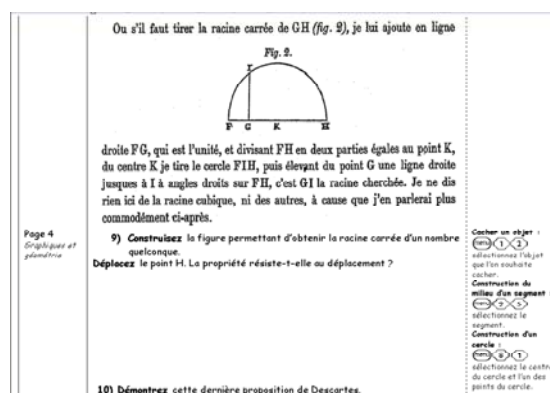


Figure 6. Last part of the Descartes resource (extracted from the pupil sheet)

Interaction between the work on different supports and the desire to keep usable records on each support

The pupils have a pupil sheet and a tns file which complies with the standards that have progressively been developed (§ 1); the task uses the two supports alternately (this is the concept of the “duo” described in § 1.2). The construction of the product of two numbers is firstly made on paper in a specific case before being tested more generally on the calculator version. This also applies for the square root. Explanations are to be written on this form which also contains a reproduction of the Descartes texts which we felt it would be useful to give the pupils “as is”. The tns file is structured into four pages, one per construction, and therefore allows a record of all of the constructions performed to be kept.

Attention to the pupil-work problematisation and the progressive devolution of responsibilities for proof.

The task does not involve solving a geometrical problem in the usual sense of the term but discovering and understanding historical construction procedures. This is of course not a matter of questioning Descartes constructions but of asking one’s self how they can be visualised dynamically on the calculator, adapting a layout to a given situation and of course understanding what makes these constructions work. The pupil sheet and sequence scenario alternating group work and collective work are designed to devolve these questions to the pupil. As this happens to be the first geometry work done in class, the responsibilities for proof are progressively devolved. For the first construction, pupils have to explain the phenomenon and the corresponding proof is produced collectively. Pupils must then adapt it independently to show why the construction of the quotient works. For the square root, initial work is planned leading to identifying the three triangles, rectangles and the three associated possibilities of using Pythagoras theorem. The pupils are responsible for the rest of the work.

The actual process

The resources were examined experimentally in Paris in the first experimental class and shortly after in the second one, and the information obtained for the first used to “adjust the sights” for the second. It is not possible here to go into the details of these processes (these appear in the observation reports annexed to the resources– cf. § 1), but we would like to highlight some major points.

Two sessions were required in the two classes. The first went as intended: the contrast between history and modernity had the desired effect and the pupils were very interested from

the start. Most had already examined alone the Geometry application. Use of the pointer and moving the points did not pose any problems. Intercept theorem⁶ was identified without difficulty and the first demonstration was produced collectively, as intended. The information given for the second construction was sufficiently clear although some pupils had not finished at the end of the session and they were asked to complete the construction at home. The second session was more laborious and the teachers had to deal with various instrumental problems: how to “seize” the variables to calculate the quotient? Why does the requested calculation not display? Why is the displayed quotient value not what is expected? Points were created inadvertently which to a greater or lesser extent were superimposed on the points of the construction and could invalidate the measurements, segments were too short and were difficult to handle on the calculator. The question of the reciprocal was handled collectively in the second class after the pupils had discovered the strategy. The numerical approximations raised interesting questions: is the construction valid or not if the values displayed differ in their last decimal place? All the pupils were not able to construct the square root just with the information given and once more, a segment of length 1 complicated the manipulation. In the second class after, the teacher noticed some lost pupils and decided to show a pupil’s calculator screen on an overhead projector to the entire class; this pupil then, ran the collective work (cf. the concept of the Sherpa pupil introduced in Guin and Trouche 2002). This restored confidence and everyone finished the construction. The last demonstration which was started collectively was given as homework.

Overall these two sessions were extremely rich in the two classes, in both instrumental and mathematical terms, around a resource which clearly could be further optimised.

2.2 Use of dynamic interaction possibilities between applications: a “trivial” situation renovated by the TI

First introduction of mutual sharing

The Paris group was again inspired by a resource (“Equal areas”) constructed by the Montpellier group: starting from a pupil sheet designed to be used in a one hour test the Paris group decided to construct a research session. The support for this was an algebraic problem with geometrical roots; it consists of finding a length such that the areas of two surfaces are equal. (fig. 7). The expression of the two areas as a function of this length are first and second degree expressions (they could both be second degree without changing the type of problem). It has a single solution with an irrational value. This therefore falls outside of the scope of the equations which the pupils being observed are able to solve independently.

The pupils’ work was guided by a sheet following the stages of geometrical exploration and first estimate of this solution, refining the exploration using the spreadsheet to end in a result for the solution to the nearest 0.01, use of the CAS to obtain an exact solution and the corresponding algebraic proof in paper/pencil, guided by the indirect data from the canonical form.

⁶ Intercept theorem called Thalès theorem in France.

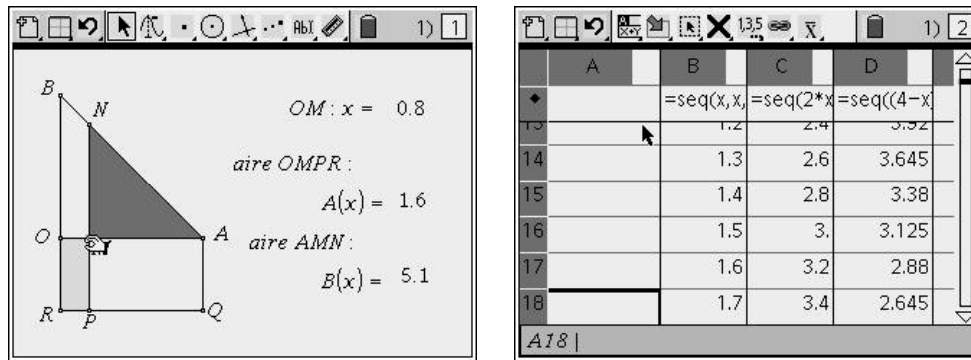


Figure 7. Graphical and numerical displays of a problem

The “rule of thirds” of the observation

We tried to go beyond the descriptive level of the initial reports and sought to identify a few questions which would give the points of interest to the observation. These questions had not all been identified a priori, although we felt they were important in light of our observation to understand the resource, identify its potentials, help the teacher to anticipate what could happen, prepare his/her activities and possibly adapt the resource to his/her specific objectives.

The questions related to the following points:

- pupil engagement in solving the problem;
- relationship with the calculator;
- complexity of the task;
- articulation between the different approaches to the problem;
- the teacher’s work.

Progress in mutual sharing

Starting from an evaluation of this session and the one hour class review conducted by the teacher we attempted to identify factors allowing mutual sharing to advance. We used the previous questions:

- Pupil engagement in solving the problem

In the observed group, but also in the other groups, pupils became engaged in the given work and maintained their engagement during the two hours of the session. The problem solving in the use made by the two teams in Montpellier and Paris was, however, intended to be as independent as possible. It would undoubtedly also be interesting for this problem to include a scenario involving more conventional class management alternating the pupil research phases with collective discussions led by the teacher, and obtaining a report of the observations from this scenario.

- Relationship with the calculator

Observation of this session showed that the calculator at this time of the year (February, i.e. mid-term in France) is a non-marginal part of the mathematical workspace of pupils and that its use is co-ordinated with paper-and-pencil work, although the balance between the two varies between pupils. The level of familiarity also varies between the pupils and it appears to be still limited at the time of the year when this session took place (we shall return to this point in § 3 in the analysis of the questionnaires and interviews). The pupils, for example, only used the symbolic calculation when it was explicitly asked as, for example for the “Solve” function. Instrumental knowledge about other parts of the symbolic calculation allowing, for example, calculations and factorisations to be confirmed or simplifications to be tested were visibly not yet available. In view of these findings we considered a “variant” of

the resource which could be worked on and shared: this or a similar problem would be given without an imposed solution trajectory and addressed to the pupils at the end of 10th grade for example simply asking them to observe the variations of the two areas and to find one or more values of the variable for which they are equal. We might then be able to study the mathematical and instrumental knowledge available and their interaction.

- Complexity of the task

Observing the session and a group of four pupils in particular, very clearly showed the complexity of the task asked to the pupils, this complexity had not been taken correctly into account by the brief *a priori* mathematical and instrumental analysis which had been performed.

Mathematically, we see very clearly, for example, all of the elements used to calculate the two areas, a calculation which appears very simple: the simplification of expressions, problems raised by moving from estimates to enclosing ranges, problems raised using the results of the symbolic calculation and the instruction to move to the canonical form of the expression in order to solve the equation. The emphasis in the class review was placed on the difference between the two tasks “obtain an approximate solution for an equation $A(x) = B(x)$ ” and “obtain a range within which this solution lies”.

Instrumentally, as the figure had been given, geometrical investigation did not raise any problems but the use of the spreadsheet in the second phase required actual knowledge, whether the pupils recopied the formula or whether they adapted the column formulae which they had previously used in class in this situation. The possibilities offered by the symbolic calculation which could assist them at different times in the session were not used and the articulation between exact and approximate calculation, needed to exclude a solution lying outside of the interval, had to be introduced by the teacher. A comparison was made in the review between the spreadsheet solution and the graphical solutions.

By analysing the complexity, we see a richness in mathematical work potentially involved in solving this problem with the calculator, which was hidden by the simplicity of the problem and its usual resolution in the paper-pencil environment. Many areas of work emerged between which the teacher clearly has to select from depending on his plan, the time dedicated to the session, his/her position in the progression etc. Explaining this also appears to allow progress in mutual sharing.

- Articulating between the different approaches to the problem

The proposed form was intended to interlink the approaches to the problem and in its designers' view these approaches complemented and mutually enriched each other. These inter-relationships however remained implicit and one might wonder whether the pupils reconstructed them and arrived at an overall understanding of the process. As might have been expected the pupils did not spontaneously go down this route and tended to see each sub-task as an isolated problem. Here again, it was up to the teacher to restore the missing links, particularly for incoherence. In this respect, other questions emerge which have to be mutually shared: “What relationships will the pupils establish between these different approaches?” and “What mediation is required from the teacher to enable these relationships?”

- The teacher's work

Even though the proposed task has real potential to encourage independent pupil activity, the teacher's work is not limited to devolving and institutionalising activities (Brousseau 1997). The successful progression of this session requires many mediation activities on the part of the teacher. The collaborative work of the three groups provided considerable assistance to the work before preparing the session, although in the session itself the mediation from the teacher and its possible effects were not studied in great detail. They were however examined in detail retrospectively and the analysis of the teacher's work, both during the

session and in the review session helped to enrich the resource and make it again “mutually sharable”.

2.3 Initial review of the observations

We have described only two observations here. This only gives a very incomplete view of the input from the different observations made by members of the e-CoLab group from the start of the experiment onwards. Their primary objective was to record their findings in order to improve understanding of the potentials offered by the TI-*n*spire for teaching and learning mathematics. It also formed part of the joint project contributing to collective work on resources: the groups designed and used their own resources, but also those produced by other groups, adapting them as necessary. They questioned their possible evolution. This dimension is the specific topic of this article and we would like to highlight the extent to which the observations contributed to this collective work. As we have tried to show, the preparation of the observations led to fundamental work on the different components of a resource, whether it was used or was adapted *per se*. In addition, each of the observations revealed new uses, new questions and new avenues to explore. It improved understanding of the pupils’ and teachers’ activities, and the real sharing of responsibilities within the class following the production phases, which a teacher cannot do alone in his/her class.

Contributing to the collective work on resources (§ 1) involved finding means of expressing the contributions to the resources, themselves based on the observation reports. We found initial reports which were mostly descriptive and chronological, unsatisfactory. We gradually designed a structured approach as follows:

1. Context
2. Questions specific to the observation
3. *A priori* analysis of the proposed tasks
4. Overview of the process execution
5. Structured analysis around the questions identified in 2
6. Review and suggestions on the concerned resource, its uses, improvements and possible enrichments.

This structure appeared to us to identify the force lines in an observation (as described above) and by doing this, to help a teacher who wished to use the resource to make better use of it, to identify its potentials, to anticipate mathematical or instrumental difficulties which may be encountered in its use, and to think about adapting the resource to his/her specific context. The obtained feedbacks appear to show that this was the case at least within the e-CoLab team.

Whilst the observations such as those described above suggesting avenues to examine the pupils’ mathematical activity were found to be essential tools for joint advancement of the e-CoLab work, they were not able to take account of other equally essential factors in a project in which the question of introducing a new technological tool played a central role. Apart from feedback on the pupils’ mathematical activity, feedback on instrumented activity providing information about the relationship between pupils and the instrument was essential. The next section is dedicated to this through an examination of the pupils’ views of this new mathematics experience.

3. The pupils' views

During the experiment we were interested in hearing the pupils' views on the use of the calculator. In order to do this we designed a questionnaire⁷ (available on [EducMath - http://educmath.inrp.fr](http://educmath.inrp.fr)). The pupils taking part in the experiment answered the questionnaire in the month of December 2006 (the pupils had had the calculator since the Autumn holidays) and in the month of June 2007. This allowed us to record changes in the pupils' opinions of the device and also on the use in and outside of the mathematics class.

We also interviewed a few pupils chosen by the teachers at the end of the year using explanatory interview techniques (Vermersch 1990); the choice was made in order to interview pupils who had succeeded in mathematics or not, and pupils who had succeeded with the technological instrument, or not. These pupils had already used the calculator throughout the entire educational year and the interview concerned aspects of the uses of the calculator (personal impression, use, instrumentalisation and organisation).

We will present the most striking opinions from an analysis of these questionnaires and interviews in this section.

Access to the tools

Of the questionnaires completed, more than 96% of pupils had a computer at home and 75% used it almost daily; several pupils stated that they used it for their schoolwork although of course also in their spare time and for communication. In the previous years, few of the pupils had used dynamic geometry software in class (59.9%) and even fewer had used computer algebra systems (80.7%)⁸.

In the interviews the pupils said that they had access to a computer at home and they gave their comparative views between the use of computer and of calculator, describing the advantages of the two tools. One of the advantages of the calculator highlighted by the pupils was its extreme portability and dynamic applications: on the other hand the computer was preferred for ease of mouse handling and for internet access, as illustrated by this commentary extract:

“I like the TI more than the computer. You can draw graphs and use “Cabri”. The calculator is small, I like handling it and I can take it anywhere. The only bad thing compared to the computer is that it can't send e-mails” (interview, January 2007)

Instrumentalisation and instrumentation⁹

It can be seen in the answers to the questions from the section “relationship between calculators and paper-pencil” in the questionnaire that the calculator is used in parallel with the paper-and-pencil environment. The influence of teachers and instructions given in the class or assessment activities is apparent in the questionnaires and interviews. When talking about their first approach to the calculator, the interviewed pupils clearly reported that these initial contacts had been difficult because of the novelty and complexity of the calculator commands and that they overcame these difficulties during the year. According to the pupils, the ease and mastering which they achieved was explained by the fact that they used the

⁷ It should be noted that as with all of the resources used in the experiment this questionnaire was constructed through an interaction between the different groups using the EducMath workspace.

⁸ The term “formal calculation” is not always fully understood by pupils as is shown in apparently contradictory answers: pupils said firstly that they had never used the formal calculation although did say that they had mastered factorisation or solving equations on the calculator.

⁹ The concepts of instrumentalisation and instrumentation are described in the article by Hivon et al (in this journal)

calculator very regularly in the mathematics class, the assistance given by the teachers and the advice available on the pupil form (in the “Use and Instructions” column – § 1):

“It’s easy, our mathematics teacher showed us the main points, the main places to change a file, create a file and everything like that, and then it was very easy: insert a page, go to the main menu, go to a file, move and everything else ...” (interview, May 2007)

“To begin with it was pretty complicated, I didn’t manage to use it and all that, and with time ... now it’s very easy [...]. My teacher and friends helped us and the sheet he gave us to help us Record, use the spreadsheet and things like that” (interview, May 2007)

The ease in creating document directories allowed the pupils to organise their own directories in a personalised way.

“It’s easy, I do it for everything, insert a file for all subjects, English, French, even those which aren’t of any use, for fun” (interview, May 2007)

“Yes, I named the file to be able to find it because I use it in physics for calculations, and also in biology and for the subjects, graphs, spreadsheet that’s how I organise it” (interview, May 2007)

The link between learning mathematics and using the calculator is a difficult question to formulate and the analyse of pupil’s answers is a delicate issue as the answers are sometimes contradictory in the same interview: a few points can nevertheless be highlighted from the pupils’ answers which provide us with some information, particularly about motivational issues:

“No it is ... it can be interesting because you can create graphs quickly, for example, by hand if you need to make a graph that takes a little time but with a calculator I find that it’s a bit easier because I can change the sizes and I can see by changing the equations I can do anything I can even superimpose the so I can create other pages, I can do loads of things with it, it’s neat that ... no I don’t find it very different except that it helps me do a problem slightly faster, for example, if I need to draw curves it’s a little faster because the curve draws itself ... compared to that otherwise I don’t see any difference” (interview, May 2007).

“Actually I’m not very good at maths but after, with the calculator that did motivate me a bit more even so ... Because I wasn’t very motivated it helped me learn maths. It’s a way of learning maths a little bit ... it helped me understand things which I hadn’t understood at the beginning of the year when I hadn’t yet used it” (interview, May 2007).

“This is the first time I have been able to do maths and understand almost everything” (questionnaire, June 2007)

Use of the calculator

The analysis of questionnaires and pupil interviews showed that they often did not use the calculator at home or in other disciplines:

“Just for simple calculation ... simple things in other subjects I don’t just calculation” (interview, May 2007)

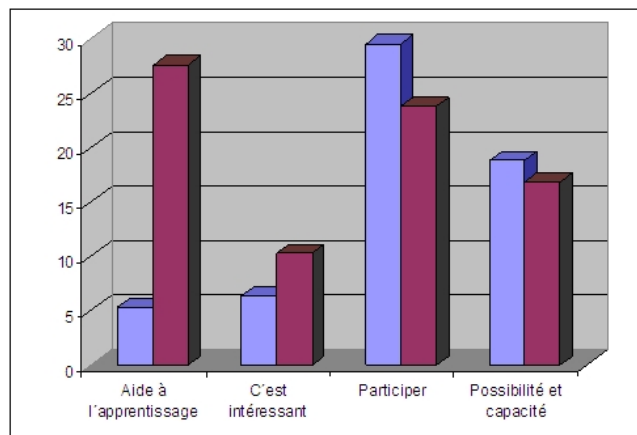
“It’s for division and things like that which are hard to do by hand” (interview, May 2007)

The calculator was often used in mathematics even though its use fell off, explained by the fact that it had to be used at almost every lesson during the learning phase whereas, as the year progressed, the calculator was seen more as a tool available in the class:

“It helps to solve a problem and after that I had it worked out. What was useful from the start was that compared to the first drawing I can change the points, I can move the points so that the measures changed and I can see [...] and after that it’s all linked to the point cloud or to curves, we can see the curves which change and all that, that’s not bad” (interview, May 2007)

“For example the geometrical figure, I can see changes which I can’t see on the sheet on the sheet you have to do a lot of drawings to see the changes on the calculator you can change different things which lets you see how the figure changes each time” (interview, May 2007)

The level of satisfaction is demonstrated by the change in favourable or very favourable opinions from 72.5% in December to 82.2% in June. The reasons for satisfaction were several factors including taking part in an experiment, possibilities and abilities of the calculator, more vague description described by a phrase such as “it’s interesting” and aid to learning mathematics (graph. 1).



Learning Aid It's interesting Taking part Possibility and ability

Graph 1. Change in pupils opinions from December 2006 to June 2007

The pupils’ comments support this graph:

“Despite my difficulties in maths I found this experiment very valuable, it helped me to gain mathematics knowledge, better than using paper and pen, that motivates me !!!” (questionnaire, June 2007)

“Because it helped me to understand some more subtle things in maths. It also helped me discover a new way of discovering mathematics” (questionnaire, June 2007)

Difficulties

Negative points raised by the pupils related more to the interface which only gave a limited amount of information because of the use of a prototype, which as we have already written required improvements that have been taken into account in the current version of the calculator. The instrumentalisation difficulties were often minimalised by the pupils in the interviews:

“...it’s accessible, it’s good... I can understand it easily... Me, I think I can understand it easily” (interview May 2007)

They are however clearly seen in their responses to the questionnaires: in December 21.6% of pupils said that the difficulties of use of the calculator was a hindrance to its integration into the mathematics lessons. This percentage fell significantly during the year only 12.4% of pupils stressed this difficulty in June in response to the same question.

Changes

There was a considerable advance in the opinion that the calculator was an aid to learning mathematics. During the first six months of the experiment, the pupils' favourable opinions of the calculator related to the ergonomic and numerical features:

"I like the fact that it has letters. It has a big screen, the calculations and graphics are clear. You can do a lot of things with it ... it has a lot of functions and the computer type files are really good. It has every possible function" (questionnaire December)

Whereas in the second six months the emphasis was far more on the possibilities for symbolic calculation and new potentials:

"It is able to create geometrical figures. Doing symbolic calculations. Drawing curves" (questionnaire June)

The discovery of new potential uses is intimately linked to the calculator's integration into the mathematics class. We can assume that introducing the calculator promotes real awareness of its uses in learning mathematics, at the same time as manipulating the main functionalities becomes increasingly familiar. It takes us back to the creation of resources which need to take into account of this delicate instrumentalisation process.

4. Difficulties, tools and benefits of collaborative work

As already stated the three groups in the e-CoLab project worked jointly using a workspace on the EducMath site. The operation, organisation and evolution of the workspace took place as it was being used until the emergence of a model (fig. 9), which facilitates communication by offering clear organisation and an ergonomic interface. This space therefore facilitated the exchange and sharing of documents (fig. 8).

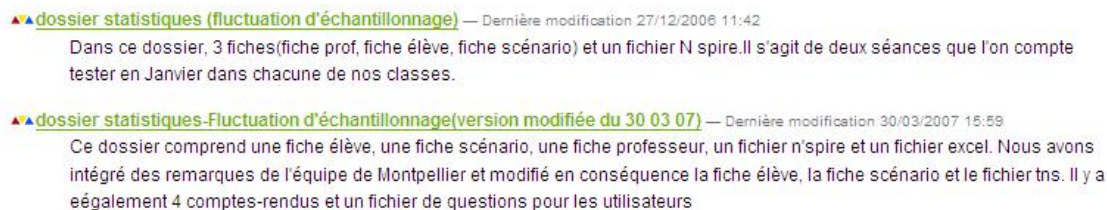


Figure 8. Extract of a few sections from the workspace

The progressive construction of the space from the omissions, redundancies and imperfections of an initial structure took into account the difficulties of use and needs expressed as the resources were collectively constructed, and also the tools used to observe the experiments. Based on the experience from this work, the difficulties in using the space can be classified into two quite distinct categories:

- firstly, the difficulties sharing work which is known to be unfinished: whilst it is easy to share when people are face to face, sharing is far more difficult when this involves "circulating part of the work, even in a private space. Joint working practices and the often unspoken comments between colleagues used when teachers are working

together are also obstacles to mutual understanding and require clarification. Although this may seem complex and difficult, it advances the description and sharing of a common resource as we saw in the initial section of this article;

- secondly, the general ergonomics of a workspace even if considered *a priori* in the context of a precise working objective cannot be achieved without testing the reality of the exchanges. Ease of use, and an interface which is both complete and rapidly usable are not self-defining and experience shows the needs for joint construction to make this type of workspace operational.

Each of the three groups could use their own “in process resources” page to offer the seed for a resource which was considered, criticised and enriched by colleagues until a sufficiently stable status was achieved to allow it to be proposed to the entire group. As shown for the “Descartes” resource, in-class use enables it to be refined and enriched from the pupils’ reactions, effects of the didactic variables and the instrumental distance (§ 2.1). Clearly in our experiment, distant work was an essential obligatory pre-requisite to construct the resources and the collaborative work, firstly allowed to share a detailed *a priori* analysis of the situations, and in parallel to establish a common resource structure.(§ 1).

Two significant examples highlighting the benefits of this work can be demonstrated, firstly to construct an observation tool linked to the research questions and secondly to create, develop and improve a resource:

- the questionnaire, the results of which are discussed in the previous section, gave rise to a collective construction from a seed proposed by one group, which was then reviewed, increased, criticised and finished by all of the people involved. In this example, the collaborative work enabled rapid, bi-directional discussion to construct and a cross- analysis of questions depending on the intended populations. In addition, the collective constructions and the versions stored in the space facilitated the establishment of a blank slate of an interview based on the questionnaire and the successive comments which led to its construction.
- one illustration in the construction of resource which can be given is the example of “You(r) bet!”, already described in the body of this article. Originally proposed by the Paris group, the resource was refined firstly from comments from everyone but also from the reports of the observation in the initial classes. The presence of this resource history also facilitated it being updated, to “transfer” the activity from the prototype version of TI-*nspire* to the current commercial-available version. One can clearly imagine this transposition to other software: all of the successive records of the work performed and the reasons for the choices made are a significant aid to transposition.

The current version of the workspace (fig. 9), which is still evolving, takes into account these comments and is constructed along a design shown below which is not only a workspace map but which allows the person to determine at any time where he/she is within the space.

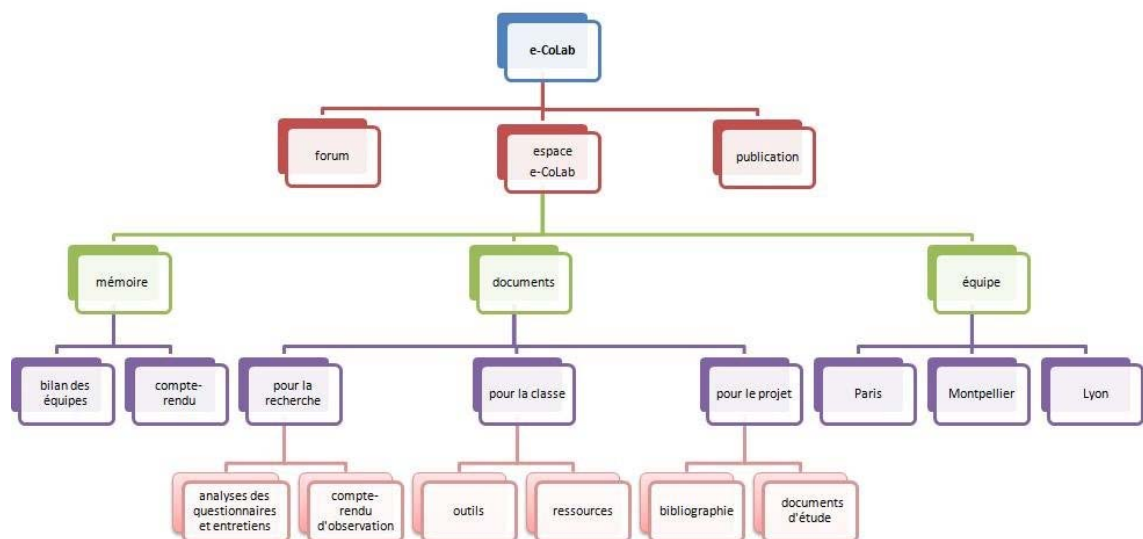


Figure 9. Structural tree of the common workspace on EducMath

This article and the intermediate study report by the e-CoLab group (Aldon et al 2007) are examples of achievements made possible by the use of this type of space.

5. Conclusion

The question of the “co-operative ” in the field of education is not new. From the point of view of the person learning, the translation of the work by Vygotski undoubtedly played a significant role in the expansion of research in which the *individual* feature of the pupil stressed in Piaget’s genetic epistemology gave way to seeing the pupil as an epistemic *fruit of his/her social interactions*. Whilst elements of the theoretical work of both Piaget and Vygotski considered to be directly relevant (Brun 1994; Rogalski 2006) can be found in the French mathematic education writings, the role of the social environment in the learning process and the suggestion of an epistemic community appears to be widely accepted within the field and their heritage seen in many didactic theories (Brousseau 1986).

With the spiralling development of ICT and particularly the emergence of new communication means, the utility of collective work in the educational world has increased and much research has shown mathematics teaching to have emerged in this light. The Repères journal has been particularly aware of this question, which is still current (Kuntz 2007).

Whilst this article has also examined the articulation between collective work and new technologies, this is seen at different levels, with particular emphasis on community work from *Humans-with-Media* (Borba et Villarreal 2005) in which, extending the concept of the epistemic community, learning in a broad sense of the word occurs *through and throughout the interaction process*.

This article primarily describes work conducted by three groups (Lyon, Montpellier and Paris) around a common project, the introduction of the TI-nspire calculator into 10th grade classes.

We have seen that from its specific features (described in the introduction) which distinguish it from other calculators and as had been envisaged *a priori*, the introduction of this new tool was not without difficulty and required considerable initial work on the part of the teachers,

both to allow rapid familiarisation on their part and those of the pupils but in particular to optimise the use of the potentials offered by this new tool in mathematics activities. Judging from the impressions of the pupils who took part in this experiment we can state that the three challenges described in section 1 were achieved with some degree of success.

Firstly, instrumentally, the difficulties encountered by the pupils in their initial handling of the new tool appear to have been rapidly overcome (§ 3). The teachers, aware of the complexity of the tool and in light of their own familiarity with the device did not under-estimate the instrumentation difficulties and dedicated an important part of their time helping and explaining the instrument in the context of resources which were created (§ 2).

In terms of the subsequent mathematics activity, apart from a permanent desire to offer resources able to bring out the richness of a mathematical work which could be marked by the banal nature of the problem and usual practice of resolving it in the paper-and-pencil environment, use of the articulation between the different boundaries, something which the teachers were particularly aware of, appears to have borne fruit: a large number of pupils referred to this to illustrate their feelings about the calculator as an aid to mathematics learning.

Returning to the second part of the articulation between community and new technologies, the *Humans in Humans-with-Media*, the work in its current form and the summary we have provided would never have come about without an underlying collaborative spirit.

The e-CoLab project is not just a question of examining a common question on three different sites punctuated by simple exchanges and sharing of experience. It involves living through a project collectively or rather making the project a collective one. In other words it involves jointly constructing this rich experience; accepting the numerous challenges which it represented and collectively solving the challenges – and, this is the point we have sought to emphasise in this article – involved jointly constructing the necessary tools.

We have seen that collaborative work is a *sine qua non* condition for producing the type of resources we have described. These resources are profoundly evolutionary in nature and are the fruit of exchanges between groups, suggestions proposed by their different members, reports of experiments which took place on different sites (with their distinct features) and also the progressive instrumentation of the different people involved.

It was because the Lyon, Montpellier and Paris groups all had the same project (and through this very fact that the mutual sharing of this experiment formed its core) that a resource model was gradually introduced, the components of which were also intended to evolve. We have shown particularly here the extent to which the collective work influences the evolution of the resources (§ 1) and also led to the development of observation reports (§ 2) which themselves informed the evolution of the resources, etc. etc. ...

As the tool enables distant mutual sharing (§ 4) it was also constructed collectively and based on the different results which emerged progressively. The workspace was both a useful tool to work in e-CoLab and, as this article suggests it, was one of the subjects of the study illustrating the somewhat cyclical nature of the concept of *Humans-with-Media* which we have tried here to illustrate.

We have tried in this article both to point out the difficulties which collective work may involve as experienced in e-CoLab and to describe all of its rich content and potentials. The examples of the different parts of the project which we have described here suggest that this community represents a true epistemic co-operative, it is indeed the network of contributions from which the production of knowledge originates in the broadest sense of the term.

In *Blizzard sur Québec* (Parizeau, 1987) Alice Parizeau states: “When we study history we realise that it is individuals who advance communities”. In light of our experience in e-

CoLab we would tend to say that “it is communities which advance individuals”. We would therefore like to add a fifth dimension to the four dimensions to which this special edition is dedicated: learning, self-training, experimenting, creating resources TOGETHER: *evolving TOGETHER*.

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Annexes

References:

- “The helicopter”
- “The shortest path”
- “Sangaku”
- “You(r) bet!”
- “The sign”

These resources are available on the EducMath website






Annex 1

An example of learning the machine syntax given in the pupil sheet.

<p>Page 3 <i>Editeur/Calculs</i></p>	<p>Pour la suite, il va être nécessaire d'apprendre à la TI-<i>nspire</i> la formule permettant le calcul de l'aire $A(x)$, pour x appartenant « exactement » à l'intervalle $[0,8]$.</p> <p>Définissez la fonction <i>aire</i>.</p> <p>La syntaxe de cette définition est la suivante : « quand x est compris entre 0 et 8, l'aire est $x^2 - 4x + 32$, sinon elle n'est pas définie »</p> <p>Passer à la page 5 du classeur.</p>	<p>Define $a(x)=\text{when}(x \geq 0 \text{ and } x \leq 8, x^2 - 4x + 32, \text{undef})$</p> <p>Attention : aucun espace n'apparaît dans l'instruction précédente, à l'exception de celui situé après la commande Define et des deux qui entourent le mot « and ».</p> <p>Le message « Terminé » s'affiche : cela signifie que la tâche demandée a été réalisée.</p>
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Annex 2

An example of technical aid given in the pupil sheet.

<p>Page 5 <i>Tableur / Graphiques et géométrie</i></p>	<p>La construction du nuage de points faite sur le graphique ci-dessus peut être réalisée par la TI-<i>nspire</i>.</p> <p>Pour obtenir dans cette page l'affichage visuel des colonnes contenant la distance AM et l'aire, sélectionnez⁽¹⁾ l'application <i>Tableur</i> dans la partie gauche de la page 5 (si elle ne l'est pas), puis sélectionnez la cellule D1.</p> <p>a) Sélectionnez l'application graphique de la fenêtre droite de la page 5 ; affichez la ligne de saisie puis la fenêtre de liaison des coordonnées des points.</p> <p>(1) Dans un affichage partagé de l'écran, on reconnaît qu'une application est sélectionnée à la présence d'une bordure supérieure noire et épaisse.</p>	<p>Pour cela amenez le curseur dans cette partie de l'écran et pressez la touche  :</p> <p>Tapez    :Afficher ligne de saisie</p> <p></p>
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Annex 3

An example of a teacher sheet.

This session is part of the progression for a 10th grade class after dealing with descriptive statistics. The pupils must know how to collect data.

I Objectives of the session

To observe variability of results when reproducing a random experiment and expressing these in terms of sample variation moving to frequencies.

To observe that the amplitude of variations falls when the sample size increases. To introduce the concept of probability law as giving sense to this situation even if the concept of probability is not formally in the curriculum.

To introduce simulation techniques on the calculator and the associated programming using simple language.

.....

II Our choices

Historically, human beings have become familiar with the concept of arbitrary events through chance games and we have therefore chosen to conduct this session on the subject of games. The idea of the bet links both to this subject and to our aim of introducing the concept of probability. The second game is also known as it led to an article by Alembert in the Encyclopédie Méthodique (Noughts and Crosses article) where he describes different possible mathematisations.

The words “probability” or “chance” are part of the second grade pupils’ vocabulary. We are therefore covering an “intuitive” area of knowledge of the concept of probability. The many research studies which have been conducted on this intuitive probability knowledge show that many erroneous beliefs exist although that it may be assumed that second grade pupils in simple situations, such as throwing assumedly unloaded dice, do not doubt the equal probability of the different faces showing. In order to give meaning to the process with the pupils we have avoided a “tossing a coin or rolling a dice” type of experiment and deliberately chosen an experiment in which the model goes outside of their intuition but uses the same initial intuition in order that the simulation which forms the basis of the experiment is credible. We have therefore chosen the difference in values shown when two dice are thrown.

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Annex 4

A scenario sheet

Example of a scenario containing session 1 from the resource “You(r) bet!”

SESSION 1		
Done by the teacher	Done by the pupils	Time
<p>Devolving the problem</p> <p>Two entirely equal cubic dice with six surfaces numbered 1 to 6 are thrown. The number obtained on the upper surface of each die is then recorded and the difference between this number is calculated (taking the smallest away from the largest to obtain a positive number)</p> <p>Ask the question: “What difference would you bet on?”</p> <p>Record the bets on the board without commenting on them.</p>	<p>Before “actually” playing, bet on a difference.</p> <p>Record the class bets on the pupil form</p>	<p>5 min</p> <p>5 min</p>
<p>Conduct of the session</p> <ul style="list-style-type: none"> • I game in pairs if the session is performed with the whole class or individually if modular. <p>In the collective review discuss the only possible values for the difference and reiterate work done in descriptive statistics to organise the results as a distribution.</p> <p>Record the results of the different pairs on the table in this form.</p> <p><u>Ask the questions:</u> “do you want to change your bet?” “what you do to be really sure?” (we hope to see the idea of increasing sample size emerge)</p>	<p>Perform the experiment (play) ten times in pairs.</p> <p>Copy the distributions onto the pupil form.</p> <p>Pupil answer on the form</p> <p>The pupils choose the sample sizes, carry out simulations with the sample function and record the results on the pupil form.</p>	<p>5 min</p> <p>10 min</p> <p>10 min</p> <p>5 min</p>

TO INSERT INTO PAGE 4 (From diagram)

A TP REFINED FUNCTIONS

Objective:

The activity involved 4 successive problems

Consider displaying the whole activity

NEXT

B Question

What does the equation $Y = a.x. + b$ remind you of ?

CONTINUED

Answer

C Move the points to change a and b

Work Observe the impact of a and b on the appearance of the line.

D Problem 2

The purpose of problem 2 is a more detailed study of the co-efficient b from equation $y = a.x. + b.$

E CONTINUED

Question

What would you say about an equation line $y = a.x. + b$ in which the coefficient b is 0 (where necessary use the next page).

Answer

TO INSERT INTO PAGE 5 (From diagram)

Ti-nspire Sequence 3

Dossier: Functions

File: Teaching

The light sign

What is the function of the sign?

**First page
index**

Instructions

Page 1

Graphics and Geometry

In order to reach more clients the “Games zone” video games shop ordered a new light sign. This involves a moving geometrical shape consisting of a square and a triangle with a common apex.

Open the “Sign” file from the “Functions” directory where the shape of the sign is shown. Look at it.

This sequence ask you to study the area of the figure and its variations with movement.

Go to page 2 of the file organiser.

Page 2

Graphics and Geometry

Figure data

ABCD is a square of side 8 cm, m is a segment point [AR]. The square AMNP and the triangle DNC, the interiors of which have been shaded, make up the sign.

Move point M on [AB] and note the changes in the area of the shaded surface which occur as a result. Describe below the changes seen.

(Reduced scale figure)

Note x the distance AM and $A(X)$ the area of this surface

Describe below the **interval** in which x varies.

TM x varies in the interval

TO INSERT INTO PAGE 6 (From diagram)

Choose another position for point B (on AX)

Does this confirm your thoughts? If not change it.

TM

Explain this last thought:

TM

Theorem

TO INSERT INTO PAGE 9 (From diagram)

To draw the square root of GH (fig. 2) I add to it a straight line FG which is unity and dividing FH into two equal parts at point K . From the centre K I draw a circle FIH and then raising a point G at a straight line up to L at right angles to FH is GI the desired root. I do not say anything about the cubic or other roots as it will be easier for me to talk about this later.

9) Construct the figure allowing the square root of a number to be obtained.

Move point H . Does the point resist being moved.

10) Show this last Decartes proposal

Mark an object – Select the object to be marked.

Contracting the middle of a segment – select the segment

Contracting a circle – select the centre of a circle and a point on the circle.

